Extended Moduli Spaces

and a corresponding Moduli Space Size Conjecture

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based on work with **Philipp Henkenjohann** and **Lukas Witkowski**

Outline

- Recall the Weak Gravity Conjecture for axions: $f < M_P$.
- We try to circumvent this extending the moduli space with fluxes ('winding trajectories').
- If we do not address inflation, SUSY-breaking, moduli-stabilization, this can be done very explicitly.
- Nevertheless, a 'Moduli Space Size Conjecture' appears to hold.

Introduction

- The Weak Gravity Conjecture,

Arkani-Hamed/Motl/Nikolis/Vafa '06

$$m < gM_P \qquad \text{or} \qquad \Lambda < gM_P \ ,$$

has recently been revisited by many authors:

Cheung/Remmen; Rudelius; de la Fuente/Saraswat/Sundrum . . . '14

Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler; Bachlechner/Long/McAllister; AH/Mangat/Rompineve/Witkowski; Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala; Harlow; AH/Rompineve/Westphal; . . . '15

Conlon/Krippendorf; Ooguri/Vafa; Freivogel/Kleban; Banks; Danielsson/Dibitetto; . . . . . . '16

Introduction (continued)

- For recent work concerning the derivation of the WGC in various contexts see e.g.

  Cottrell/Shiu/Soler '16
  Fisher/Mogni '17

  Soler/Hebecker '17

  Hod '17

Motivation (continued)

- A particularly timely aspect of it is the axionic case,

$$g \equiv 1/f \ ,$$

  relevant for natural inflation.

- Another important motivation:
  Learning general lessons about quantum gravity.

- Expect relations to Ooguri-Vafa swampland conjecture
  ['Going long distances in moduli space lowers the cutoff exponentially.']

Ooguri/Vafa, '05, '06

(see also Klaewer/Palti, '16)

Let us first recall the Generalized WGC:

General Action: $$S \sim \int_d \frac{1}{g^2} F_{p+1}^2 + \int_p A_p + T \int_p (*1)$$

WGC: $$g > \frac{T}{M_P^{d/2-1}} \sim \frac{\Lambda^p}{M_P^{d/2-1}}$$

- Specifically for an axion in $d = 4$ this implies

$$\frac{1}{f} > \frac{S_{inst.}}{M_P} \qquad \text{or even} \qquad f < M_P \,.$$

- This case is very special since the cutoff Λ drops out. But this is too quick – we will see at the end that Λ makes a comeback.

It is known that:

- $f < M_P$ is consistent with all simple stringy examples.

  Banks et al. '03

- It is consistent in spirit with the swampland conjecture ('no large distances in moduli space').

  see especially Klaewer/Palti '16

- It is challenged by Monodromy.

  McAllister/Silverstein/Westphal

- It is also challenged by KNP.

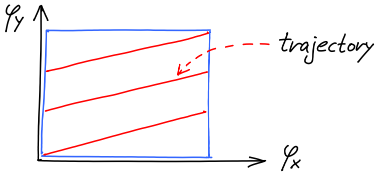
  Kim/Nilles/Peloso '04

- Here, we want to use the 'Winding inflation' realization of the last idea to see whether we can beat the WGC for axions.

  AH/Mangat/Rompineve/Witkowski

- Even in a small field space a long trajectory can be realized if the potential is appropriate.

Kim/Nilles/Peloso '04 (Berg/Pajer/Sjors '09; Ben-Dayan/Pedro/Westphal '14)



- The possibly simplest way to achive this is via gauging à la Dvali (cf. also KS/KLS), as in 'Winding Inflation'.

AH/Mangat/Rompineve/Witkowski '14

$$|F_0|^2 \quad \to \quad |F_0 + \varphi_x + N\varphi_y|^2$$

- This is can be realized very explicitly in the flux landscape, with N being the flux number.

An Aside:

- Recently, the same gauging idea of Dvali has been discussed as a way to evade the WGC for 1-forms.

  Saraswat '16

- Our personal feeling is that

  (a) This is very interesting to explore further.

  (b) In the end, it won't work since the UV theory will not permit $N \gg 1$ together with $\Lambda \sim M_P$, as required.

- The technical reason might be as follows:

$$N \gg 1 \;\Rightarrow\; \text{Ratio of certain radii is large (e.g. } R_A/R_B \gg 1\text{)}$$
$$\Rightarrow\; \Lambda \ll M_P.$$

(This logic is not applicable in the axionic case since Λ does not enter. We may have an interesting answer to this....)

