7. Electromagnetic field

7.1 Motivation from gauge invariance

- Recall the Lagrangian of a complex scalar

$$\mathcal{L} = \partial_\mu \phi \, \partial^\mu \phi^* - m^2 \phi \phi^*$$

with its $U(1)$-symmetry $\phi(x) \to e^{i\alpha} \phi(x)$.

- We want to promote this to a local or gauge symm.

$$\phi(x) \to e^{i\alpha(x)} \phi(x).$$

- The Lagrangian is not inv. since $\partial_\mu \phi \;\to\; e^{i\alpha(x)}\big(\partial_\mu \phi(x) + i(\partial_\mu \alpha(x))\phi(x)\big)$

$$\neq e^{i\alpha(x)} \partial_\mu \phi(x).$$

- This can be corrected by introducing a gauge connection $A_\mu(x)$ and ∂_μ by the covariant derivative $D_\mu = \partial_\mu + iA_\mu(x)$.

Now $D_\mu \phi \;\to\; D_\mu' \phi' = (\partial_\mu + iA_\mu')\, e^{i\alpha}\phi$

$$= e^{i\alpha}\,(\partial_\mu + iA_\mu' + i\partial_\mu \alpha)\,\phi$$

$$\underset{\uparrow}{=} e^{i\alpha} D_\mu \phi$$

$$\text{if } A_\mu \to A_\mu' = A_\mu - \partial_\mu \alpha .$$

- The latter statement is equivalent to $D_\mu \to e^{i\alpha} D_\mu e^{-i\alpha}$.
(From this form of the gauge trf. of A_μ it is obvious that $D_\mu \phi \to e^{i\alpha} D_\mu \phi$ and that $\mathcal{L}$ is inv.)

- Having introduced A_μ, we need to add to $\mathcal{L}$ an A_μ-dependent piece which specifies the dynamics of A_μ.

- This is most easily done by observing that

$$[D_\mu, D_\nu] = (\partial_\mu + iA_\mu)(\partial_\nu + iA_\nu) - (\partial_\nu + iA_\nu)(\partial_\mu + iA_\mu)$$

$$= iA_\mu \partial_\nu + i\partial_\mu A_\nu - i\partial_\nu A_\mu - iA_\nu \partial_\mu$$

$$= i\big((\partial_\mu A_\nu\underset{\uparrow}{)} - (\partial_\nu A_\mu\underset{\uparrow}{)}\big)$$

This bracket symbolizes that, here, ∂_μ & ∂_ν do <u>not</u> act to the right. (In the following we will drop such brackets)

$\Rightarrow$ $[D_\mu, D_\nu]$ is <u>not</u> a differential operator but just a fct.

- We define: $F_{\mu\nu} = \frac{1}{i}[D_\mu, D_\nu]$ (field strength).
- A_μ is a vector field, in the sense that $(A^\mu = \eta^{\mu\nu} A_\nu)$

$$A^\mu(x) \xrightarrow{\Lambda \in SO(1,3)} \Lambda^\mu{}_\nu A^\nu(\underset{\uparrow}{\Lambda^{-1}x})$$

vector with components $x'^\mu = (\Lambda^{-1})^\mu{}_\nu x^\nu$.

(as opposed to a scalar

$$\phi(x) \xrightarrow{\Lambda \in SO(1,3)} \phi(\Lambda^{-1}x).)$$

<u>Problem:</u> Show that using Λ^{-1} instead of Λ as specified above is necessary in order to realize the group trf. law

$$\begin{tikzcd} A_\mu \arrow[r, "\Lambda_1"] \arrow[rr, bend right, "\Lambda_3 = \Lambda_2 \Lambda_1"'] & A_\mu' \arrow[r, "\Lambda_2"] & A_\mu'' \end{tikzcd} ,$$

i.e. the commutativity of the above diagram.

($F^{\mu\nu}$ is a tensor, in obvious generalization of the above concepts of scalar & vector.)

- We now have arrived at the lagrangian of scalar QED:

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)(D^\mu\phi)^* - m^2\phi\phi^*,$$

which is invariant under $\phi \to e^{i\alpha}\phi$ & $A_\mu \to A_\mu - \partial_\mu\alpha$.

