The Weak Gravity Conjecture through the eyes of Cosmic Strings and Axionic Black Holes

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based on work with **Philipp Henkenjohann** and **Lukas Witkowski**

and with **Pablo Soler**

Outline

- The magnetic Weak Gravity Conjecture for axions
- Cosmic string solutions and possible implications

- Axionic Black Holes
- Attempt at 'deriving' a Weak Gravity Conjecture

Motivation

- The Weak Gravity Conjecture,

Arkani-Hamed/Motl/Nikolis/Vafa '06

$$m < gM_P \qquad \text{or} \qquad \Lambda < gM_P \ ,$$

has recently been revisited by many authors:

Cheung/Remmen; Rudelius; de la Fuente/Saraswat/Sundrum . . . '14

Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler; Bachlechner/Long/McAllister; AH/Mangat/Rompineve/Witkowski; Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala; Harlow; AH/Rompineve/Westphal; . . . '15

Conlon/Krippendorf; Ooguri/Vafa; Freivogel/Kleban; Banks; Danielsson/Dibitetto; . . . . . . '16

Motivation (continued)

- A particularly timely aspect of it is the axionic case,

$$g \equiv 1/f \ ,$$

relevant for natural inflation.

- The standard ('electric') logic is

$$m < g\, M_P \qquad \Rightarrow \qquad S_{inst} < M_P/f \ ,$$

such that the instanton-induced potential

$$V_{inst} \sim e^{-S_{inst}} \cos\varphi$$

is unsuppressed. This threatens slow-roll inflation.

Motivation (continued)

- An important concern is that the underlying 'black hole stability argument' can not be made for instantons.
- Another is a set of loopholes related to the prefactors of the instanton terms and the 'mild vs. strong' forms of the WGC.

  de la Fuente/Saraswat/Sunderum; Rudelius; Brown/Cottrell/Shiu/Soler; (cf. AH/Mangat/Rompineve/Witkowski for a stringy realization)

- Thus, it might be worthwhile to approach the problem from the 'magnetic side' ....

What is the magnetic WGC for axions?

- The basic underlying assumption is that "The minimally charged magnetic objects should exist in field theory," (i.e. not yet be a black hole).

- This can be insured if the monople UV-completes at a scale

$$\Lambda < g\, M_P$$

or, in a p-form gauge theory in d dimensions,

$$\Lambda < \left(g^2 M_P^{d-2}\right)^{1/(2p)} \,.$$

- It is useful to rewrite this in terms of the 'electric strong-coupling-scale' Λ_e of the p-form gauge theory:

$$g^2 \equiv g_e^2 \equiv \Lambda_e^{2(p+1)-d} \,.$$

What is the magnetic WGC for axions?

- The parametric situation is shown on the right.

- One finds

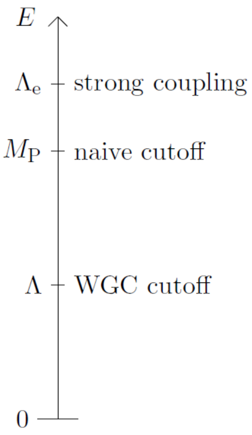
$$\frac{\Lambda}{M_P} < \left(\frac{M_P}{\Lambda_e}\right)^{\frac{d-2(p+1)}{2p}}$$

- In our case of interest,

$$\Lambda_e \equiv f > M_P \,, \quad p \to 0 \,.$$

- Thus, one is tempted to conclude:

  $\Lambda = 0$, i.e. the theory does not exist!



Magnetic WGC for axions – another naive argument

- This 'analytic continuation in p' is clearly somewhat naive.
- Another (also naive) argument supports the conclusion:
- Indeed, $p = 0$ means the magnetic object has codimension two (a string in $d = 4$).
- But the gauge-field contribution to the tension of a string diverges logarithmically (both near the string and at infinity):

$$\sigma \sim f^2 \ln(\Lambda_{UV}/\Lambda_{IR})\,.$$

- The deficit angle is

$$\Delta\varphi = \sigma/M_P^2\,,$$

  such that for $f > M_P$ one does not expext a sensible string-spacetime to exist.

Magnetic WGC for axions – string solutions

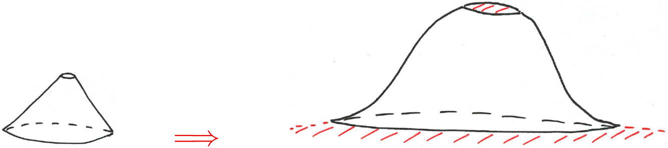
- Now, by the standard logic,

  not having a field-theoretic magnetic object

  means

  the theory should not exist.

- This can be made more precise by studying the Cohen-Kaplan string spacetime



- The latter can indeed not be extended to $f > M_P$, since the outer singularity moves inwards and meets the region where $\Delta\varphi > 2\pi$.