Our example:

- Type IIB on $T^6/\mathbb{Z}_2$ with 64 O3 planes.
- Using standard technology, we can generate

$$W = (M\tau_1 - N\tau_2)\,(\tau - \tau_3)$$

Kachru/Schulz/Trivedi '02
Gomis/Marchesano/Mateos '05
...

(The explicit F_3/H_3 will appear in a moment.)

- $D_{\tau_i} W = 0$ ensure $W = 0$ together with

$$M\tau_1 = N\tau_2 \qquad \text{and} \qquad \tau = \tau_3\,.$$

- If, for example, $M = 1$, $N \gg 1$, this gives exactly our previous winding picture with

$$\varphi_x \equiv \mathrm{Re}\tau_1 \qquad \text{and} \qquad \varphi_y \equiv \mathrm{Re}\tau_2\,.$$

Comments:

- Many authors have considered monodromy & backreaction.
- Back-reaction induced, logarithmic limits on field-space distances have been in particular been suggested by

Klaewer/Palti '16

- What we do here is very different:

(1) No real monodromy – just an extended peridic field space.
(2) No backreaction – our field space is 'SUSY Minkowski'.

Still, a logarithm will emerge...

- Recent work related in spirit includes...

Bielleman/Ibanez/Valenzuela '15
Conlon/Krippendorf '16

- We will ignore τ, τ_3 and all Kahler moduli.
  (We do not care about pheno - only about the WGC.)

- On the 4-dimensional τ_1/τ_2 moduli space, we have the constraint $\tau_1 = N\tau_2$.

- Parameterize the remaining 2-dimensional space using just τ_1:

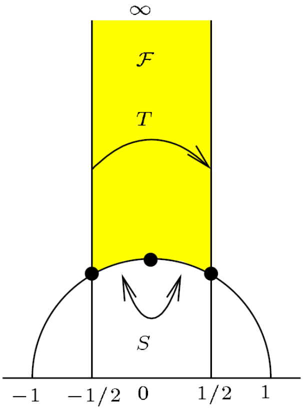
$$\mathcal{L} \quad \supset \quad \frac{(\partial\tau_1)^2}{|\tau_1 - \overline{\tau}_1|^2} + \frac{(\partial\tau_2)^2}{|\tau_2 - \overline{\tau}_2|^2} \quad \sim \quad \frac{(\partial\varphi)^2}{\mathrm{Im}\tau_1^2}$$

  with $\varphi \equiv \mathrm{Re}\tau_1 \in (-N/2, N/2)$.

- With the tadpole constraint $MN \leq 16$, this allows us $N = 16$ and hence, with $\mathrm{Im}\tau_1 \simeq 1$ we get $f_{\mathrm{eff}}/M_P \simeq 16$.

  (Much more should be doable on CYs in the large-complex-structure limit.)

- Before claiming victory, we should revisit the other moduli.
- Dismissing τ, τ_3 and Kahler moduli may be OK – their spaces factorize. But $\text{Im}\tau_1$ is really part of our game...
- Most naively, τ_1 describes T^2 and lives in the fundamental domain of $SL(2, \mathbb{Z})$.
- Of course, we already know that the horizontal periodicity must somehow be enlarged N times.



- To make this explicit, let us spell out the flux:

$$
\begin{aligned}
F_3 &= (-M\,dx_1 \wedge dy_2 + N\,dy_1 \wedge dx_2) \wedge dx_3 = +A \wedge dx_3 \\
H_3 &= (+M\,dx_1 \wedge dy_2 - N\,dy_1 \wedge dx_2) \wedge dy_3 = -A \wedge dy_3\,.
\end{aligned}
$$

- The 2-form A lives only on the first two tori:

$$
A = A_{ij}\,d\xi_1^i \wedge d\xi_2^j \qquad \text{with} \qquad \xi_{1,2}^i = \begin{pmatrix} y_{1,2} \\ x_{1,2} \end{pmatrix}.
$$

- The essential flux information is in the matrix

$$
A_{ij} = \begin{pmatrix} 0 & N \\ -M & 0 \end{pmatrix}.
$$

- Under $R_1 \in SL(2,\mathbb{Z})$, the T_1^2 and the flux transform as

$$\tau_1 \quad \to \quad \tau_1' = R_1(\tau_1) = \frac{a\tau_1 + b}{c\tau_1 + d}\,,$$

and

$$A \quad \to \quad A' = R_1 A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & N \\ -M & 0 \end{pmatrix}.$$

- To map this back to the original configuration, we need $R_2 \in SL(2,\mathbb{Z})$ of T_2^2:

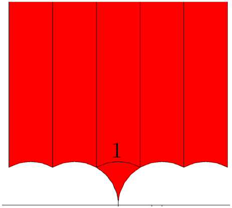
$$A' = R_1 A R_2^T = A$$

- But this is only possible if $b = 0\,(\mathrm{mod}\ N)$ and $c = 0\,(\mathrm{mod}\ M)$.

