From the Weak Gravity Conjecture to Wormholes and Baby Universes

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including original work with

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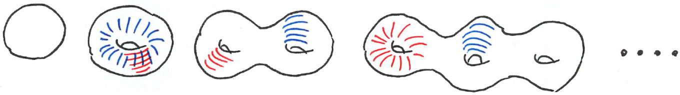
and work in progress with P.Soler/T.Mikhail and D.Junghans/E.Palti/A.Schachner

Outline

- The Landscape/Swampland paradigm and the The Weak Gravity Conjecture
- Applications (especially to Large-Field Inflation)
- Circumventing the Weak Gravity Conjecture
- Gravitational Instantons and Wormholes

Landscape vs. Swampland

- The superstring in $d = 10$ is a remarkably consistent and elegant candidate for a theory of quantum gravity.
- However, in $d = 4$ one faces a flux-induced, exponentially large number of solutions (EFTs).



Bousso/Polchinski '00, Giddings/Kachru/Polchinski '01 (GKP)
Kachru/Kallosh/Linde/Trivedi '03 (KKLT), Denef/Douglas '04

Landscape vs. Swampland (continued)

- While the simplest solutions are SUSY-Minkowski or -AdS, there is strong evidence for meta-stable de-Sitter vacua.

- Personal expectation: This is not 'going away' due to some ovelooked fine technical point.

Comment:

- recently, generic objections against meta-stable dS vacua have been raised.

  Vafa et al. '18

- Indeed, we could be overlooking deep issues in all uplifting proposals and the dS landscape could be an illusion.
- However, I am very confused by the idea that an extra light quintessence field could save the day.
- I know of no example where such a field makes things simpler.
- Thus, the logical conclusion for me would be to drop string theory. But for this I do not see strong enough arguments.

Landscape vs. Swampland (continued)

- While the simplest solutions are SUSY-Minkowski or -AdS, there is strong evidence for meta-stable de-Sitter vacua.
- Personal expectation: This is not 'going away' due to some ovelooked fine technical point.
- The 'old' number ($\sim 10^{500}$)
  has recently been significantly updated to $\sim 10^{272,000}$

  Taylor/Wang '15
- Eternal inflation ('bubbles nucleating within other bubbles') appears to populate all those vacua.
- Yet, due to the measure problem, we do not know even in principle how to make (even just statistical) predictions.

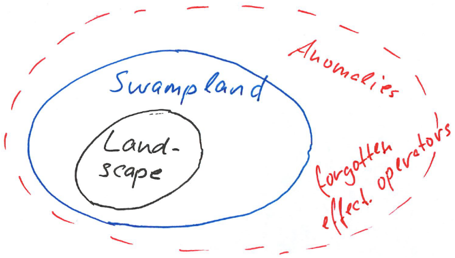
Landscape vs. Swampland (continued)

- Thus, while we must keep struggling with the above problems, a different question comes to mind:

  Does ‘anything go’ in the landscape?

  Are there general criteria for a given model not to be in the landscape?

  Can we formulate and prove such citeria in ‘consistent quantum gravity’ (rather than specifically in string theory)?



Concrete 'Swampland Criteria'

- Specific quantum-gravity consistency citeria have been discussed since a long time ....

No exact global symmetries

Completeness

see e.g. Banks/Seiberg '10 and refs. therein

[the charge lattice is fully occupied]

The swampland conjecture

[infinite distances in moduli space come with exponentially light states]

Vafa '05, Ooguri/Vafa '06

The weak gravity conjecture

Arkani-Hamed/Motl/Nicolis/Vafa '06

- If any of those criteria were relevant experimentally...
  $\rightarrow$ unique opportunity to confront quantum gravity & reality!

The weak gravity conjecture (WGC)

- Roughly speaking: 'Gravity is always the weakest force.'
- More concretely (mild form):

  For any U(1) gauge theory there exists a charged particle with

  $$\boxed{q/m > 1} \qquad \text{(with } q = gQ \text{ and } M_P = 1\text{)}\,.$$

- Strong form:

  The above relation holds for the lightest charged particle.

- Cutoff form:

  The weakly-coupled 4d EFT breaks down at $\Lambda \sim g \equiv gM_P$.

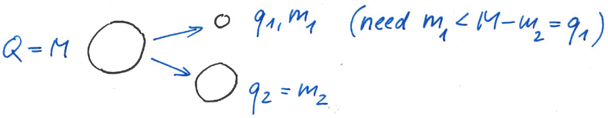
Weak gravity conjecture (continued)

- The historical supporting argument:

  In the absence of sufficiently light, charged particles, extremal BHs are stable. Such remnants are believed to cause inconsistencies.

  see e.g. Susskind '95

  Indeed, the boundary of stability of extremal black holes is precisely $q/m = 1$ for the decay products.

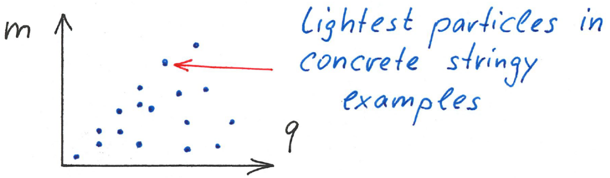


Weak gravity conjecture (continued)

- Another (possibly stronger?) supporting argument:

  Quantum gravity forbids global symmetries. We should not be able to take the limit of small gauge couplings.