[Note: We do not include the $F\tilde{F}$-term since it is a total derivative]

- It is easy to see that a field redefinition $A_\mu \to eA_\mu$ leads to the more familiar form

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)(D^\mu\phi)^* - m^2\phi\phi^*,$$

with $D_\mu = \partial_\mu + ieA_\mu$.

Comment: Using the language of differential forms, we have

A – 1-form ; $A \xrightarrow{\text{gauge trf.}} A + \underset{\substack{\uparrow \\ \text{0-form or fct.}}}{d\alpha}$

$F = dA$ – 2-form.

$\mathcal{L} \sim F \wedge \underset{\substack{\uparrow \\ \text{Hodge-operator}}}{*} F$ (The normalization depends on conventions, which vary from author to author.)

7.2 Gupta-Bleuler quantization

Focus on the free action for A_μ: $S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu}\right)$

and attempt to go to the Hamilton formulation:

$$\pi^\mu = \frac{\partial \mathcal{L}}{\partial \dot{A}_\mu} = \frac{\partial}{\partial(\partial_0 A_\mu)} \left(-\frac{1}{4} F_{\rho\nu} F_{\sigma\tau} \eta^{\rho\sigma} \eta^{\nu\tau} \right)$$

$$= -\frac{1}{2} F_{\rho\nu} \eta^{\rho\sigma} \eta^{\nu\tau} \frac{\partial}{\partial(\partial_0 A_\mu)} \left(\partial_\sigma A_\tau - \partial_\tau A_\sigma \right)$$

$$= -\frac{1}{2} F_{\rho\nu} \left(\eta^{\rho 0} \eta^{\nu\mu} - \eta^{\rho\mu} \eta^{\nu 0} \right) = F^{\mu 0} .$$

- We find $\pi^0 = 0$, which represents a problem.

- It can be overcome by <u>fixing the gauge</u>, i.e. by

(Lorentz or covariant gauge)

demanding $\partial A \equiv \partial_\mu A^\mu = 0$ & using $\mathcal{L} = -\frac{1}{4} F^2 - \frac{\lambda}{2} (\partial A)^2$.

(Deriving EOMs from this new Lagrangian and setting $\partial A = 0$ is equivalent to the usual EOMs with $\partial A = 0$.)

- Setting $\lambda = 1$ for simplicity and repeating our previous analysis, we get:

$$\pi^\mu = -\partial^0 A^\mu + \ldots$$

$$= \underline{\underline{-\dot{A}^\mu}} + \ldots$$

$\uparrow$ terms without time-derivatives of A^ν.

(Such terms are not important for the quantization since they do not affect $[\pi^\mu, A_\nu]$ as long as $[A^\mu, A_\nu] = 0$.)

- Thus, it looks as if we are dealing simply with 4 indep. real fields A^μ (labelled by "μ").

- We can demand: $[A, A] = [\pi, \pi] = 0$

$$\& \quad [A_\mu(t, \bar{x}), \pi^\nu(t, \bar{y})] = i \eta_\mu{}^\nu \delta^3(\bar{x} - \bar{y}). \qquad (\eta_\mu{}^\nu = \delta_\mu{}^\nu)$$

- We could now, following the scalar case, Fourier transform A & π and introduce a & a^+ as lin. combinations of these Fourier-transformed fields.
- However, it is easier to guess the result and to "work backwards", checking that the proper commutation relations of A & π follow from those of a & a^+:

$$A_\mu(x) = \int d\tilde{k} \left(a_{\bar{k},\mu}\, e^{-ikx} + a^+_{\bar{k},\mu}\, e^{ikx} \right) \quad ; \quad k^0 = \sqrt{\bar{k}^2} = |\bar{k}|$$

- Motivated by the scalar case, we postulate that

$$[a_{..}, a_{..}] = [a^+_{..}, a^+_{..}] = 0 \quad \& \quad [a_{\bar{k},\mu}, a^+_{\bar{k}',\nu}] = -\eta_{\mu\nu}\, 2k^0 (2\pi)^3 \delta^3(\bar{k}-\bar{k}')$$

- It is straightforward to check that this leads to the π_μ-A_ν-commut. relations introduced above.

- We now define $|0\rangle$ as the state annihilated by all a's and obtain our Fock-space basis by applying (all types of) a^+'s to $|0\rangle$. It appears that we are "done". However....

