16 Kaluza-Klein compactification, dualities, AdS/CFT, towards the real world

16.1 Kaluza-Klein compactifications

- Both for practical reasons (10d → 4d "real world physics") as well as for conceptual reasons (see below), it is important to understand how d-dim. theories can follow from D-dim.. theories ($d < D$).

- Let us look at a very simple toy model; i.e.

$$S = -\int d^5x \; \frac{1}{2}(\partial_M \varphi)(\partial^M \varphi) \quad ; \quad M = 0,1,2,3,5$$

(it would be strange to talk about the "5-th dimension" x^4)

- often convenient: $x^5 \equiv y$,
  $x \equiv \{x^\mu\}$, $\mu = 0,\dots,3$

- consider this theory on geometry $\mathcal{M} = \mathbb{R}^{1,3} \times S^1$ (radius R)

- w.l.o.g. $\varphi(x,y) = \sum_{n=0}^{\infty} \varphi_n^c(x) \cos(ny/R) + \sum_{n=1}^{\infty} \varphi_n^s \sin(ny/R)$

$$\Rightarrow \quad S = -\pi R \int d^4x \Big[(\partial^\mu \varphi_0^c)(\partial_\mu \varphi_0^c) + \frac{1}{2}\sum_{n=1}^{\infty} \Big\{ \underset{\uparrow \; \partial^\mu(\dots)\partial_\mu(\dots)}{(\partial\varphi_n^c)^2} + m_n^2 (\varphi_n^c)^2 \Big\} + \dots$$

$$\dots + \frac{1}{2}\sum_{n=1}^{\infty} \Big\{ (\partial\varphi_n^s)^2 + m_n^2 (\varphi_n^s)^2 \Big\} \Big]$$

where $m_n = \frac{n}{R}$ arises from $\partial_5 \cos(ny/R) = -\frac{n}{R}\sin(ny/R)$

and we used orthogonality of sin/cos modes under y-integration.

$\Rightarrow$ 4d theory of one zero mode (φ_0^c) and a tower of massive KK-modes.

(**Note**: Exp-ansatz also works. See problems.)

Note: This also works with more complicated theories, including gravity. The famous historical "Kaluza-Klein theory" is gravity on $\mathbb{R}^{1,3} \times S^1$:

$$S \sim \int d^5x \sqrt{-G}\, M_5^3 \mathcal{R}[G_{MN}] \qquad G_{MN}(x^0,\dots,x^3,x^5) - \text{5d metric}$$

$$\sim \int d^4x \sqrt{-g}\, \Big(\underbrace{M_5^3 R\, \mathcal{R}[g_{\mu\nu}]}_{\substack{\text{4d-metric} \\ \equiv \text{ zero mode of } G_{\mu\nu}}} + \underbrace{(\partial_\mu \varphi)(\partial^\mu \varphi)}_{\substack{\sim G_{55} \\ \text{(size of } S^1 \\ \text{or "variable" } R)}} + \underbrace{F_{\mu\nu} F^{\mu\nu}}_{\substack{A_\mu \sim \text{ zero mode} \\ \text{of } G_{5\mu}}} + \text{massive} \Big)$$

- This would also work for e.g. IIB on $\mathbb{R}^{1,3} \times T^6$, but we would get a highly-SUSY theory in 4d with lots of massless scalars (corresponding e.g. to the moduli of T^6).

