7 Wick-Theorem & Feynman Rules

7.1 Time ordering vs. normal ordering

- We have learned to express scatt. amplitudes ${}_{out}\langle p_1 \cdots | k_1 \cdots \rangle_{in}$ through Heisenberg-field correl. fcts. $\langle 0 | T\varphi(x_1) \cdots \varphi(x_n) | 0 \rangle$ and the latter through Interaction-pict.-field (i.e. free-field) expressions like

$${}_I\langle 0 | T\varphi_I(x_1) \cdots \varphi_I(x_n)\, e^{i\int d^4x\, \mathcal{L}_{int}(\varphi_I(x))} | 0 \rangle_I \,.$$

[Here we used $-i\int d\tau\, H_{int} = i\int d^4x\, \mathcal{L}_{int}$.]

- Since this whole section deals with free fields, we will drop the index "I": $\varphi_I \to \varphi$; $|0\rangle_I \to |0\rangle$.

- Since $\mathcal{L}_{int}$ is a polynomial in φ we can expand the exponential and reduce the last expression to a sum of free-field correl. fcts.:

$$\langle 0 | T\varphi(x_1) \cdots \varphi(x_m) | 0 \rangle \quad ; \quad \varphi \text{ free.}$$

- We have already given a name to the two-field expression:

$$\langle 0 | T\varphi(x)\varphi(y) | 0 \rangle = D_F(x-y) = \text{"Feynman propagator"},$$

and we could easily evaluate it since we know $\langle 0|\varphi(x)\varphi(y)|0\rangle$.

- But, to be able to generalize to the multi-field case, it is useful to introduce a more general concept in this simple case:

$$\varphi(x) = \int d\tilde{k} \left(a_{\vec{k}}\, e^{-ikx} + a_{\vec{k}}^{+}\, e^{ikx} \right) \equiv \underbrace{\varphi^a(x)}_{\text{annihilator part}} + \underbrace{\varphi^c(x)}_{\text{creator part}}$$

- We define the normal-ordered form of any operator expression $a_{\bar{k}_1} a^+_{\bar{k}_2} a^+_{\bar{k}_3} a_{\bar{k}_4} \cdots a^+_{\bar{k}_n}$

by

$$:(a_{\bar{k}_1} a^+_{\bar{k}_2} a^+_{\bar{k}_3} a_{\bar{k}_4} \cdots a^+_{\bar{k}_n}): \;\equiv\; \underbrace{a^+_{\bar{k}_2} a^+_{\bar{k}_3} a^+_{\bar{k}_n} \cdots}_{\text{all creators first}} \underbrace{a_{\bar{k}_1} a_{\bar{k}_4} \cdots}_{\text{then all annihilators}}$$

- Since φ is a lin. comb. of a, a^+, this def. extends to any product of φ's. In particular $:\varphi^a(x)\varphi^c(y): = \varphi^c(y)\varphi^a(x)$.

- <u>Note:</u> In complete generality, $\langle :\hat{O}: \rangle = 0$ for any operator $\hat{O}$.*) Our earlier definition of H_0 by dropping the cosm. constant can be expressed as

$$H_0 = :\tfrac{1}{2}\int d^3x \left(\pi^2 + (\bar\nabla\varphi)^2 + m^2\varphi^2\right):$$

- For two fields, $\varphi(x)\varphi(y)$ and $:\varphi(x)\varphi(y):$ obviously differ only by commutators, i.e. <u>numbers</u>:

$$\varphi(x)\varphi(y) = (\varphi_x^a + \varphi_x^c)(\varphi_y^a + \varphi_y^c) =$$

$$= \varphi_x^a\varphi_y^a + \varphi_y^c\varphi_x^a + \varphi_x^c\varphi_y^a + \varphi_x^c\varphi_y^c + [\varphi_x^a, \varphi_y^c]$$

$$= :\varphi(x)\varphi(y): + \text{"number"}$$

$$\Rightarrow \langle 0|\varphi(x)\varphi(y)|0\rangle = \text{"number"}$$

*) which is a polynomial or series in a, a^+ <u>without</u> constant term.

$$\Rightarrow \quad \varphi(x)\varphi(y) = :\varphi(x)\varphi(y): + \langle 0|\varphi(x)\varphi(y)|0\rangle$$

<u>in particular:</u>

$$\boxed{T\varphi(x)\varphi(y) = :\varphi(x)\varphi(y): + \langle T\varphi(x)\varphi(y)\rangle}$$

Convenient notation: $\langle T\varphi(x)\varphi(y)\rangle \equiv \overbracket{\varphi(x)\varphi(y)}$

Hence: $T\varphi(x)\varphi(y) = :\varphi(x)\varphi(y): + \underbrace{\overbracket{\varphi(x)\varphi(y)}}$

This is called a "contraction".