Problem ①: The "wrong sign" of the a_0-a_0^+-commutator renders our space non-pos.-definite. This is clear from the following simple QM-example:

- Consider space $|0\rangle$, $a^+|0\rangle$, $(a^+)^2|0\rangle$ etc. with $[a, a^+] = -1$.
- Define as usual $\| |0\rangle \|^2 = \langle 0|0\rangle = 1$
- $\| a^+|0\rangle \|^2 = \langle 0| a a^+ |0\rangle = \langle 0| [a, a^+] |0\rangle = -\langle 0|0\rangle = -1$.

This is physically inacceptable. It is also unavoidable at the technical level since the sign-change between $[a_0, a_0^+]$ & $[a_i, a_i^+]$ comes from $\eta_{\mu\nu}$ and hence from Lorentz-symm.

Problem ②: We have not yet implemented $\partial A = 0$ in our quantum theory. Moreover, we can't simply declare $\partial A = 0$ an operator equation since $[A_0, \partial A] = [A_0, \dot{A}_0] \neq 0$.

Solution: (Gupta, Bleuler) (cf. Nachtmann's book)

- Call our full Fock space $\mathcal{H}$ and define $\mathcal{H}_1 \subset \mathcal{H}$ by

$$\partial A^v |\psi\rangle = 0 \quad \Leftrightarrow \quad |\psi\rangle \in \mathcal{H}_1 .$$

- This implies $\langle\psi|\partial A|\psi\rangle = \langle\psi|(\partial A^v + \partial A^e)|\psi\rangle$

$$= \langle\psi|(\partial A^v + (\partial A^v)^+)|\psi\rangle = 0,$$

which is a satisfactory implementation of our gauge condition.

- It will turn out that $\mathcal{H}_1$ is pos.-semi-definite (i.e. it still contains zero-norm states). Thus, we will define $\mathcal{H}_{phys.} \equiv \mathcal{H}_1/\sim$ where $\{|\psi\rangle \sim |\psi'\rangle \Leftrightarrow \| |\psi\rangle - |\psi'\rangle \| = 0\}$.

- To work all this out in detail, it is convenient (not necessary) to think in terms of polarizations. Hence a small detour:

A general 1-photon state is $-\varepsilon^\mu(k)\, a^+_{k,\mu}|0\rangle$ (& analogously for many photons). Obviously, there are 4 indep. polarizations for any given k and many possible basis choices. For example, we can demand (covariant) orthonormality

$$\varepsilon^{(\lambda)}_\mu(k)\left(\varepsilon^{(\lambda')\mu}(k)\right)^* = \eta^{\lambda\lambda'}$$

Problem: Show that this implies the completeness relation

$$\sum_{\lambda,\lambda'} \eta_{\lambda\lambda'}\, \varepsilon^{(\lambda)}_\mu(k)\left(\varepsilon^{(\lambda')}_\nu(k)\right)^* = \eta_{\mu\nu}$$

To make a concrete choice, we introduce some fixed vector n with $n^2 = 1$ & $n_0 > 0$ (otherwise arbitrary) and demand:

$$\varepsilon^{(0)} = n \ ; \quad \varepsilon^{(i)} \cdot n = 0 \ ; \quad \varepsilon^{(1)} \cdot k = \varepsilon^{(2)} \cdot k = 0 .$$

(This is consistent with our previous orthon. relation and essentially fixes the ε's.)

Now let us go to a coord. system where $n = (1, \bar{0})$ & $\bar{k} \parallel \hat{e}_3$.

$$\Rightarrow \quad \varepsilon^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} ; \quad \varepsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} ; \quad \varepsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} ; \quad \varepsilon^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

(up to rotations in 1-2-plane

Alternatively, we can choose an n with $n^2 = 0$ and demand:

$$\varepsilon^{u} = n \ ; \quad \varepsilon^{L} \sim k \ ; \quad \varepsilon^{(1)} \ \& \ \varepsilon^{(2)} \text{ orth. to } n \ \& \ k .$$

(This is not consistent with our previous orthog. rel. since $(\varepsilon^{u})^2 = (\varepsilon^{L})^2 = 0$. We demand instead $\varepsilon^{u} \cdot \varepsilon^{L} = -1$.)