  The WGC quantifies this on the basis of stringy examples.



Weak gravity conjecture (continued)

- Several 'versions' exist:

  The strong 'lightest particle' version

  The strong 'minimal charge' version

  The (sub-)lattice version

  Heidenreich/Reece/Rudelius '15

  The geometric version

  AH/Rompineve/Westphal/Witkowski '15

- Some of them have counterexamples, but it may still be true that they can not be violated parametrically

Weak Gravity Conjecture (continued)

- For recent work concerning the derivation of the WGC in various contexts see e.g.

Cheung/Remmen '14 / Cheung/Liu/Remmen '18
['Infrared consistency' / q/m-ratio of small extremal BHs]

Harlow '15 [AdS/CFT, two entangled CFTs]

Cottrell/Shiu/Soler '16
Fisher/Mogni '17 [BH entropy]

Soler/Hebecker '17 [paradox in axionic BH evaporation]

Crisford/Horowitz/Santos '17 [cosmic censorship]

Hod '17 ['universal relaxation bound']

(Saraswat '16; Montero '17) [favoring 'superweak' version]

- It is not obvious how the WGC could impact phenomenology.

- Some interesting proposals have been made

  See, in particular, talk by. L. Ibanez and the recent work of his group.

  (WGC $\rightarrow$ vacuum stability $\rightarrow$ '3d SM' $\rightarrow$ SM spectrum)

  Ooguri/Vafa '16

- One of the widely accepted applications is to constraining large-field inflation.

- Again, the connection is not obvious, but will become clear in few slides ...

  Cheung/Remmen; de la Fuente/Saraswat/Sundrum . . . '14
  Rudelius; Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler/..
  ..Staessens/Ye; Bachlechner/Long/McAllister; AH/Rompineve/Witkowski;
  Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala;
  Harlow; AH/Rompineve/Westphal; . . . '15
  Ooguri/Vafa, Conlon/Krippendorf . . . '16
  Dolan/Draper/Kozaczuk/Patel; AH/Henkenjohann/Witkowski/Soler . . . '17

Introduction: slow-roll inflation

Starobinsky '80; Guth '81
Mukhanov/Chibisov '81; Linde '82

- The simplest relevant action is

$$S = \int d^4x \sqrt{g} \left[\frac{1}{2} R[g_{\mu\nu}] + \frac{1}{2} (\partial\varphi)^2 - V(\varphi) \right] .$$

(We use $M_P \equiv 1$ here and below.)

- (Slow-roll) inflation requires

$$\epsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2 \ll 1 \quad \text{and} \quad |\eta| = \left| \frac{V''}{V} \right| \ll 1 .$$

- To gain some intuition, assume that

$$V \quad \sim \quad \varphi^n \quad \text{or} \quad \ln(\varphi) \qquad \text{(or some combination thereof).}$$

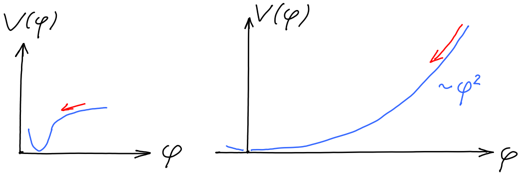
- This implies

$$\epsilon \sim \eta \sim 1/\varphi^2 \,,$$

  such that inflation is generic if $\varphi \gg 1$.

- As a result, one can roughly distinguish

Small- and Large-Field Models



- Small field: $V(\varphi)$ has some tuned very flat region.

- Large field: 'Generic' potentials.

  But: $\Delta\varphi \gg 1$ may lead to problems with quantum gravity.

Recently, the focus has been on large-field models
for two reasons....

1) Observations

- Recall the relation of tensor-to-scalar ratio and field-range:

$$r \equiv \frac{\Delta_T^2}{\Delta_R^2} = 16\epsilon \quad \Leftrightarrow \quad \Delta\varphi \simeq 20\sqrt{r}$$

Lyth '96

- The Planck/BICEP bounds are now somewhere near $r \simeq 0.07$.
- This will improve and we will see the discovery or demise of large-field models.
- If we manage (see below) to show that string theory forbids $\Delta\varphi > 1$, we can hope to rule out string theory!

...reasons for interest in large-field models...

2) Fundamental

- On the one hand, large-field models are more 'robust'

- On the other hand, there may be generic arguments against large-field models in consistent quantum gravity theories

  see e.g. Arkani-Hamed/Motl/Nicolis/Vafa '06 .... Conlon '12

  ....

  see however Kaloper/Kleban/Lawrence/Sloth '15

- This goes hand in hand with persistent problems in constructing large-field models in string theory.

- However, triggered by BICEP and bulding on earlier proposals

  Kim, Nilles, Peloso '07
  McAllister, Silverstein, Westphal '08
  Kaloper, Sorbo '08

  new promising classes of stringy large-field models have been constructed (e.g. F-term axion monodromy)

  Marchesano, Shiu, Uranga '14
  Blumenhagen, Plauschinn '14
  AH, Kraus, Witkowski '14

- At the same time, there are ongoing efforts to sharpen the 'no-go arguments' as well as to refute them

  Rudelius '14...'15
  Ibanez, Montero, Uranga, Valenzuela '15
  Brown, Cottrell, Shiu, Soler '15
  AH, Mangat, Rompineve, Witkowski '15
  ...