- In mathematical terms: R_1 must be in one of the Congruence Subgroups of $SL(2,\mathbb{Z})$.

- These subgroups have a larger fundamental domain, corresponding to the Extended Moduli Spaces of fluxed tori.

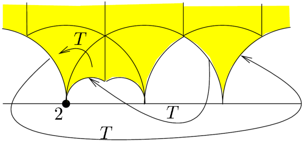
- As a simple example, consider $M = 1$ and $N = 5$, leading to the congruence subgroup $\Gamma^0(5)$ with fundamental domain:



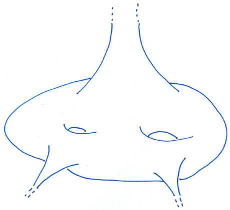
- The horizontal extension at $\mathrm{Im}\tau_1 \gg 1$ was of course expected, but the structure near the real axis can be complicated...

- To appreciate this, consider e.g. part of the domain of $\Gamma^0(7)$, with the appropriare identifications indicated:

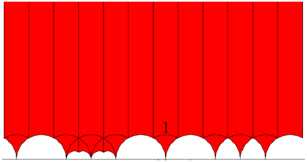
Helena A. Verrill, 2001
see also her code 'fundomain'



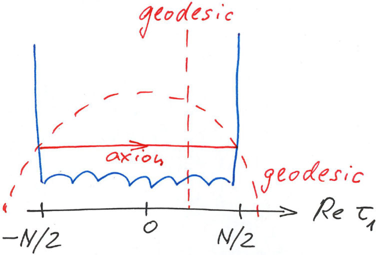
- A sketch of the actual full geometry of such extended moduli spaces might look as follows:



- Let us finally look at a case where the 'upper' throat (cusp) is extended even more, $N = 12$.



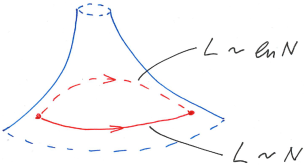
- One can clearly feel uneasy about our extended axionic direction: It is very different from a geodesic.



- Indeed, the distance between two maximally separated points on the longest axion-trajectory grows $\sim N$.

- By contrast, the actual (geodesic) distance grows only $\sim \ln N$.

- This is not too surprising: Our geometry is locally always that of the hyperbolic plane.



- I skip further analytical work (the paper is in the process of being written) and formulate our precise conjecture...

- Choose an $\epsilon \ll 1$. Restrict the moduli space of a given model by demanding $\Lambda / M_P > \epsilon$.

  [Masses of KK or string states should not fall below Λ. This cuts off the infinite throats at a distance $\sim \ln(1/\epsilon)$.]

- Moduli Space Size Conjecture:
  The resulting moduli space has physical diameter $\lesssim \ln(1/\epsilon)$.

  This requires a number of comments....

- First, concerning distances along the throat, this is basically the Ooguri-Vafa swampland conjecture.

- Second, concerning axionic directions without flux, this is just 'Banks et al.'

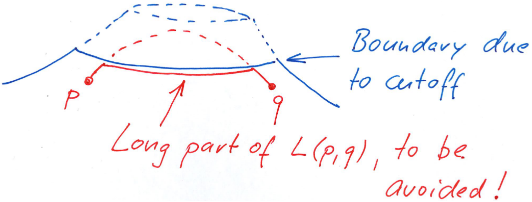
- But, including axionic directions and fluxes, this may be new and interesting, also mathematically (cf. congruence subroups and their domains).

- Finally, our term 'physical diameter' D has to be discussed.
- First, as in math,

$$D \equiv \sup_{p,q} \inf_{L} \int_{L(p,q)} ds \, ,$$

where $L(p,q)$ is a smooth curve connecting points p and q.

- But second, in contrast to the standard math definition, we allow for curves L which jump from one boundary point to another.



- In this way, we are sure that raising ϵ does not make the manifold larger.

Summary/Conclusions

- Axionic directions may be extended in fluxed geometries, in apparent conflict with the WGC.
- But the corresponding, appropriately defined, moduli-space distances do not grow faster than logarithmic.
- This can be formalized in a Moduli Space Size Conjecture.
- Interesting mathematical structures (fundamental domains of congruence subgroups) arise as descriptions of the relevant Flux-Extended Moduli Spaces.
- The fate of large field inflation entirely depends on effects destroying the moduli space (instantons, SUSY breaking).