Now let us go to a coord. system where $n = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$ & $\bar{k} \parallel \hat{e}_3$.

$$\Rightarrow \quad \varepsilon^{u} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} ; \quad \varepsilon^{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} ; \quad \varepsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} ; \quad \varepsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} .$$

Finally, we note $\langle \varepsilon', k' | \varepsilon, k \rangle = \varepsilon'^{\mu}(k')^{*} \varepsilon^{\nu}(k) \langle 0 | a_{\bar{k}', \mu} \, a^{+}_{\bar{k}, \nu} | 0 \rangle$

$$= \varepsilon'^{\mu}(k')^{*} \varepsilon^{\nu}(k) \left[-\eta_{\mu\nu} \, 2k^0 (2\pi)^3 \delta^3(\bar{k} - \bar{k}') \right]$$

$$= -(\varepsilon' \cdot \varepsilon) \, 2k^0 (2\pi)^3 \delta^3(\bar{k} - \bar{k}') ,$$

i.e. $-(\varepsilon)^2$ "measures the norm".

- It is now immediately clear that our "phys. space condition" really means

$$\partial \cdot A^{\nu} | \varepsilon, q \rangle = 0 \quad \Leftrightarrow \quad k \cdot a_{\bar{k}} | \varepsilon, q \rangle = 0 \quad \Leftrightarrow \quad k \cdot \varepsilon(q) = 0 \ \text{ if } \bar{k} = \bar{q}$$

$$\Leftrightarrow \quad k \cdot \varepsilon(k) = 0 .$$

- This is violated for ε^u and <u>only</u> for ε^u. (Hence "u" for unphysical.)

- We now define $\alpha^+_{\bar k,(u,L,1,2)} \equiv \varepsilon^{(u,L,1,2)\mu}(k)\, a^+_{\bar k,\mu}$ and build our Fock space using α^+'s (this is of course fully equivalent).

- We see immediately that $\mathcal{H}_1$ is simply the subspace of $\mathcal{H}$ built by applying only $\alpha^+_{L,1,2}$ to $|0\rangle$.

[In more detail: $k\cdot a_{\bar k}$ (product of $\varepsilon_{\bar k}\cdot a^+_{\bar k}$) $|0\rangle$ is evaluated by commuting $k\cdot a_{\bar k}$ through all $\varepsilon_{\bar k}\cdot a^+_{\bar k}$. A non-zero term arises only if $k\cdot\varepsilon_{(\bar k)} \neq$ for one of the ε's.]

- Furthermore, consider vectors $|\psi\rangle \in \mathcal{H}_1$ of the form (Product of $\alpha^+_{L,1,2}$'s) $|0\rangle$. Obviously $\| |\psi\rangle \| = 0$ if (and only if) at least one α^+_L is present in this product. This is furthermore identical to the statement $|\psi\rangle \sim 0$.

- <u>Also:</u> States $|\psi\rangle$ of this type do not affect any (by def. gauge inv.) observable. We only illustrate this using an example:

$$H = :\int d^3x\,(\pi^\mu \dot A_{,\mu} - \mathcal{L}): \;=\; \cdots \;=\; \int d\tilde k\, k_0 \left(-a^+_{\bar k,\mu}\, a_{\bar k}{}^{\mu}\right)$$

$$\underset{\substack{\uparrow\\ \text{check!}}}{=} \int d\tilde k\, k_0 \left(\sum_{i=1}^{2} \alpha^+_{\bar k,i}\,\alpha_{\bar k,i} - \underbrace{\left[\alpha^+_{\bar k,u}\,\alpha_{\bar k,L} + \alpha^+_{\bar k,L}\,\alpha_{\bar k,u}\right]}_{\substack{\text{this drops out in any phys. state,}\\ \text{so that our claim becomes obvious.}}}\right)$$

In summary

Any $|\psi> \in \mathcal{H}$: $\quad |\psi> = \sum \left(\alpha^+_{\bar{k},i} \, \alpha^+_{\bar{q},u} \, \alpha^+_{\bar{p},L} \cdots \right) |0>$

$|\psi> \in \mathcal{H}_1$: $\quad$ no terms involving α^+_u allowed

$|\psi> \sim |\psi'>$: $\quad |\psi>$ & $|\psi'>$ differ only by terms involving at least one α^+_L.

$$\mathcal{H}_{phys.} = \mathcal{H}_1 / \sim .$$

Note: $|\varepsilon, k> \; \rightarrow \; |\varepsilon, k> + \text{const.} \times |\varepsilon^L, k>$ corresponds to a gauge-trf. since $A_\mu \rightarrow A_\mu + \partial_\mu \alpha$

is $A_\mu \rightarrow A_\mu + (\sim k_\mu)$

in Fourier space.