- This debate has now been ongoing for several years....

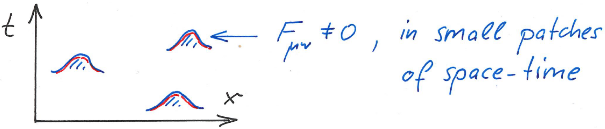
Natural (axionic) inflation in string theory

Freese/Frieman/Olinto '90

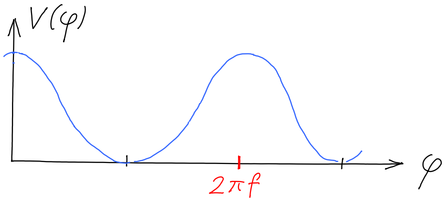
- In 4d effective theories of string compactifications, axion-like fields are abundant:

$$\mathcal{L} \supset -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{32\pi^2}\left(\frac{\varphi}{f}\right)\mathrm{tr}(F\tilde{F})\,.$$

- The shift symmetry is generically broken by instantons:



$$\Rightarrow \quad V_{eff} \sim \cos(\varphi/f)\,, \qquad \varphi \equiv \varphi + 2\pi f\,.$$



- **Problem:** $f \ll 1$ in perturbatively controlled regimes.

- **Illustration:** 5d $\rightarrow$ 4d compactification with $\varphi \sim \int_{S^1} A_5$

  One finds $f \sim 1/R$, such that perturbative control restricts one to sub-planckian f.

- Based on many stringy examples,
  this appears to be a generic result (cf. Banks et al.)

- Three ideas about how to enlarge the axionic field range without losing calculational control:

  (a) KNP   Kim/Nilles/Peloso '04

  (b) N-flation   Dimopoulos/Kachru/McGreevy/Wacker '05

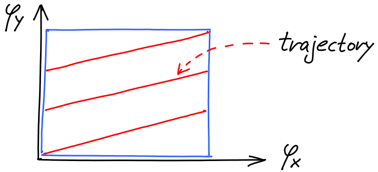
  (c) Axion-Monodromy   McAllister/Silverstein/Westphal '08

- No-Go arguments (to be explained) challenge these possibilities.

(a) KNP / Winding inflation

Kim/Nilles/Peloso '04; Berg/Pajer/Sjors '09; Ben-Dayan/Pedro/Westphal '14

- Consider a 'winding' trajectory on a 2d periodic field space:



- Clearly, such a trajectory can be much longer than the (naive) field range
- But: It is hard to realize the required potential in concrete string models
- Thus, even getting only an effective trans-planckian axion appears to be difficult. Is there a fundamental reason?

...to see this, the previously discussed WGC needs to be generalized:

Generalizations of the weak gravity conjecture

- The basic lagrangian underlying the above is

$$S \sim \int (F_2)^2 + m \int_{1-dim.} d\ell + q \int_{1-dim.} A_1 \,.$$

- This generalizes to charged strings, domain walls etc. Crucially, the degree of the corresponding form-field (gauge-field) changes:

$$S \sim \int (F_{p+1})^2 + m \int_{p-dim.} dV + q \int_{p-dim.} A_p$$

with

$$F_{p+1} = dA_p \,.$$

Generalizations to instantons

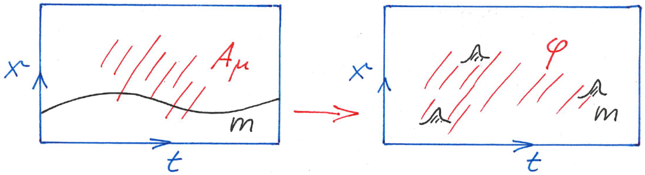
- One can also lower the dimension of the charged object, making it a point a in space-time:

$$S \;\sim\; \int (d\varphi)^2 \;+\; m \;+\; q\,\varphi(x_{inst.})\,.$$

This should be compared with

$$\text{cf.}\quad S \sim \int (d\varphi)^2 + \int \mathrm{tr}(F^2) + \int \left(\frac{\varphi}{f}\right) \mathrm{tr}(F\tilde{F})\,,$$

$$\text{where}\quad \int \mathrm{tr}(F^2) \;\sim\; S_{inst.} \;\sim\; m\,.$$



WGC for instantons and inflation

- The consequences for inflation are easy to derive.

- First, recall that the instantons induce a potential

$$V(\varphi) \sim e^{-m} \cos(\varphi/f)\,.$$

- Since, for instantons, $q \equiv 1/f$, we have

$$q/m > 1 \qquad \Rightarrow \qquad \boxed{m\,f < 1}\,.$$

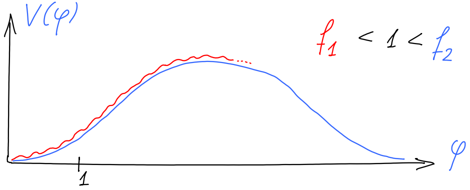
- Theoretical control (dilute instanton gas) requires $m > 1\,.$

- This implies $f < 1$ and hence
  large-field 'natural' inflation is in trouble.