7.2 Wick-Theorem

Theorem: $\boxed{T\varphi(x_1)\cdots\varphi(x_n) = :\varphi(x_1)\cdots\varphi(x_n): + \text{all contractions of } :\varphi(x_1)\cdots\varphi(x_n):}$

sum over all possib. to put one or more "$\sqcap$" over the fields

Example: Let $\varphi_i \equiv \varphi(x_i)$.

$$T\varphi_1\varphi_2\varphi_3\varphi_4 = :\varphi_1\varphi_2\varphi_3\varphi_4: + \left\{ :\overbracket{\varphi_1\varphi_2}\varphi_3\varphi_4: + 5 \text{ analogous terms} \right\}$$
$$+ \left\{ :\overbracket{\varphi_1\varphi_2}\,\overbracket{\varphi_3\varphi_4}: + \underbrace{2 \text{ analogous terms}}_{(*)} \right\}$$

Can be dropped in these 3 terms since $:\#: = \#$.

$$(*) = \varphi_1\varphi_2\varphi_3\varphi_4 + \varphi_1\varphi_2\varphi_3\varphi_4 = D_F(x_1-x_3)D_F(x_2-x_4) + D_F(x_1-x_4)D_F(x_2-x_3)$$

Relevance: Applying $\langle\cdots\rangle$, only "complete contractions" survive on r.h. side. Hence we get an expression for $\langle\cdots\rangle$ in terms of D_F's.

Proof: by induction – $n=1$, trivial – $n=2$, cf. Sect. 7.1

Step from n to $n+1$: (W.L.O.G. the $(n+1)$st field can be taken to be the one with largest time argument), i.e. $x^0 > x^0_i$, $\forall i$.)

$$T\overset{\varphi(x)}{\downarrow}\!\!\!\varphi\,\varphi_1\cdots\varphi_n = \varphi\, T\varphi_1\cdots\varphi_n + \varphi\,(\text{all contractions of } :\varphi_1\cdots\varphi_n:)$$

Here we have already used the Theorem at step "n" on the r.h. side. Next, we introduce the <u>Lemma:</u>

$$\varphi : \varphi_1 \cdots \varphi_m : \; = \; : \varphi \varphi_1 \cdots \varphi_m : + : \overline{\varphi \varphi_1} \cdots \varphi_m : + \cdots + : \overline{\varphi \varphi_1 \cdots \varphi_m} : .$$

With this lemma, the claim of the Theorem at step "n+1" immediately follows from the previous expression. □

<u>Proof of lemma:</u>

$$\varphi : \varphi_1 \cdots \varphi_m : \; = \; \varphi^c : \varphi_1 \cdots \varphi_m : + \varphi^a : \varphi_1 \cdots \varphi_m :$$

$$= \varphi^c : \varphi_1 \cdots \varphi_m : + : \varphi_1 \cdots \varphi_m : \varphi^a + [\varphi^a , : \varphi_1 \cdots \varphi_m :]$$

$$= : \varphi^c \varphi_1 \cdots \varphi_m : + : \varphi_1 \cdots \varphi_m \varphi^a : \; + \qquad -\text{''}-$$

$$= : \varphi \varphi_1 \cdots \varphi_m : + [\varphi^a , : \varphi_1 \cdots \varphi_m :]$$

- Now use $[A, B_1 \cdots B_m] = [A, B_1] B_2 \cdots B_m + B_1 [A, B_2] \cdots B_m + \cdots$
$$\cdots + B_1 \cdots B_{m-1} [A, B_m]$$

- Use also $[\varphi^a , \varphi_i] = [\varphi^a , \varphi_i^c] = \langle \varphi \varphi_i \rangle = \langle T \varphi \varphi_i \rangle = \overline{\varphi \varphi_i}$

recall that $x^0 > x_i^0$

<u>Thus:</u>

$$[\varphi^a , : \varphi_1 \cdots \varphi_m :] = : \overline{\varphi \varphi_1} \varphi_2 \cdots \varphi_m : + : \overline{\varphi \varphi_1 \varphi_2} \cdots \varphi_m : + \cdots + : \overline{\varphi \varphi_1 \cdots \varphi_m} :$$