Magnetic WGC for axions – string solutions

- The Gregory string spacetime avoids the outer singularity by allowing for an expansion along the string worldsheet (the transverse part of the solution remains static).

- However, this solution also breaks down for $f > M_P$.

- Thus, the only way out appears to be the (completely non-static) topological-inflation-spacetime.

  Linde/Vilenkin '94

- In our case, one takes $\Phi \in \mathbb{C}$ and

  $$\mathcal{L} \;\sim\; |\partial\Phi|^2 + m^2|\Phi|^2 - \lambda|\Phi|^4\,,$$

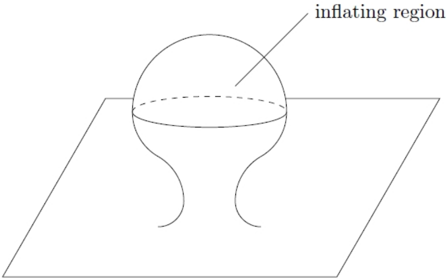
  and demands that the resulting abelian-Higgs-model string has

  $$(1/H_{core}) < R_{core}\,.$$

  cf. also Dolan/Draper/Kozaczuk/Patel '17

Magnetic WGC for axions – string solutions

- This provides a (rather exotic and not purely field-theoretic) UV completion of a string with $f > M_P$.



- Two conclusions are possible:

  (A) We insist on a static, horizonless magnetic object. Then $f > M_P$ is forbidden.

  (B) A topologically inflating region provides the magnetic object for axionic theories with $f > M_P$.

Part II:

The WGC through the eyes of **Axionic Black Holes**

- Most recent work (including Part I of this talk) was about interpreting the WGC

- Very little progress has been made towards a possible Weak Gravity Theorem.

  see however Cottrell/Shiu/Soler '16

- Let us also try to make some progress in this direction, even if (at first) only in an 'exotic' case ....

  cf. Montero/Uranga/Valenzuela '17
  (technically related, but conceptually different)

Weak Gravity Conjecture for 2-forms

- We will study the dual side of the 'natural inflation case':

$$\int \frac{1}{f^2} |dB_2|^2 \; + \int_{\text{string}} B_2 \qquad \text{for} \qquad f \ll M_P \,.$$

- Formally extending the WGC to this case implies

(a) Electric: Light strings with tension $\sigma < f \, M_P$

or

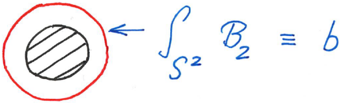
(b) Magnetic: A cutoff $\Lambda < \sqrt{f \, M_P}$.

Let us now consider

Axionic Black Holes

Bowick/Giddings/Harvey/Horowitz/Strominger '88

- In the simplest case, these are just Schwarzschild BHs with a non-zero 'B_2-Wilson-line':

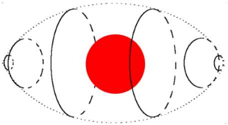


- Since the BH effectively induces a non-zero 2-cycle of space-time, such a non-zero $\langle b \rangle$ can be added at no cost to a standard BH solution.

Axionic Black Holes (continued)

- The non-zero 'Wilson-line' b can in principle be measured by strings 'lassoing' the BH.

Illustration from recent paper by Dvali/Gußmann:



- There is some controversy concerning the observability of this effect, but we believe this does not affect our parameter ranges.

Preskill/Krauss '90; Coleman/Preskill/Wilczek '92

Axionic Black Hole evaporation – explosive

- Now let the BH Hawking-radiate, as usual.

- R goes down, T goes up,
nothing unusual happens before they reach

$$R_c \equiv 1/\sqrt{\sigma} \qquad \text{and} \qquad T_c \equiv \sqrt{\sigma}$$

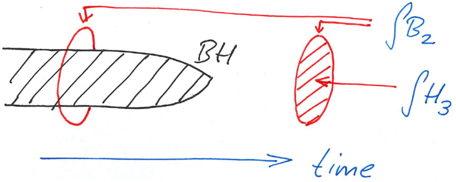
or, alternatively,

$$R_c \equiv 1/\Lambda \qquad \text{and} \qquad T_c \equiv \Lambda\,.$$

- Let us assume that, at this moment, the BHs life ends on a short time scale $\sim R_c$
(e.g. due to a KK or string tower-of-states).
cf. 'Lattice WGC' of Heidenreich/Reece/Rudelius

Axionic Black Hole evaporation – explosive (continued)

- With the BH gone, the non-zero B_2 integral **must** be supported by field-strength (flux) of $H_3 = dB_2$



- Using $b = \oint B_2 = \int H_3$, we can estimate the energy of the resulting field configuration as

$$E \sim \frac{1}{f^2} \int |H_3|^2 \sim \frac{b^2}{f^2 R_c^3} \sim \frac{1}{f^2 R_c^3} \,.$$

Axionic Black Hole evaporation – explosive (continued)

- The necessary condition $E < M(R_c) \sim R_c M_P^2$ then immediately gives

$$\frac{1}{f^2 R_c^3} < R_c M_P^2 \qquad \text{and hence} \qquad \frac{1}{R_c^4} < f^2 M_P^2 \,.$$

- Recalling that $R_c = 1/\sqrt{\sigma}$, we now have

$$\sigma < f\, M_P \qquad \text{or} \qquad \Lambda^2 < f\, M_P \,,$$

i.e. precisely what is expected from the WGC.