A Loophole

Rudelius, Brown/Cottrell/Shiu/Soler, '15

- Suppose that only the mild form of the WGC holds.
- In this case, we can have one 'sub-planckian' instanton maintaining the WGC, together with a lighter 'super-planckian' instanton realizing inflation:



For other arguments and loopholes see e.g.
de la Fuente, Saraswat, Sundrum '14
Bachlechner, Long, McAllister '15.

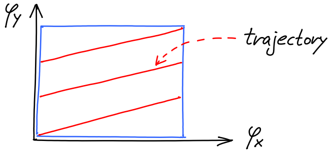
String theory appears to realize this loophole...

AH/Mangat/Rompineve/Witkowski '15

- The fields φ_x and φ_y are two 'string theory axions', both with $f < 1$ (obeying the WGC).
- They are also moduli. Hence, fluxes (e.g. $\langle F_3 \rangle \neq 0$ on the compact space) can be used to stabilize them.
- A judicious flux choice allows for stabilizing just one linear combination, forcing the remaining light field on the winding trajectory:

$$V \supset (\varphi_x - N\varphi_y)^2 + e^{-M}\cos(\varphi_x/f) + e^{-m}\cos(\varphi_y/F)$$

with $N \gg 1$.



for an alternative approach
by Shiu/Staessens/Ye see below...

Concrete realization at (partially) large complex stucture

- Let $z_1, \cdots, z_n, u, v$ be complex structure moduli of a type-IIB orientifold, let $\text{Im}(u) \gg \text{Im}(v) \gg 1$.

$$K = -\log\left(\mathcal{A}(z, \bar{z}, u - \bar{u}, v - \bar{v}) + \mathcal{B}(z, \bar{z}, v - \bar{v})e^{2\pi i v} + \mathrm{c.c.}\right)$$

$$W = w(z) + f(z)(u - Nv) + g(z)e^{2\pi i v}$$

- Without exponential terms, it is clear that W leaves one of the originally shift-symmetric directions $\text{Re}(u)$ and $\text{Re}(v)$ flat
- If $N \gg 1$, this direction is closely aligned with $\text{Re}(u)$
- The exponential terms induce a long-range cosine potential for this light field φ:

$$e^{2\pi i v} \quad \rightarrow \quad \cos(2\pi\varphi/N)$$

Let us take a

Conceptual view on the above 'Winding Inflation' model

To do so, recall how gauging/Higgsing works in general:

$$\textbf{(}p\textbf{)} \qquad \mathcal{L}_p \;=\; \int_d |F_{p+1}|^2 \quad \text{with} \quad F_{p+1} = dA_p$$

$$\textbf{(}p-1\textbf{)} \qquad \mathcal{L}_{p-1} \;=\; \int_d |F_p|^2 \quad \text{with} \quad F_p = dA_{p-1}$$

$$\textbf{(Higgsed)} \qquad \mathcal{L}_{p\,/\,p-1} \;=\; \int_d \; |F_{p+1}|^2 + |F_p + A_p|^2 \,.$$

The most familiar example is, of course, $p = 1$ and $p - 1 = 0$:

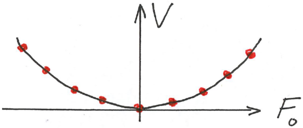
$$\textbf{(Higgsed)} \qquad \mathcal{L}_{1\,/\,0} \;=\; \int_d \; |F_2|^2 + |d\varphi + A_1|^2 \,.$$

Conceptual view (continued)

- The above includes the slightly special case of (-1)-forms:

  $$(p=-1) \qquad \mathcal{L}_{-1} \;=\; \int_d |F_0|^2 \qquad \text{where, by flux quantization,}$$

  $$F_0 \in \alpha \times \mathbb{Z}$$



- All the dynamics is a discrete set of vacua with domain walls coupled to A_3 of $*F_0 = F_4 = dA_3$.

- Crucially, one can use this theory to Higgs a 0-form, i.e. an axion

  Dvali '05; Kaloper/Sorbo '08
  (also: Quevedo/Trugenberger '97)

$$\mathcal{L}_{0\,/\,-1} \;=\; \int_d |F_1|^2 + |F_0 + A_0|^2 \;=\; \int_d (\partial\varphi)^2 + |F_0 + \varphi|^2$$

Conceptual view (continued)

$$\mathcal{L}_{0\,/\,-1} \;=\; \int_d (\partial\varphi)^2 + |F_0 + \varphi|^2$$

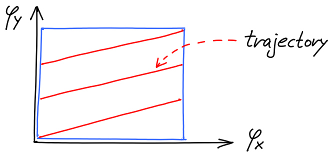
- This is of course just 'the gauge-theory perspective' on axion monodromy.
- Since 'turning on fluxes corresponds to gauging', the flux landscape gives mass to the (axionic components of) moduli in precisely this way.
- In 'Winding Inflation' we used $W \sim u - Nv$,
with complex-structure moduli u and v.
- This corresponds to Higgsing a specific linear combination of $\varphi_x = \text{Re}\, u$ and $\varphi_y = \text{Re}\, v$:

$$\mathcal{L}_{0\,/\,-1} \;=\; \int_d (\partial\varphi_x)^2 + (\partial\varphi_y)^2 + |F_0 + \varphi_x + N\varphi_y|^2$$

Conceptual summary of Winding Inflation

$$\mathcal{L}_{0\,/\,-1} \;=\; \int_d (\partial\varphi_x)^2 + (\partial\varphi_y)^2 + |F_0 + \varphi_x + N\varphi_y|^2$$

- The underlying idea it to generate a transplanckian axion by Higgsing a linear combination of two subplanckian axions.