□

<u>7.3 The Feynman propagator</u>

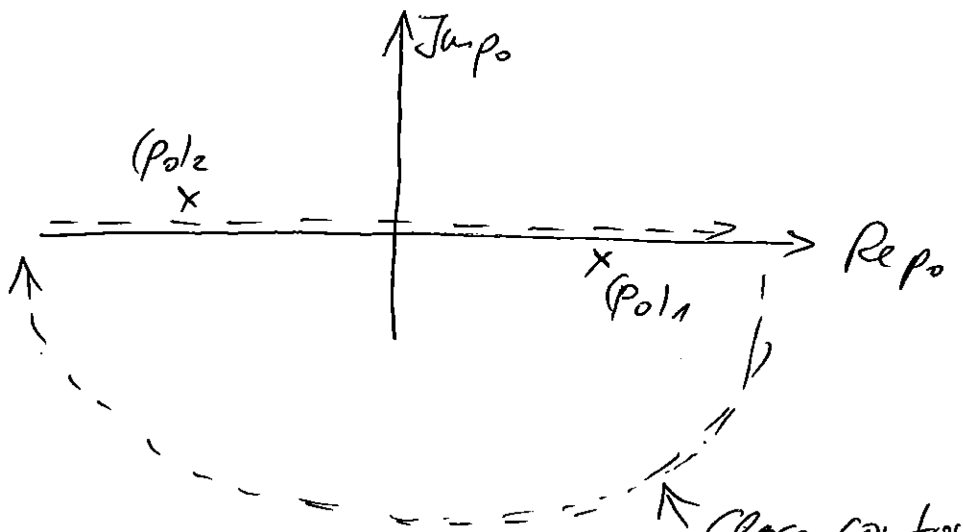
<u>Claim:</u> $$D_F(x-y) = \int \frac{d^4 p}{(2\pi)^4} \cdot \frac{i}{p^2 - m^2 + i\varepsilon} \cdot e^{-ip(x-y)} \Big|_{\varepsilon \to 0}$$

<u>Derivation:</u> Let $x^0 > y^0$; perform p_0-integration first;

Write denominator as $(p_0 - (p_0)_1) \cdot (p_0 - (p_0)_2)$

with $(p_0)_{1,2} = \pm\sqrt{\vec{p}^2+m^2-i\varepsilon} = \pm\left(\sqrt{\vec{p}^2+m^2}-i\varepsilon'\right)$.

View p_0-integration in complex p_0-plane:



Close contour such that integrand is suppressed

Pick up residue; compare to $\langle\varphi(x)\varphi(y)\rangle$.

Let $x^0 < y^0$; need to close contour above; compare to $\langle\varphi(y)\varphi(x)\rangle$. $\square$ (Details: $\to$ problems)

7.4 Feynman rules...

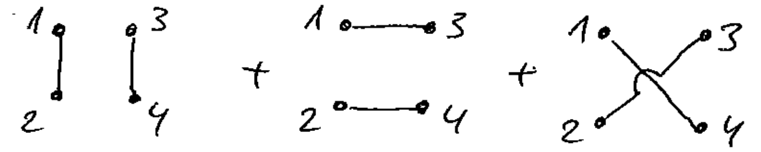
.. allow for systematically writing down mathematical expressions of terms in the pert. expansion of Correl. or Green's fcts. We will start with examples. Consider

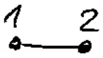
$$\langle T\varphi_1 \cdots \varphi_4\, e^{i\int d^4x\, \mathcal{L}_{int}(\varphi)}\rangle$$

and work order-by-order in λ.

Order $(\lambda)^0$: $\langle T\varphi_1 \cdots \varphi_4\rangle = \varphi_1\varphi_2\,\varphi_3\varphi_4 + 2$ more terms

Wick theorem

$=$ 

where 

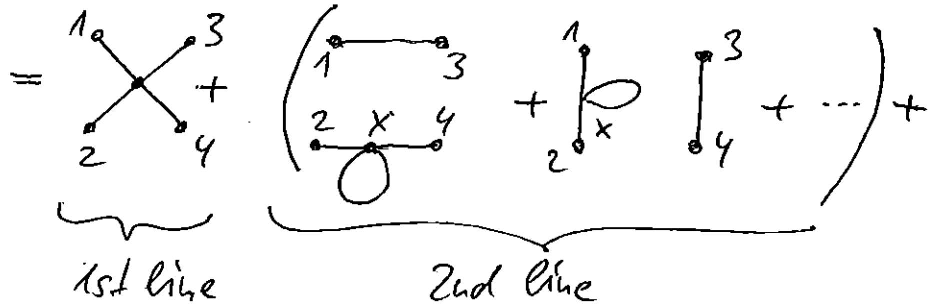
$\equiv D_F(x_1-x_2)$

Order $(\lambda)^1$:

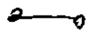
$$\langle T\varphi_1\cdots\varphi_4\left(-\frac{i\lambda}{4!}\int d^4x\,\varphi(x)^4\right)\rangle = \left(-\frac{i\lambda}{4!}\right)\int d^4x\,\overline{\varphi_1\varphi}\,\overline{\varphi_2\varphi}\,\overline{\varphi_3\varphi}\,\overline{\varphi_4\varphi}\cdot 4!$$

+ terms in which $\overline{\varphi\varphi}$ appears once

+ terms with $\overline{\varphi\varphi}\;\overline{\varphi\varphi}$

$=$ 

$+\ \underbrace{\infty\cdot\left(\text{Result of ``order }(\lambda)^0\text{''}\right)}_{\text{3rd line}}$

↑ Clearly, this picture will correspond to the correct result precisely if; in addition to  $\equiv D_F$, we define  $\equiv (-i\lambda)\int d^4x$.

