# String Theory (Arthur Hebecker, WS'24)

## 1. Introduction

- With very minor exceptions, our world is perfectly described by
$$\mathcal{L} = \mathcal{L}_{grav.} + \mathcal{L}_{SM}$$

- Here $\mathcal{L}_{SM}$ specifies a "conventional" QFT (scalars, fermions, gauge fields) and the most important exceptions are DM, baryon asymmetry & cosmol. inflation.

- As you know, $\mathcal{L}_{grav} = \frac{1}{2} M_p^2 \mathcal{R}[g_{\mu\nu}]$, may be viewed as specifying a classical FT, with the field being $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

- Taking $\eta_{\mu\nu}$ as the "Minkowski" background, the fluctuation field $h_{\mu\nu}$ may be quantized. Without going much beyond what you learned in your QFT lectures, one arrives at a "less conventional" QFT, namely one with spin-2 particles $\equiv$ gravitons.

- Symbolically, one finds
$$\mathcal{L}_{grav} \sim \frac{1}{2} M_p^2 \left( (\partial h)^2 + h(\partial h)^2 + h^2(\partial h)^2 + \cdots \right)$$

or, after $h \to h/M_p$
$$\mathcal{L}_{grav} \sim \frac{1}{2}(\partial h)^2 + \frac{1}{2M_p} h(\partial h)^2 + \frac{1}{2M_p^2} h^2(\partial h)^2 + \cdots$$

- We can now discuss the key difference between $\mathcal{L}_{SM}$ & $\mathcal{L}_{grav}$:
- In the former, couplings are dimensionless, allowing for the key feature of <u>renormalizability</u>.
- Example: $\mathcal{L}_{SM} \supset \frac{1}{2g^2} \operatorname{tr} F_{\mu\nu} F^{\mu\nu}$ ("QCD")

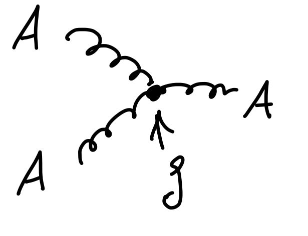
$$\underbrace{F_{\mu\nu}}_{-i[D_\mu, D_\nu] \,\in\, \mathrm{Lie}(SU(3))}$$

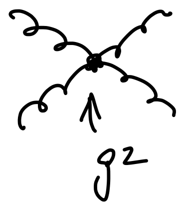
$$D_\mu = \partial_\mu + i \underbrace{A_\mu}_{\text{"gluon field"}}$$

Symbolically: $\mathcal{L}_{SM} \supset \frac{1}{2g^2}\left((\partial A)^2 + A^2 \partial A + A^4\right)$

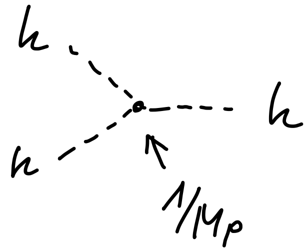
or, after $A_\mu \to g A_\mu$,

$$\mathcal{L}_{SM} \supset \frac{1}{2}(\partial A)^2 + \frac{1}{2} g A^2 \partial A + \frac{1}{2} g^2 A^4 .$$

$\Longrightarrow$ 

; 


- By contrast, in gravity we have analogous diagrams, but with dimensionfull coupling:



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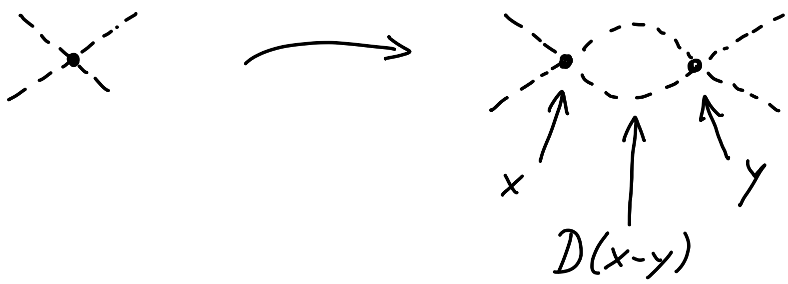
(often: $\kappa \equiv \frac{1}{M_P}$)

- You should remember from QFT why this dimensionful coupling is problematic for renormalization:

SM (more precisely: QCD) : higher-loop-corrections
$\Rightarrow$ higher powers of $g$ (dim.less!)
$\Rightarrow$ no reason for divergence to become worse than "$\ln \Lambda$".

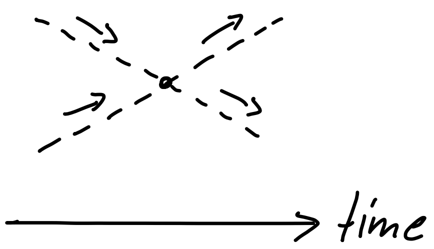
gravity : higher-loop-corrections
$\Rightarrow$ higher powers of $1/M_P$
$\Rightarrow$ higher powers $\Lambda$ for dim. reasons and hence worse & worse UV divergence
$\Rightarrow$ gravity OK as "effective QFT" at $E \ll M_P$, but total loss of control at $E \sim M_P$.