- Whether this will actually work for inflation is still under disussion

see e.g. Palti '15 and Blumenhagen/Herschmann/Wolf '16

also: Shiu/Staessens/Ye '15, Shiu/Staessens '18

An Aside:

- Return, for a moment, to the more conventional WGC with 1-forms rather than 0-forms (axions).

- Here, the same gauging idea can apparently be used.

Saraswat '16

$$\mathcal{L}_{1\,/\,0} \;=\; \int_d (F_x)^2 + (F_y)^2 + |d\varphi + A_x + NA_y|^2$$

with $F_x = dA_x$, $F_y = dA_y$,

and with the surviving light gauge field $A_{\rm light} \sim NA_x - A_y$ having gauge coupling

$$g_{\rm light} \sim \frac{1}{\sqrt{N^2+1}} \,.$$

- This is very interesting to explore further!
- In the end, it might fail since the UV theory will not permit $N \gg 1$ together with $\Lambda \sim M_P$, as required.
- A technical problem might arise as follows:

$$N \gg 1 \quad \Rightarrow \quad \text{Ratio of certain radii is large (e.g. } R_A/R_B \gg 1\text{)}$$
$$\Rightarrow \quad \Lambda \ll M_P.$$

(This logic is not applicable in the axionic case since Λ does not enter. More on this will follow shortly....)

- If, however, Higgsing can indeed avoid the WGC, it might even do so exponentially due to the clockwork mechanism.

Choi/Kim/Yun/Im, Kapla/Rattazzi, ...
(see, however, Ibanez/Montero '17)

- motivated by the above, let us now turn to a more modest and maybe more treatable set of problems:

Extended Moduli Spaces
and a corresponding Moduli Space Size Conjecture

Idea:

- It is thought that Quantum Gravity (e.g. via the WGC) implies $f < M_P$.
- We try to circumvent this extending the moduli space with fluxes ('winding trajectories').
- If we do not address inflation, SUSY-breaking, moduli-stabilization, this can be done very explicitly.
- Still, a 'Moduli Space Size Conjecture' appears to hold.

cf. Ooguri/Vafa, Palti, Palti/Grimm/Valenzuela, ...

A simple, torus-based model for transplanckian axions

(toy-model for winding inflation)

- Type IIB on $T^6/\mathbb{Z}_2$ with 64 O3 planes.
- Using standard technology, we can generate

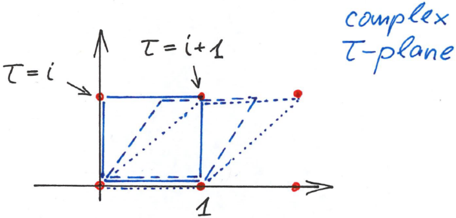
  $$W = (M\tau_1 - N\tau_2)\,(\tau - \tau_3)$$

  Kachru/Schulz/Trivedi '02
  Gomis/Marchesano/Mateos '05
  ...

  (The explicit F_3/H_3 is easy to state.)

- In the interests of time, the rest will be described in pictures...

- Recall that a torus can be viewed as a lattice in $\mathbb{C}$ and its shape is parametrized by $\tau \in \mathbb{C}$.



- There are many identifications
  (e.g. $\tau = i$ and $\tau = i + 1$ correspond to the same torus)

- Moreover, the metric in the τ-plane (both in math in the 4d EFT with a complex modulus field τ) reads

$$ds^2 = \frac{d\tau\, d\overline{\tau}}{4\,(\mathrm{Im}\tau)^2}$$ ‘Hyperbolic plane’

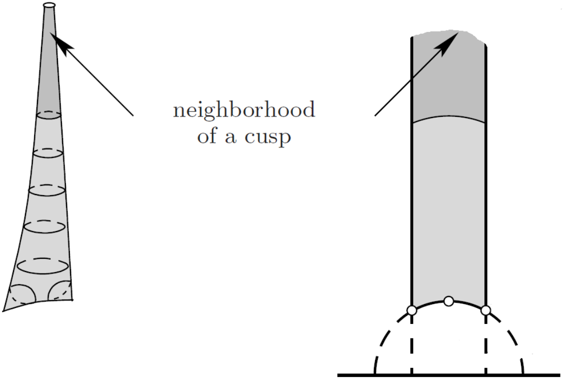
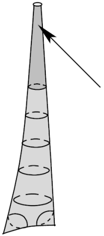


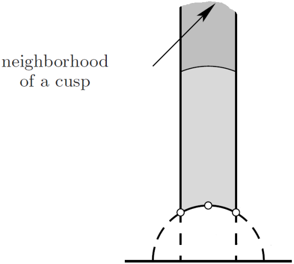
Fig. from A. Zorich, 'Flat surfaces'

- The fundamental domain is an infinitely long, vertical strip with $i \times \infty$ corresponding to a very thin torus.