16.2 T-duality

- Return to bosonic string (just for simplicity) and replace our target space $\mathbb{R}^{1,25}$ by $\mathbb{R}^{1,24} \times S^1$.
  $\Rightarrow$ X^{26} is periodic with period $2\pi R$

- Work out the mass-shell constraint in any of the approaches we used before.

$$\Rightarrow \quad -p^2 = \frac{2}{\alpha'}(N + \tilde{N} - 2) + \underbrace{\frac{n^2 R^2}{\alpha'^2}}$$

this new term arises because we can choose $X^{26}(\tau,\ell) = X^{26}(\tau,0) + 2\pi n R$ (i.e. a "winding string")

Also, by the KK-mode logic above:

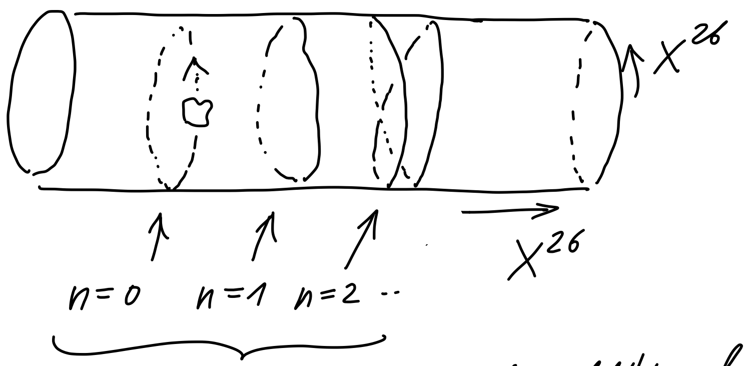
$(p^M)^2 = (p^\mu)^2 + (p^{25})^2$

with $p^{25} = \frac{m}{R}$ ("KK momentum") & $M = 0, \cdots, 24, 26$
$\mu = 0, \cdots, 24$

$\Rightarrow$ The mass spectrum of the 25d effective theory is

$$(\text{mass})^2 = -(p^\mu)^2 = \frac{2}{\alpha'}(N + \tilde{N} - 2) + \frac{m^2}{R^2} + \frac{n^2 R^2}{\alpha'^2}$$

- Vizualization:



all of them can in addition have non-zero KK-momentum m.

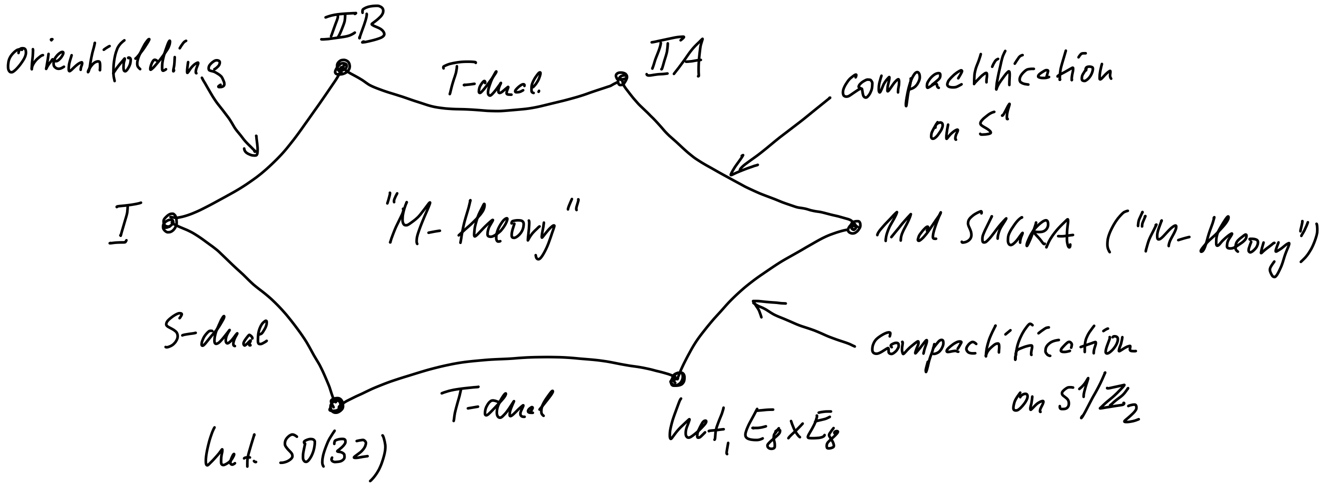
- Key observation: Spectrum (and in fact the 25d theory) is invariant under $m \leftrightarrow n$, $R \leftrightarrow R' \equiv \frac{\alpha'}{R}$.
  This is called "T-duality".