- We will comment on the prefactors

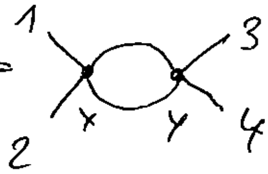
Order $(\lambda)^2$:

$$\langle T\varphi_1\varphi_2\varphi_3\varphi_4\,\frac{1}{2!}\left[-\frac{i\lambda}{4!}\int d^4x\,\varphi(x)^4\right]^2\rangle = \left\{\text{write } \int d^4x\int d^4y\,\varphi_x^4\varphi_y^4\right\}$$

$$= \frac{1}{2!}\left(-\frac{i\lambda}{4!}\right)^2\int_x\int_y \overline{\varphi_1\varphi_x}\,\overline{\varphi_2\varphi_x}\,\overline{\varphi_x\varphi_y}\,\overline{\varphi_x\varphi_y}\,\overline{\varphi_y\varphi_3}\,\overline{\varphi_y\varphi_4}\cdot\#\ +\ \cdots$$

This numerical factor is most naively expected to be $4!\cdot4!\cdot2$, from re-shuffling $\varphi_x^4\varphi_y^4$, but is actually slightly different

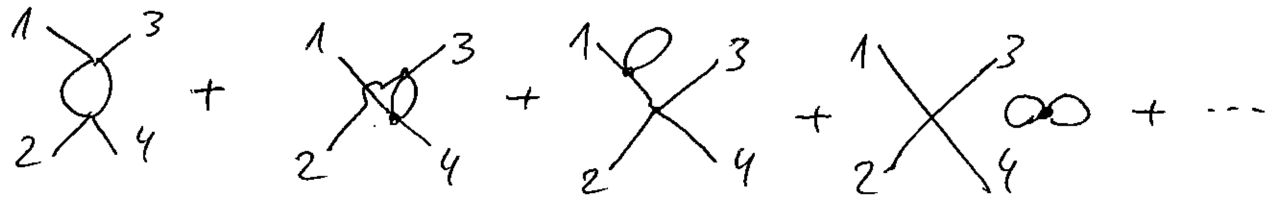
other full contractions (giving a different picture)

= 

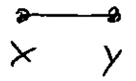
+ ....

- The dots stand for <u>all other</u> "Feynman diagrams" (truely <u>different</u> pictures) which can be built from 6 propagators (•—•) and two vertices (✕). The rules for this are: → Each end of each propagator attaches either to an external point (1,2,3,4) or to a vertex
  → Each external point accepts one and each vertex four propagator endpoints.

- Examples for these other diagrams:



- This extends to all orders in λ, always using the same <u>Feynman Rules</u>: (i.e. rules by which each building block of the diagrams is associated with an analytical expression)

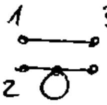
	$= D_F(x-y)$	propagator
	$= (-i\lambda)\int d^4x$	vertex

<u>Prefactors:</u> For the "generic" diagram, blindly applying these rules to the relevant picture gives

precisely the correct result. The reason is that the factors $1/4!$ take care of the possibilities of reshuffling the 4 "ends" of each vertex and the $1/n!$ of the exp-expansion take care of reshuffling vertices.

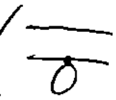
- Unfortunately, many diagrams are "non-generic" in the sense that they have <u>symmetries</u>, in which case a non-trivial prefactor or "symm. factor" arises.

<u>Example of a symmetry factor:</u>

- Consider the diagram . The numerical prefactor is unambiguously determined by <u>Wick's theorem</u>. Thus, write down the 8 fields $(\varphi_1 \varphi_3 \varphi_2 \varphi \varphi_4 \varphi \varphi \varphi)$ and count in how many ways one can place the "$\sqcap$" to get <u>this</u> specific diagram:

$$\varphi_1 \varphi_3 \; \varphi_2 \varphi \; \varphi_4 \varphi \; \varphi \varphi$$

$$\text{prefactor} = \frac{1}{4!} \cdot 4 \cdot 3 = \underline{\underline{\frac{1}{2}}} \;\Longleftarrow\; 1 \quad 4 \quad 3 \quad 1 \text{ choice}$$

- Result: () $= \frac{1}{2} \cdot$ (Result of blindly applying Feynm. rules)

"Symm. factor", associated with the fact the diagram does not change if the two "downward-pointing" ends of the vertex are swapped.