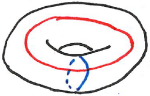
- The modulus space has an infinite extension, but the cutoff comes down exponentially fast if one goes there (due to light winding strings).



- The 'axionic' horizontal direction is at most $\mathcal{O}(1)$ in size ($f \lesssim M_p$)

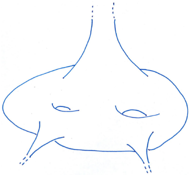


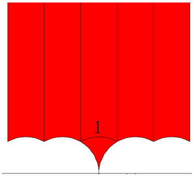
- Now, if the torus carries flux (think of rubber bands marking the cycles), the picture changes.



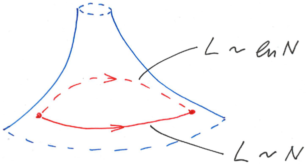
- Some of the identifications are lost and the fundamental domain increases ($\equiv$ fund. domain of congruence subgroups of $SL(2, \mathbb{Z})$).

- The cusp or 'throat' becomes much wider (super-planckian f),





...but the geodesic distances remain short ($\sim \ln(1/\text{cutoff})$)



- We formulate this in a 'moduli space size conjecture' which tries to unify the axionic WGC and Swampland Conjecture

Intermediate summary

- It appears that the swampland conjecture extends in a non-trival way to axions.
- This extension does not preclude transplanckian f.
- Implications for large-field inflation are not a priori negative.
- One needs more detailed explicit stringy models and/or finer conjectures (work in Progress Palti, Junghans, Schachner...) [Crucially: we need to stabilize the saxion!]
- We will next return to 'generic quantum gravity' and how it breaks shift symmetry in a 'conjecture-independent' way ....

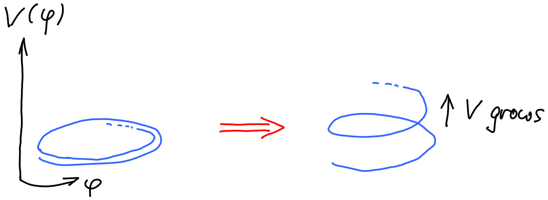
  But before doing so: A digression on Monodromy...

A relevant aside: Monodromy inflation

Silverstein/Westphal/McAllister '08

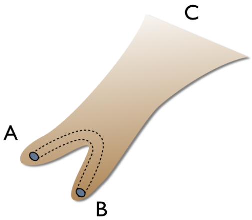
Very general but simple-minded definition:

- Start with a single, shift-symmetric, periodic inflaton φ
- Break the periodicity weakly by the scalar potential



The ‘classical’ model ...

$$S_{\mathrm{NS5}} \sim \int \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu} + C_{\mu\nu})}$$



Bifid throat with shared 2-cycle
(figure from Retolaza et al. ’15)

... has issues with the explicit geometry and quantitative control.

For recent progress see e.g. McAllister/Silverstein/Westphal/Wrase ’14
...
Retolaza/Uranga/Westphal ’15

F-term axion monodromy

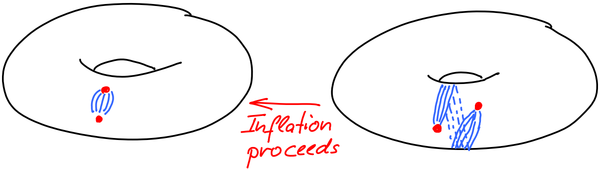
- More recently, classes of monodromy models with 4d supergravity description and stabilized compact space have emerged.

  Marchesano/Shiu/Uranga '14
  Blumenhagen/Plauschinn '14
  AH/Kraus/Witkowski '14

- One option is that inflation corresponds to brane-motion

  Dvali/Tye '98....Dasgupta et al. '02....Lüst et al. '11

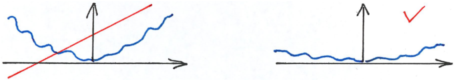
- The monodromy arises from a flux sourced by the brane



Challenges in axion monodromy

- It remains controversial whether one can (e.g by tuning) make the monodromy as small as necessary for moduli stabilization

  cf. recent work by Blumenhagen, Valenzuela, Palti, Marchesano,... (and by our group)



- The WGC applies only indirectly (in its domain-wall version), but the constraints are not strong enough for inflation

  Brown/Cottrell/Shiu/Soler, Ibanez/Montero/Uranga/Valenzuela, AH/Rompineve/Westphal '15

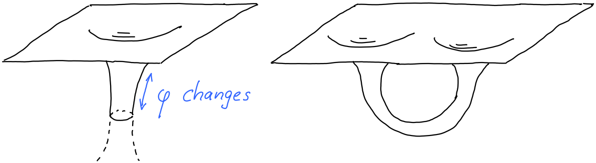
- It has been attempted to use the Swampland Conjecture to argue against axion monodromy inflation

  Baume/Palti, Klaewer/Palti '15 ... '16

A distinct but WGC-related tool: (Gravitational) instantons

- In Euclidean Einstein gravity, supplemented with an axionic scalar φ , instantonic solutions exist:

Giddings/Strominger '88
...



