8. Exercise Sheet for Theoretical Physics I (Mechanics)

Solutions to be handed in in the tutorials of Week 11 (19th December 2007)

Exercise 30: (5 points)

In a supplement to the pure Kepler problem (Chapter 7.5) the gravitational potential for planetary motion is modified by general relativistic corrections as follows:

$$V(r) = \frac{L^2}{2mr^2} + \frac{\eta m}{r^2} - \frac{\alpha}{r},$$

where η is a constant. Show that the additional term leads to a coefficient $C \neq 1$ of $u = \frac{1}{r}$ in the differential equation

$$u'' = -Cu + m\alpha/L^2.$$

The result of this correction is a precession of the perihelion of the planetary orbit. Calculate the displacement of the perihelion after one rotational period.

Exercise 31: (5 points)

Consider the elastic scattering of a sphere of radius r on a fixed, immobile sphere of radius R. Calculate the scattering angle θ as a function of the impact parameter, and from the result find the differential cross-section $d\sigma(\theta)/d\Omega$ and the total cross-section σ. (Here $d\Omega$ is an infinitesimal element of solid angle.)

Exercise 32: (5 points)

Little Konrad und little Cornelia roll a homogeneous solid cylinder and a homogeneous ball, both with radius R und mass M, down an inclined plane of length l und gradient ϕ. Cornelia contends that the sphere will arrive at the bottom more quickly. Konrad claims that the cylinder is quicker. Calculate the respective moments of inertia about the symmetry axes of the solid bodies. (Decompose the cylinder into infinitesimal cylindrical shells of thickness dr, and the sphere into tubular sections with wall thickness dr and variable length $L(r)$). Set up the Lagrangian and equations of motion, and find the time of transit for each of the bodies. Who was right? Compare the result with the transit time of a frictionlessly sliding point mass.