- Implication: The same 25d theory has two stringy UV-completions: If one of them has $R < \sqrt{\alpha'}$, then the other has $R > \sqrt{\alpha'}$. Thus, one may say that $l_s \sim \sqrt{\alpha'}$ is really in some the smallest distance at which our field-theoretic intuition about space holds. If we make the S^1 smaller, control is lost due to many light winding modes. But, a new, T-dual description becomes available in which $\sqrt{\alpha'} < R$ and winding modes are heavy.

- In superstring theory, T-duality changes not just the radius but also the type of theory: IIA $\leftrightarrow$ IIB & het. $SO(32)$ $\leftrightarrow$ het $E_8 \times E_8$

16.3 M-theory

- Thanks to dualities, the 5 different superstring theories we learned about are not just different fundamental theories but different, calculationally controlled corners of one unique fundamental theory (sometimes called M-theory):



- We control only the regime of a "large" target space ($R \gg \sqrt{\alpha'}$) and/or $g_s \ll 1$. The centre of the diagram is the regime where neither of the above "control handles" applies.

- One of the corners has no stringy UV completion but a (poorly understood) UV completion through "M2-branes". This 11d SUGRA theory (the only SUGRA above $D=10$) turns into type IIA string theory upon compactification on S^1. M2-branes wrapped on S^1 turn into the fundamental strings of type IIA.

16.4 EFT lagrangians

- As already mentioned several times, at low energies in 10d we only see the massless modes. These form the field content of of 10d supergravity theories. The interaction terms of these theories are precisely reproduced by string scattering amplitudes. We display the IIA action (cf. Polch. for more details & for the other actions):

$$S_{IIA} = S_{NS} + S_R + S_{CS} + \text{fermions \& higher-derivative terms} \qquad \left(\kappa_{10}^2 = \frac{1}{2}(2\pi)^7 \alpha'^4\right)$$

with
$$S_{NS} = \frac{1}{2\kappa_{10}^2}\int d^{10}x \sqrt{-G}\, e^{-2\phi}\left(R + 4(\partial\phi)^2 - \frac{1}{2}|H_3|^2\right)$$

$$S_R = \frac{1}{2\kappa_{10}^2}\int d^{10}x \sqrt{-G}\left(-\frac{1}{2}|F_2|^2 - \frac{1}{2}|F_4|^2\right)$$

$$S_{CS} = -\frac{1}{4\kappa_{10}^2}\int \underbrace{B_2 \wedge F_4 \wedge F_4}_{\sim B_{\mu_1\mu_2}(F_4)_{\mu_3\cdots\mu_6}(F_4)_{\mu_7\cdots\mu_{10}}\,\varepsilon^{\mu_1\cdots\mu_{10}}} \qquad (\text{"Chern-Simons"})$$

$$|F_p|^2 \equiv \frac{1}{p!}(F_p)_{\mu_1\cdots\mu_p}(F_p)_{\nu_1\cdots\nu_p}\, G^{\mu_1\nu_1}\cdots G^{\mu_p\nu_p}$$

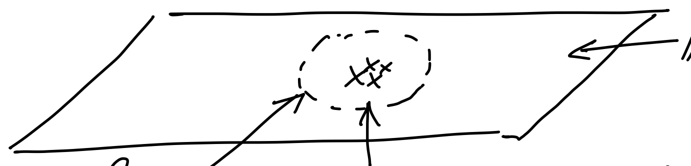
- The IIB action is similar, just with different RR-form-degrees and a different CS term.
- If D-branes are present, one also has:

$$S_{D_p} = \underbrace{-\mu_p\int d^{p+1}\xi\, e^{-\phi}\, \mathrm{tr}\sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})}}_{\substack{\text{"DBI-action"} \\ \text{(Dirac-Born-Infeld)}}} + \overbrace{\mu_p\int C_{p+1}}^{\text{"CS"}} + \text{ferm. \& higher-deriv.}$$