- The 'throat' is supported by the kinetic energy of $\varphi = \varphi(r)$, with r the radial coordinate of the throat/instanton.

- The relevance for inflation arises through the induced instanton-potential for the originally shift-symmetric field φ.

Montero/Uranga/Valenzuela '15

Gravitational instantons (continued)

- The underlying lagrangian is simply

$$\mathcal{L} \sim \mathcal{R} + f^2 |d\varphi|^2 \,, \quad \text{now with} \quad \varphi \equiv \varphi + 2\pi \,.$$

- This can be dualized $(dB_2 \equiv f^2 * d\varphi)$ to give

$$\mathcal{L} \sim \mathcal{R} + \frac{1}{f^2} |dB_2|^2 \,.$$

- The 'throat' exists due the compensation of these two terms. Reinstating M_P, allowing n units of flux (of $H_3 = dB_2$) on the transverse S^3, and calling the typical radius R, we have

$$M_P^2 \, R^{-2} \sim \frac{n^2}{f^2} R^{-6} \quad \Rightarrow \quad M_P \, R^2 \sim \frac{n}{f} \,.$$

Gravitational instantons (continued)

- Returning to units with $M_P = 1$, their instanton action is

$$S \sim n/f \qquad \text{(with } n \text{ the instanton number).}$$

- Their maximal curvature scale is $\sqrt{f/n}$, which should not exceed the UV cutoff:

$$f/n < \Lambda^2$$

- This fixes the lowest n that we can trust and hence the minimal size of the instanton correction to the potential $V(\varphi)$:

$$\boxed{\delta V \sim e^{-S} \sim e^{-n/f} \sim e^{-1/\Lambda^2}}$$

Gravitational instantons (continued)

- For gravitational instantons not to prevent inflation, the relative correction must remain small:

$$\frac{\delta V}{V} \sim \frac{e^{-1/\Lambda^2}}{H^2} \ll 1$$

- For a Planck-scale cutoff, $\Lambda \sim 1$, this is never possible
- However, the UV cutoff can in principle be as low as H
- Then, if also $H \ll 1$, everything might be fine....

$$\frac{\delta V}{V} \sim \frac{e^{-1/H^2}}{H^2}$$

AH, Mangat, Rompineve, Witkowski '15

- At least for high-cutoff models:

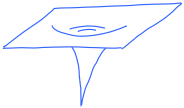
  Can one obtain a reasonably model-independent bound from gravitational instantons?

AH/Mangat/Theisen/Witkowski '16

Note:

- Our analysis also includes the closely related issue of (singular) 'cored instantons', which have been brought up by

Heidenreich, Reece, Rudelius '15



- For recent work on the emedding in string theory see...

Hertog/Trigiante/Van Riet '17

Very rough summary of results

- Look at the case where we expect the strongest bound:
  A string model with $g_s = 1$ on T^6 at self-dual radius.

- Need to decide when to trust a wormhole / extremal instanton

  (i.e., what is the smallest allowed S^3-radius r_c)

The following two choices appear 'natural':

$$2\pi^2 r_c^3 = \mathcal{V}_{self-dual}^{1/2} \quad \Rightarrow \quad r_c M_P \simeq 1.3 \quad \Rightarrow \quad e^{-S} \simeq 10^{-68}$$

$$2\pi r_c = \mathcal{V}_{self-dual}^{1/6} \quad \Rightarrow \quad r_c M_P = \simeq 0.56 \quad \Rightarrow \quad e^{-S} \simeq 10^{-13}$$

Surprisingly weak bounds!

...However, beyond inflation, wormholes remain very interesting, both conceptually and phenomenologically

Gravitational instantons - Small-f axions

see e.g. Alonso/Urbano '17

- For example, for a QCD axion with (relatively) high f, the wormhole effect might be relevant:

$$V(\varphi) = \Lambda_{QCD}^4 \cos(\varphi) + r_c^{-4} e^{-S_w/2} \cos(\varphi + \delta)\,.$$

- It turns out that for $f \gtrsim 10^{16}$ GeV the solution to the strong CP problem is lost.

- Interesting positive observational consequences exist in the context of black-hole superradiance and ultralight dark matter.

Example: Fuzzy Dark Matter

Alonso/Urbano '17
AH/Mikhail/Soler '18

- Fuzzy Dark Matter is, by definition, so light that its de Broglie wavelength affects sub-galactic-scale structure: $m \lesssim 10^{-21}$ eV

- If wormholes are the universal, model-independent effect of shift symmetry breaking, then this fixes f by the relation

$$m^2 \sim \exp(-1/f)$$

- At the same time, the abundance of Fuzzy Dark Matter is given by

$$\Omega_{FDM} h^2 \approx 0.1 \left(\frac{f}{10^{17}\mathrm{GeV}} \right)^2 \left(\frac{m}{10^{-22}\mathrm{eV}} \right)^{\frac{1}{2}}$$

- Together, these two relations lead to a slight clash (between wormhole/WGC and Fuzzy Dark Matter pheno):

One finds $\quad m \gtrsim 10^{-19}\,\mathrm{eV}\,,\quad$ ...slightly too high...

- Clearly, there are ways around this...

