7. Exercise Sheet for Theoretical Physics I (Mechanics)

Solutions to be handed in in the tutorials of Week 10 (12th December 2007)

Exercise 27: (5 points)

Consider a thin spherical shell with radius R and total mass M. Show that the force due to gravity inside the shell vanishes

a) by direct integration of the infinitesimal contributions to the potential

b) by considering the symmetries of the problem and using Gauss' law

c) by analysing the cancellation of forces induced by (carefully chosen) infinitesimal surface elements of the shell (similar triangles!).

Draw the potential $V(r)$, $0 < r < \infty$. (If you want to manage without the explicit calculation of a), you must make it clear whether or not the potential $V(r)$ is continuous at $r = R$.)

Exercise 28: (5 points)

Little Konrad would very much like to visit his friend Cornelia in New Zealand. For this purpose he digs a tunnel through the geometrical centre of the Earth. After his visit he returns by dropping himself down the tunnel. During the journey he flies through the tunnel (as is typical for him) without air resistance. Determine the time taken by Konrad on his way back. (Assume that the Earth is a sphere with constant density of approximately 5.5 g/cm^3. The rotation of the Earth may be neglected.)

Exercise 29: (5 points)

Show that a curve defined in polar coordinates by $r = p/(1 + e\cos\phi)$ $(p, e > 0)$ can be described by $(x/a)^2 \pm (y/b)^2 = 1$ in suitably chosen Cartesian coordinates. (The different signs of the second term correspond to the two cases $e < 1$ and $e > 1$.) State a and b as functions of p and e in each case. Verify also that, in the marginal case $e = 1$, one obtains the familiar definition of a parabola in Cartesian coordinates.