(More specifically, since $k^2 = 0$ in all of the above, the transformed A_μ still satisfies $\partial \cdot A = 0$. Hence, this is residual gauge freedom. In other words, our final step of modding out "$\sim$" corresponds to removing residual gauge freedom.)

Comment: It is a non-trivial and important fact that our quantization procedure (i.e. $\mathcal{H}_{phys}$ and all observables) does not depend on the choice of our reference vector n_μ ($n^2 = 1$).

7.3 Propagator

$$<0| A_\mu(x) A_\nu(y) |0> \; = \; <0| \int d\tilde{k} \, d\tilde{k}' \, e^{-ikx + ik'y} \, a_{\bar{k},\mu} \, a^+_{\bar{k}',\nu} \, |0>$$

↑ ↑

only a $\quad$ only a+

are important

$$\langle A_\mu(x) A_\nu(y) \rangle = -\eta_{\mu\nu} \int d\tilde{k}\, e^{-ik(x-y)}$$

- Following precisely the scalar case, it is easy to show that

$$\langle T A_\mu(x) A_\nu(y) \rangle = \int \frac{d^4k}{(2\pi)^4} \cdot \frac{-i\eta_{\mu\nu}}{k^2+i\varepsilon}\, e^{-ik(x-y)}$$

$$= -\eta_{\mu\nu}\, D_F(x-y,\, m^2=0)$$

$\uparrow$ Note that, for the physical polarizations, the propagator is <u>exactly</u> as in the scalar case.

<u>Fact:</u>
If we had kept general λ, we would have found

$$-i\left(\frac{\eta_{\mu\nu}}{k^2+i\varepsilon} + \frac{1-\lambda}{\lambda} \cdot \frac{k_\mu k_\nu}{(k^2+i\varepsilon)^2}\right) \text{ instead of } \frac{-i\eta_{\mu\nu}}{k^2+i\varepsilon}.$$

(This will be easy to derive later on, in the path-integral approach.)

<u>Note:</u> λ drops out in all results for physical scattering amplitudes.

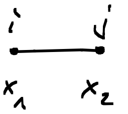
<u>7.4 Feynman rules for scalar QED</u>

Just to get used to models with different types of particles, we first consider a theory with N different scalars:

$$\mathcal{L} = \sum_{i=1}^{N} \frac{1}{2}\left((\partial\varphi^i)^2 - m^2(\varphi^i)^2\right) - \frac{\lambda}{4}\left(\sum_{i=1}^{N}(\varphi^i)^2\right)^2$$

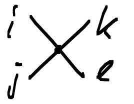
$\uparrow$ no factorial!

- It is easy to show that $\overbracket{\varphi_1^i \varphi_2^j} = \delta^{ij} D_F(x_1 - x_2) =$



- To derive the vertex, consider

$$\langle T \varphi_1^i \varphi_2^j \varphi_3^k \varphi_4^l \int d^4x \left(\frac{-i\lambda}{4}\right) \left(\delta_{mn} \varphi_x^m \varphi_x^n\right) \left(\delta_{pq} \varphi_x^p \varphi_x^q\right) \rangle .$$

- When calculating the diagram 

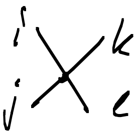
, there are clearly 2×2 equal terms corresponding to $m \leftrightarrow n$ & $p \leftrightarrow q$.

- Then, one still has two possibilities for attaching φ_1^i
two possibilities for attaching φ_2^j

to the same "pair" as φ_1^i → φ_3^k, φ_4^l unambiguous

to the other "pair" → two possibilities for φ_3^k, φ_4^l

$\Rightarrow$ 6 possibilities, giving



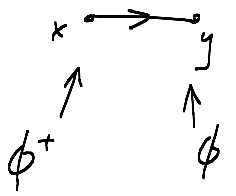
$$= -2i\lambda \left(\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}\right)$$

- Next, we consider one <u>complex</u> scalar:

Now, $\overbracket{\phi_x \phi_y} = 0$ (ϕ contains particle creation & antiparticle annih. operators, a^+ & b)

and $\overbracket{\phi_x \phi_y^+} = \overbracket{\phi_x^+ \phi_y} = D_F(x-y)$.