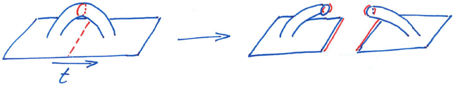
Gravitational instantons / wormholes - conceptual issues

- Motivated by the above, it is worthwhile revisiting some very fundamental conceptual issues of (euclidean) wormholes.

  Hawking '78..'88, Coleman '88, Preskill '89
  Giddings/Strominger/Lee/Klebanov/Susskind/Rubakov/Kaplunovsky/..
  Fischler/Susskind/...

  Recent review: AH, P. Soler, T. Mikhail '18

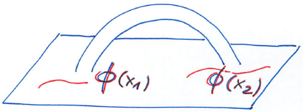
- First, once one allows for wormholes, one has to allow for baby universes.



- Second, with baby universes comes a 'baby universe state' (α vacuum) encoding information on top of our 4d geometry.

Conceptual issues (continued)

- Crucially, α-parameters remove the disastrous-looking bilocal interaction.



$$\exp\left(\int_{x_1}\int_{x_2}\Phi(x_1)\Phi(x_2)\right) \quad \to \quad \int_\alpha \exp\left(-\frac{1}{2}\alpha^2+\alpha\int_x \Phi(x)\right)$$

- In our concrete (single-axion) case, an α parameter now governs the naively calculable $e^{-S}\cos(\varphi/f)$-term.

- But, what is worse, all coupling constants are 'renormalized' by α parameters are hence not predictable in principle.

Conceptual issues (continued)

- Most naively, 4d measurements collapse some of the many α parameters to known constants.

- But in a global perspective, both different 4d geomtries and α parameters have to be integrated over.

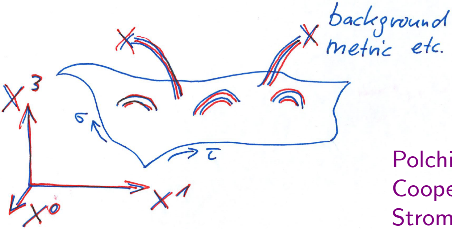
- But this leads to the 'Fischler-Susskind-Kaplunovsky catastrophy'.

- The problem is that, through certain higher operators, high densities of even very large wormholes are rewarded;
  $\rightarrow$ exponential suppression overcome.

- Finally, just integrating over the α parameters is clearly not sufficient - one needs to consider their full quantum dynamics.

Conceptual issues (continued)

- Indeed, consider the case of 1+1 dimensions with a number of scalar fields (in addition to gravity).
- This is, of course, well known as string theory and the α parameters characterize the geometry the target space.



Polchinski, Banks/Lykken/O'Loughlin, Cooper/Susskind/Thorlacius, Strominger '89...'92

- The latter has a quantum dynamics of its own, the analogue of which in case of 3+1 dimensions is completely unknown.
- All this raises so many complicated issues, that one might want to dismiss wormholes altogether.

Conceptual issues (continued)

- But this is not easy, for example because we know that in string theory wormholes correspond to string loops and are a necessary part of the theory.

- Thus, forbidding for example topology change in general does not appear warranted.

- Is there a good reason to forbid topology change just in $d > 2$?

- Arguments have been given that the euclidean Giddings-Strominger solution has negative modes and should hence be dismissed.

  Rubakov/Shvedov '96, Maldacena/Maoz '04, see however Alonso/Urbano '17, ...

- But, while this is even technically still an open issue, it does not appear to be a strong enough objection ....

Conceptual issues (continued)

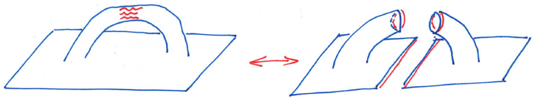
- Indeed, once a non-zero amplitude

  universe $\rightarrow$ universe $+$ baby-universe

  is accepted, the reverse process is hard to forbid.

- As a result, one gets all the wormhole effects.

- The negative mode issue may be saying:
  'Giddings-Strominger' does not approximate the amplitude well.



- ..hard to see, how it would dispose of the problem altogether..

For further problems (and possible resolutions) see e.g.
Bergshoeff/Collinucci/Gran/Roest/Vandoren/Van Riet '04,
Arkani-Hamed/Orgera/Polchinski '07, Hertog/Trigiante/Van Riet '17

Summary/Conclusions

- Thinking in terms of the landscape/swampland paradigm is one promising way of trying to relate quantum gravity and observations.
- Of the relevant 'swampland criteria', the Weak Gravity Conjecture may be the most quantitative (but also most breakable).
- Its axionic (0-form) version promises to constrain large-field inflation, but will this promise be kept?
- Indeed: axionic directions may be extended in fluxed geometries, violating a possible 'subplanckian-f conjecture'.

Summary/Conclusions (continued)

- The **extended** moduli-space-size does not grow faster than logarithmic. Consequences for inflation so far open....

- Euclidean wormholes are the universal, semiclassical counterpart of instantons (they break the axionic shift symmetry independently of the WGC).
- They do not constrain inflation strongly, but may have other phenomenological applications 'at small f'.
- They come at the price of α vacua (and other disasters).
- Worthwhile reviving this fundamental unresolved issue?