- Origin of UV divergences: Point-like interactions, i.e.




$\Rightarrow$ Integral $\int d^4x\, d^4y$ diverges at $x \sim y$

- Key idea for improvement: Replace point-particles by strings

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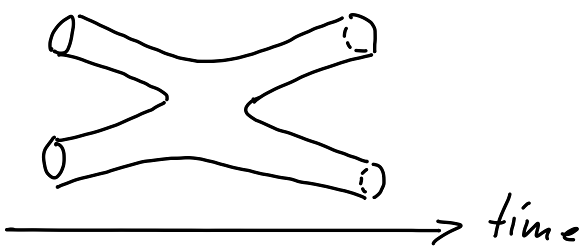
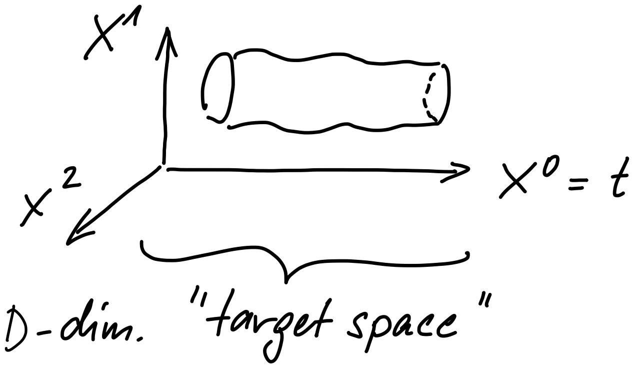


- On the right, there is no "interaction point" or "vertex". There is only an "interaction region", the size of which is set by the typical size $\ell_s \sim 1/M_s$ of the string.
  $\Rightarrow$ UV divergences automatically cut off by $M_s$.

- As a basis for such "string-string-scattering", we will need to quantize a single free string:




$\Rightarrow$ After quantization, one finds a discrete set of vibrational modes with different masses & spins.

- At low energies, only the lightest (as we will see massless) such modes are relevant. They are the particles (including a graviton) of a $d$-dimensional QFT – the low-energy EFT of string theory.

- As we will see, $D$ is not arbitrary but fixed by certain consistency requirements of the quantization process:

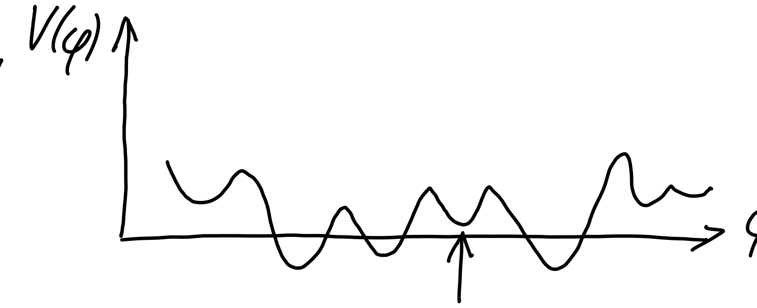
  – Bosonic string $\Rightarrow D = 26$ (nevertheless, the resulting QFT has unstable vacuum)

  – Superstring $\Rightarrow D = 10$ (more or less unique 10d QFT – a 10d "supergravity")

- To describe our world, need to "compactify" extra dims.:

- i.e.: Consider solutions of the above 10d SUGRA of type $\mathcal{M}_{10} = \mathbb{R}_{1,3} \times \mathcal{M}_6$, with $\mathcal{M}_6$ a compact manifold, in general with certain VEVs of different fields of the SUGRA.

- Since there are potentially very many (e.g. $10^{500}$) such choices of $\mathcal{M}_6$, one speaks of the "<u>string theory landscape</u>":




[ potential of the 4d EFT resulting from the compactification. The field $\varphi$ may be thought of as parametrizing the space of different manifolds $\mathcal{M}_6$ ]

a "metastable" vacuum, potentially describing our world.

- <u>There are many open problems:</u>

  - Quantitative understanding of the "landscape". (Even the existence of metastable vacua with SUSY-breaking and pos. cosm. constant – as in our world – is not established.)

  - How is the landscape populated in cosmology?

  - How to treat time-dep. gravitational backgrounds, in particular singularities, in string theory?

- <u>An independent motivation for studying string theory (ST)</u>

  - ST is an indispencible modern tool of modern research in QFT and GR.
  - It provides an explicit UV completion respecting Poincare symm. and, if desired, SUSY.
  - Via AdS/CFT, which is deeply rooted in ST, it allows for the study of non-pert. effects in both GR and QFT.

  [<u>Aside</u>: AdS/CFT is an equivalence ("duality") between GR in Anti-de-Sitter space and a specific conformal field theory (without gravity) in one dimension less.]