(Although they are equal, we can distinguish where the ϕ & ϕ^+ are and write

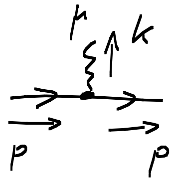


$= D_F(x-y)$.)

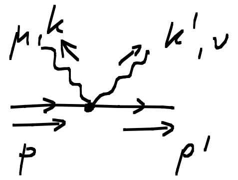
- After these preliminaries, we now directly write down the scalar-QED-Feynman rules (going directly to momentum space):

$$= \frac{i}{k^2-m^2+i\varepsilon}$$

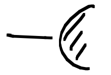
$$= \frac{-i\eta^{\mu\nu}}{k^2+i\varepsilon}$$



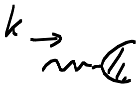
$$= -ie\,(p_\mu + p'_\mu) \qquad \text{(here } p' = p-k\text{)}$$



$$= 2ie^2\eta^{\mu\nu} \qquad \text{(here } p' = p-k-k'\text{)}$$



$$= Z_\phi^{1/2} \qquad \text{(ext. scalar)}$$



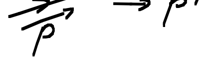
$$= Z_A^{1/2}\cdot \varepsilon_\mu(k) \qquad \text{(incoming photon)}$$

$$--- \ \varepsilon_\mu^*(k) \qquad \text{(outgoing photon)}$$

- To derive this, one has in principle to go through the whole procedure (greens-fcts., LSZ, Fourier transforms) again, keeping track of all the a's & b's, complex conjugation, the index μ, the $\varepsilon_\mu(k)$ etc. → Problems

- Here, we only give a shortcut explaining the $A\phi\phi^*$-vertex and the external polarizations:

  - Recall our naive approach to scattering and consider an "imagined" process $\phi \to \phi + \gamma$, i.e.,



– This is, of course, impossible by momentum conservation, but its good enough to "guess" the rules:

$$\langle 0| a_{\bar{p}'}\; i\int d^4x\, \mathcal{L}_{int}\; \underset{\substack{\uparrow\\ \text{scalar}}}{a^+_{\bar{p}}}\; \underbrace{\varepsilon^\mu(k)\, a^+_{\bar{k},\mu}}_{\text{photon}} |0\rangle$$

From $|D_\mu \phi|^2$, this gets a term

$$ie A^\nu \phi\, \partial_\nu \phi^+$$

contains $\int d\tilde{q}\; a_{\bar{q}}\, e^{-iqx}$

gives $-ip_\mu$

"used up" for $a^+_{\bar{p}}|0\rangle$

A^ν contains $\int d\tilde{q}\; a^\nu_{\bar{q}}\, e^{-iqx}$

"used up" for $a^+_{\bar{k},\mu}|0\rangle$

$$\Rightarrow -\eta^\nu{}_\mu (2\pi)^3 2k_0\, \delta^3(\bar{k}-\bar{q})$$

- The d^4x-integral results in $(2\pi)^4\delta^4(p'-p-k)$, which we extract as the conventional prefactor of $i\mathcal{M}_{fi}$
- Collecting all the i's and "$-$" signs, gives

$$i\mathcal{M}_{fi} = \underbrace{-iep^\mu}_{\text{vertex}}\; \underbrace{\varepsilon_\mu(k)}_{\text{incoming } \gamma}$$

- The term $\sim (\partial_\mu\phi) A^\mu \phi^+$ gives the remaining contribution $\sim p'^\mu$.
- The term $\sim A^\mu A_\mu \phi \phi^+$ gives the 2γ–2scalar vertex

<u>Comment:</u> more properly: work out $\phi^+ A_\nu [A_\mu \phi\, \partial^\mu \phi^+]\, \phi$

Naively, this gives just the derivative of D_F. However, extra non-covar. pieces are also induced in this way.

They are "cancelled" by extra pieces from $\mathcal{L}_{int} \neq \mathcal{H}_{int}$. (→ Itzykson/Zuber)