(<u>Note:</u> general formulae & computer programs for this exist (cf. "FeynArts"). For us, "by hand" will be good enough.)

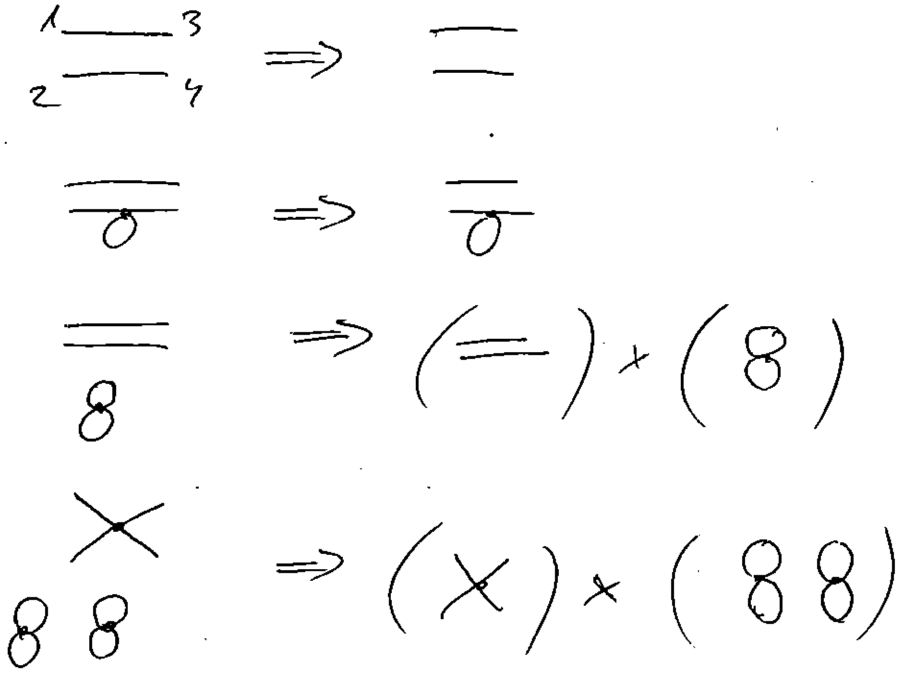
- Result so far:

$$\langle T\varphi_1 \cdots \varphi_n e^{iS_{int}} \rangle = \sum_{\substack{\text{all} \\ \text{contractions}}} \varphi_1 \cdots \varphi_n \exp(-i\lambda \textstyle\int \varphi^4)$$

$$= \left\{ \begin{array}{c} \text{Sum over all Feynm. diagrams} \\ \text{(incl. symm. factors)} \end{array} \right\}$$

- For any diagram, one can "split off" the "vacuum bubble part":

(Feynm. diagr. w/o ext. lines)



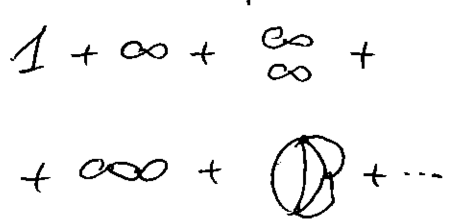
etc.

- A quite believable (but in my opinion not trivial) claim is:

$$\left\{ \begin{array}{c} \text{Sum over} \\ \text{all Feynm. d.} \end{array} \right\} = \left\{ \begin{array}{c} \text{Sum over} \\ \text{all Feynm. diagr.} \\ \text{w/o vac. bubbles} \end{array} \right\} \cdot \left\{ \begin{array}{c} \text{Sum over} \\ \text{all vac.} \\ \text{bubbles} \end{array} \right\}$$

<u>Problem:</u> Prove this using properties of "exp" & Wick's theorem!

$1 + \infty + \infty + \infty\infty + \ldots$



- Obviously, $\langle T \exp(iS_{int}) \rangle = \left\{ \begin{array}{c} \text{Sum over} \\ \text{all vac. bubbles} \end{array} \right\}$
- Thus, we finally have:

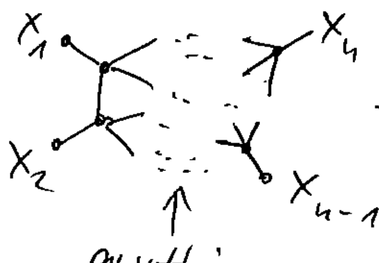
"Heisenberg"

$$\boxed{\langle T \varphi_1^H \cdots \varphi_n^H \rangle = \frac{\langle T \varphi_1 \cdots \varphi_n \, e^{iS_{int}} \rangle}{\langle e^{iS_{int}} \rangle} = \left\{ \begin{array}{c} \text{Sum over all Feyn.m. diagr.} \\ \text{w/o vacuum bubbles} \end{array} \right\}}$$