16.5 AdS/CFT

- A very remarkable duality arises from considering a stack of N

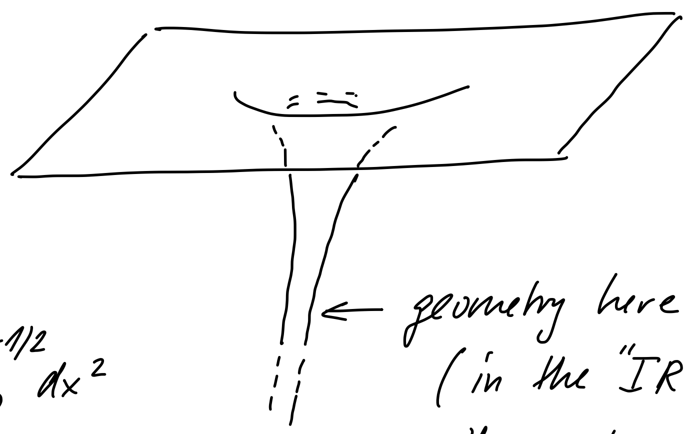
D3 branes in flat 10d space. We can only sketch this. We do not display the 4 dims. parallel to the branes and draw the 6 orthogonal dims. as $\underline{\underline{\text{two}}}$:



$\leftarrow \mathbb{R}^6$ of $\mathbb{R}^{1,3} \equiv \mathbb{R}^{1,3} \times \mathbb{R}^6$

$\int F_5 \sim N$ $\qquad$ N D3s $\Rightarrow$ localized $SU(N)$ SYM theory

The above is the perturbative ($g_s = e^\phi \ll 1$) picture. At larger g_s, backreaction effects are considerable (from the D3-mass & F_5) and the geometry gets deformed. This is very similar to a charged BH:



$\leftarrow$ geometry here is $AdS_5 \times S^5$ (in the "IR" direction, i.e. towards the center of the "Black Brane", this "throat" goes on for ever)

metric:

$$ds^2 = H_p^{-1/2} dx^2 + H_p^{1/2} dy^2$$

$$H_p(r) = 1 + \left(\frac{r_p}{r}\right)^{7-p} \; ; \; p = 3 \text{ here} \; ; \; r_3^4 \sim N\alpha'^2$$

$$dx^2 = dt^2 - d\vec{x}^2 \; ; \; dy^2 = dr^2 + r^2 d\Omega_5^2 \quad \text{for } p=3$$

- low-energy excitations of the $SU(N)$ SYM $\hat{=}$ gravitational excitations very deep in the $AdS_5 \times S^5$ throat. This can be made precise by taking the limit $E \ll 1/\sqrt{\alpha'}$ in the math. sense

$\Rightarrow$ flat-4d $SU(N)$ SYM $\equiv$ type IIB ST (i.e. gravity!) on $AdS_5 \times S^5$

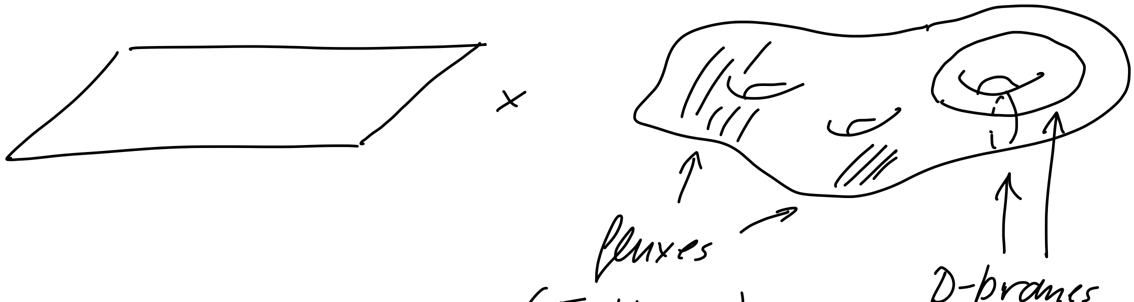
$(g_s N = R^4 / 4\pi\alpha'^2)$ $\qquad$ radius R