Axionic Black Hole evaporation – slow

- Next, let us assume that nothing dramatic happens when the BH reaches

$$R_c \equiv 1/\sqrt{\sigma} \qquad \text{and} \qquad T_c \equiv \sqrt{\sigma}\,.$$

- However, unavoidably, virtual strings will start lassoing the BH and hence the variable

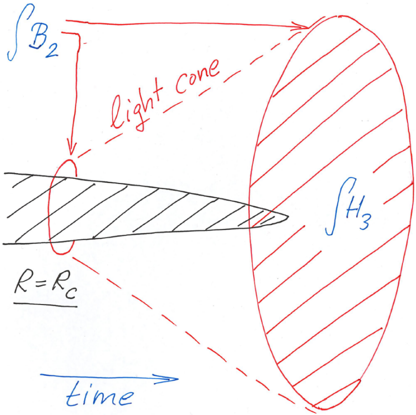
$$b(r) \equiv \int_{S^2(r)} B_2(r,\theta,\varphi)$$

  will start experiencing an effective force at $r \sim R_c$.

- $b(r)$ will develop a non-trival profile in r, and $H_3 = dB_2$ will have time to spread until the BH is gone.

Axionic Black Hole evaporation – slow (continued)

- Crucially, the resulting H_3 can be much more dilute than in the 'explosive' case:



Axionic Black Hole evaporation – slow (continued)

- The evaporation time from critical radius to 'zero' is

$$t_{ev} \sim \frac{M_c^3}{M_P^4} \sim R_c^3 M_P^2 \sim \frac{M_P^2}{\sigma^{3/2}} \,.$$

- Then H_3 can maximally spread to a radius $\sim t_{ev}$.

- Demanding that the corresponding energy satisfies $E < M(R_c)$, we now find

$$\sigma \sim \Lambda^2 \lesssim f^{2/5} \cdot M_P^{8/5} \,.$$

- This is much weaker than the naive WGC bound $\sigma < f \cdot M_P$.

- We expect a more careful analysis to give a bound in between our 'explosive' and 'slow' limits.

What if the WGC is violated only in the effective theory?

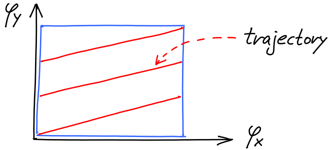
- As is well-known, an axion with large f_{eff} can in principle follow from two small-f axions.

Kim/Nilles/Peloso '04 (Berg/Pajer/Sjors '09; Ben-Dayan/Pedro/Westphal '14)

- The possibly simplest way to achive such an effective small coupling is via gauging à la Dvali (cf. also KS/KLS), as in 'winding inflation'

AH/Mangat/Rompineve/Witkowski '14

$$|F_0|^2 \quad \rightarrow \quad |F_0 + \varphi_1 + N\varphi_2|^2$$

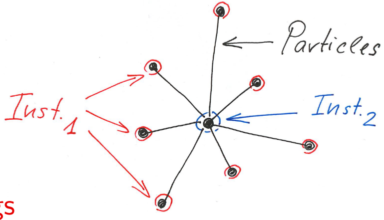


- This can of course be done more generally, trying to evade e.g. the WGC for 1-forms in the effective theory.

Saraswat '16

What if the WGC is violated only in the effective theory? (continued)

- As pointed out by Saraswat, the magnetic WGC for 1-forms is fullfilled by composite monopoles (without a low cutoff!).
- Analogous composite instantons can catalyze the problematic ABH-decay in our case.
- However, the effect is not strong enough unless the new particles are light.



- Also in Part I, composite strings exist if our f is only 'effectively' large.

cf. also Higaki,...,Takahashi '16

- But, once again, they can not be static and our earlier negative conclusion can not be avoided.

Summary/Conclusions

Part I (Cosmic strings)

- Magnetic WGC for axions: large-f string should exist.
- If this implies static or horizon-free: $f > M_P$ forbidden.
- Else: Topological inflation provides such a string.

Part II (Axionic black holes)

- We suggested a new, dynamical argument for a WGC-like-bound for 2-forms.
- Very exotic remnants are needed to avoid this.

An idea for going beyond small-f axions:
We need to make sure that topology change through shriking cycles is dynamically consistent.