7.5 Feynman rules in momentum space

- According to LSZ, scatt. amplitudes are determined by residues of poles of <u>Fourier transforms</u> of <u>T-ord. corr. fcts.</u>
- Hence, it makes sense to translate our Feynm. rules to Fourier or <u>momentum space</u>.
- With $G(x_1 \cdots x_n) \equiv \langle T \varphi_1^H \cdots \varphi_n^H \rangle$, define

$$\tilde{G}(p_1 \cdots p_n) \equiv \underbrace{\int d^4x_1 \, e^{\overset{\text{"}-\text{" for incoming}}{-ip_1 x_1}} \cdots \int d^4x_n \, e^{\overset{\text{"}+\text{" for outgoing particle}}{ip_n x_n}}}_{\text{Consistently with what we found in LSZ-formula.}} G(x_1 \cdots x_n)$$

- Recall that $G(x_1 \cdots x_n) = \left\{ \right.$  $+$ other diagrams $\left. \right\}$

& $\bullet\!\!-\!\!\bullet = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\varepsilon} \cdot e^{-ip(x-y)}$

$\times = -i\lambda \int d^4x$

- Obviously, the momentum in every external line is fixed to the ext. momentum (the argument of $\tilde{G}$) by the dx_i^4-integration
- Next, each d^4x-integration of a vertex enforces "momentum-conservation" at this vertex:

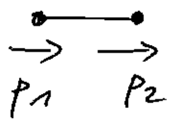
$$-i\lambda \int d^4x \, e^{-ip_1x_1 + \dots + ip_4x_4} = -i\lambda (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$$

incoming from perspective of vertex, outgoing from persp. of attached propagator

- Since each propagator end is either at a vertex or external, all $\exp(\pm ipx)$-factors are "used up" in this way. All propagator momenta which are <u>not</u> fixed by the above $\delta^4(\dots)$-fcts., are integrated over with

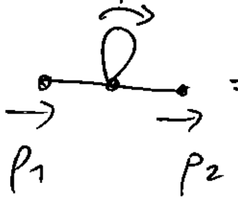
$$\int \frac{d^4p}{(2\pi)^4} \quad (\text{"loop momenta", see below}).$$

- <u>Example 1</u>



$$= \int d^4x_1 \, e^{-ip_1x_1} \int d^4x_2 \, e^{ip_2x_2} D_F(x_2 - x_1) = \frac{i}{p_1^2 - m^2 + i\varepsilon} \cdot (2\pi)^4 \delta^4(\dots)$$

- <u>Example 2</u>



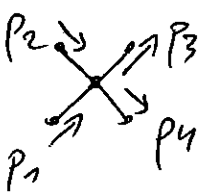
$$= \int d^4x_1 \, e^{-ip_1x_1} \int d^4x_2 \, e^{ip_2x_2} \int_x D_F(x_2 - x) D_F(x - x_1)(-i\lambda) D_F(x - x)$$

$$\sim \int d^4q \frac{i}{q^2 - m^2 + i\varepsilon}$$

$$\ldots = (2\pi)^4 \delta^4(p_2 - p_1)\left(\frac{i}{p_1^2 - m^2 + i\varepsilon}\right)^2 (-i\lambda) \int \frac{d^4 q}{(2\pi)^4} \cdot \frac{i}{q^2 - m^2 + i\varepsilon}$$

By the general rule, we would have $\delta^4(p_2 - q + q - p_1)$, which is of course the same.

- Example 3



$$= \int d^4x_1 e^{-ip_1x_1} \int d^4x_2 e^{-ip_2x_2} \int d^4x_3 e^{ip_3x_3} \int d^4x_4 e^{ip_4x_4}$$

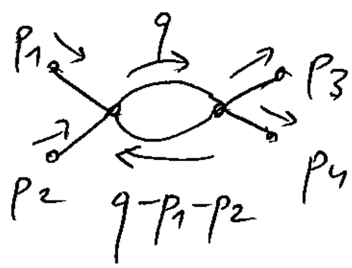
$$(-i\lambda) \int d^4x \, D_F(x - x_1) D_F(x - x_2) D_F(x_3 - x) D_F(x_4 - x)$$

$$\int \frac{d^4p}{(2\pi)^4} \cdot \frac{i}{p^2 - m^2 + i\varepsilon} \cdot e^{-ip(x - x_4)}$$

$$= (2\pi)^4 \delta^4(p_3 + p_4 - p_1 - p_2) \frac{i}{p_1^2 - m^2 + i\varepsilon} \cdot \underbrace{\frac{i}{\ldots}}_{p_2} \cdot \underbrace{\frac{i}{\ldots}}_{p_3} \cdot \underbrace{\frac{i}{\ldots}}_{p_4} (-i\lambda)$$