16.6 Towards the real world

- To realize the idea that ST UV-completes gravity + SM in our real world, the best-explored path is to consider a KK-compactification on a CY (≡ special 6d compact manifold)

  ↑ ~10^5 different topologies known, finite total number conjectured.

- e.g. type IIB on $\mathbb{R}^{1,3} \times M_6$; $M_6 = CY$



$\Downarrow$ Stabilizes "moduli"

$\Downarrow$ SM gauge group & matter

<u>Note:</u> CY's are special by being compact m/s. with $R_{\mu\nu} = 0$, such that $\mathbb{R}^{1,3} \times M_6$ is a solution of Einstein-eqs. But they can be deformed in many ways, still being CYs. This corresponds to 4d scalar fields (like $g^{55} \hat{=} \varphi$ of the 5d → 4d KK theory). "Turning on fluxes" can stabilize these moduli. It can also break SUSY (which the CY by itself preserves). Much more should be said...

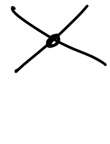
→ e.g. book "Naturalness, String-Landscape & Multiverse" & lots of other literature, e.g. "Ibanez/Uranga".

16.7. Final remarks

1) Our approach to the superstring (the "RNS superstring") was to make the WS theory supersymmetric. The alternative, known as "Green-Schwarz superstring" is to make the target space supersymmetric:
$$X^\mu(\tau,\sigma) \; + \; \Theta^a(\tau,\sigma)$$
$$\mu = 0, \dots, D-1 \qquad a = 1, \dots, 2^{D/2}.$$

2) We argued for the 10d action only perturbatively, through by adding "string-realized" particles or waves to 10d flat space. But one can instead directly consider the WS action on a non-trivial background, i.e. in the simplest case
$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h}\, G_{\mu\nu}(X) \cdot \partial_a X^\mu \partial_b X^\nu h^{ab}.$$

This 2d theory is not free (quadratic in X) any more. Instead, it is an interacting 2d QFT. After gauge-fixing, i.e. on a flat 2d WS, it is then not obvious whether we are dealing with a CFT. Explicitly, expanding around a point X_0, we have:
$$G_{\mu\nu}(X) = G_{\mu\nu}(X_0) + \underbrace{A_{\mu\nu\rho}(X-X_0)^\rho}_{\text{can be removed by coord. choice on target space}} + \underbrace{B_{\mu\nu\rho\sigma}(X-X_0)^\rho (X-X_0)^\sigma}_{\text{together with } (\partial X)^2 \text{ term, this gives rise to a vertex:}} + \dots$$



- Since Weyl inv. $\Leftrightarrow$ conf. invariance, we need to ensure all β-fcts. of our 2d QFT vanish. In particular

$$\beta_{g\sigma}(G(X,\mu)) \equiv \mu \frac{\partial}{\partial\mu} G_{g\sigma}(X,\mu) \stackrel{!}{=} 0 .$$

- This can be analyzed order by order in the expansion in $(X-X_0)$ sketched above. (This expansion is an expansion in α', as seen by redefining $(X-X_0) \to (X-X_0)\sqrt{\alpha'}$.)

- One finds e.g. $\beta_{g\sigma}(G) = \alpha' R_{g\sigma} + \cdots$ (-> Polchinski)

such that the CFT-condition enforces Ricci-flatness of the target space, i.e. Einstein's eqs.

- More generally, vanishing β-fcts. on the WS correspond to the EOMs of the 10d SUGRA theory.

This was only a start. Much more could be said. Keep studying the literature!

Thank you for your attention.