- Example 4

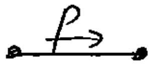
(Work out for yourself!)



$$= (2\pi)^4 \delta^4(\ldots)\, (-i\lambda)\, \frac{i}{(p_1^2 - m^2 + i\varepsilon)} \cdot \frac{i}{\ldots} \cdot \frac{i}{\ldots} \cdot \frac{i}{\ldots} \,.$$

$$\underbrace{\int \frac{d^4q}{(2\pi)^4} \cdot \frac{i}{q^2 - m^2 + i\varepsilon} \cdot \frac{i}{(q - p_1 - p_2)^2 - m^2 + i\varepsilon}}_{\text{"Loop integral"}} \cdot (\text{symm. factor})$$

?

<u>Summary:</u>  $= \dfrac{i}{p^2-m^2+i\varepsilon}$

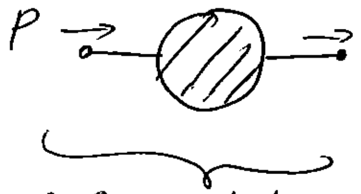
 $= -i\lambda$

<u>In addition:</u>
- assign momenta ensuring conservation at each vertex
- overall factor $\delta^4(p_f-p_i)$
- $\int \frac{d^4p}{(2\pi)^4}$ for each closed loop.

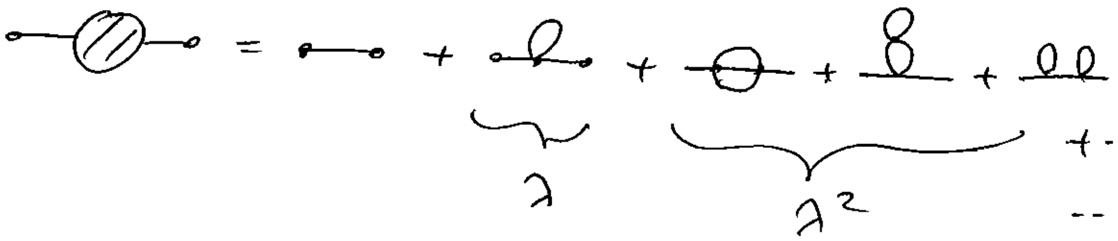
7.6. Calculating the Z-factor

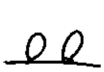
Recall $\langle T\varphi(x)\varphi(y)\rangle = Z\,D_F(x-y, m^2) + \int_{M_t}^{\infty} dM^2\,\sigma(M^2)\,D_F(x-y, M^2)$.

Fourier-trf. & drop δ-fct.

 $= \dfrac{iZ}{p^2-m^2} + \int_{M_t}^{\infty} dM^2\,\sigma(M^2)\,\dfrac{i}{p^2-M^2}$

Symbolic notation for all Feynm.-diagrams with two ext. lines, in momentum space, without vac. bubbles and without the overall δ-fct.

<u>Explicitly:</u> 

Let us introduce the notation  for those diagrams in  which <u>do not</u> fall apart after cutting any internal line (like , but <u>not</u> ).

<u>Obviously</u>: 

infinite sum

<u>Note</u>: The diagrams in  are called "<u>One-particle irreducible</u>".

Define the <u>"Self energy"</u> $\Pi(p^2)$ by

 $$\equiv \frac{i}{p^2-m_0^2}\left(-i\Pi(p^2)\right)\frac{i}{p^2-m_0^2} .$$

Clearly,

 $$= \frac{i}{p^2-m_0^2} + \frac{i}{p^2-m_0^2}\left(-i\Pi(p^2)\right)\frac{i}{p^2-m_0^2} + \frac{i}{p^2-m_0^2}(-i\Pi)\frac{i}{p^2-m_0^2}(-i\Pi)\frac{i}{p^2-m_0^2} + \cdots$$

$$= \frac{i}{p^2-m_0^2}\cdot\frac{1}{1-\frac{(-i\Pi)\,i}{p^2-m_0^2}} = \frac{i}{p^2-m_0^2-\Pi(p^2)} .$$

Hence:

$$\frac{i}{p^2-m_0^2-\Pi(p^2)} = \frac{iZ}{p^2-m^2} + \underbrace{\int_{M_t^2} dM^2\,\sigma(M)^2\,\frac{i}{p^2-M^2}}_{\text{no pole at } p^2\simeq m^2}$$

poles & residues must match $\Longleftarrow$ no pole at $p^2 \simeq m^2$

Matching poles $\Rightarrow$ $p^2-m_0^2-\Pi(p^2)=0$ at $p^2=m^2$

$$\Rightarrow \quad \underline{\underline{m^2 = m_0^2+\Pi(m^2)}}$$

Matching residues $\Rightarrow$ $$\frac{p^2-m^2}{p^2-m_0^2-\Pi(p^2)} \longrightarrow Z \quad \text{at } p^2\to m^2$$

$$\Rightarrow \quad \frac{p^2-m_0^2-\left\{\Pi(m^2)+\Pi'(m^2)\cdot(p^2-m^2)\right\}}{p^2-m^2} \rightarrow Z^{-1}$$ ← Taylor exp.

$$1-\Pi'(m^2) \rightarrow Z^{-1}$$

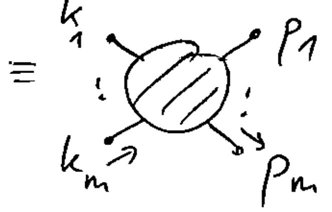
$$\underline{\underline{Z^{-1} = 1-\Pi'(m^2)}}.$$

7.7 Feynman rules for scattering amplitudes

We already know:

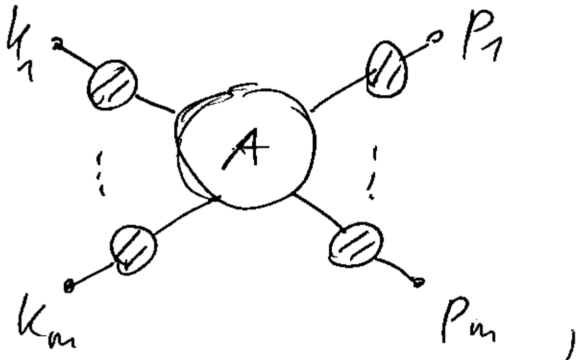
$$\underbrace{G(p_1 \cdots p_n, k_1 \cdots k_m)}_{} \sim \prod \frac{i\sqrt{Z}}{p_i^2-m^2} \cdot \prod \frac{i\sqrt{Z}}{k_i^2-m^2} \langle p_1 \cdots | k_1 \cdots \rangle .$$

$\equiv$



$\downarrow$ =

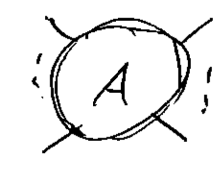
(should be obvious..)



)

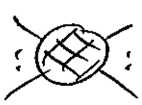
where "A" stands for "amputated diagram",

i.e.



contains only diagrams which, after cutting any internal line, will <u>not</u> fall apart in such a way that precisely one ext. line is gone. No ext. propagators are present in "A"!

Note: "A" is not the same as "1PI", i.e.

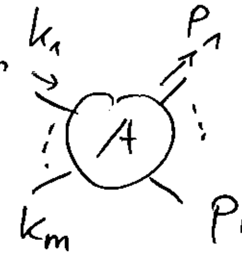
$\neq$
.

For example,  is a legitimate part of A.

- Now we also know that

 $\sim \dfrac{iZ}{p^2-m^2}$.

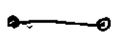
Thus:

$$\langle p_1 \cdots p_n | k_1 \cdots k_m \rangle = \left(Z^{1/2}\right)^{n+m} \Big\{$$ 

$$\Big\}$$

or, equivalently, <u>Feynman rules for $i\mathcal{M}_{fi}$:</u>

$i\mathcal{M}_{fi}$ = Sum of all amputated, connected *), diagrams w/o vacuum bubbles built from

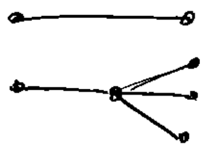
-  $= \dfrac{i}{p^2-m_0^2+i\varepsilon}$

 $= -i\lambda$ (with mom. cons. imposed)

$\int \frac{d^4p}{(2\pi)^4}$ for each loop , $Z^{1/2}$ for each ext. line
(vis. as "  ")

The other "half" is absorbed in ext. particle.

*) "Connected" means that diagrams like

"  " are not part of $2 \to 4$ scattering amplitudes, which is anyway obvious since one of the incoming particles does not change its momentum in this particular contribution.

Note:

If you start calculating cross-sects. etc. "at the loop level" with our present tools, you run into trouble with divergent loop integrals. We could just "Wick rotate" $p_0 \to i p_0$ and demand $p^2 < \Lambda^2$ for the euclidean momentum p. The procedure of removing this cutoff ($\Lambda \to \infty$) is called renormalization and will be better explained in theories other than $\lambda \varphi^4$ -- see below -- .