Large Field Inflation and the Weak Gravity Conjecture and Gravitational Instantons

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(including work with S. Kraus, D. Lüst, P. Mangat, J. Moritz, F. Rompineve, S. Theisen, A. Westphal, L. Witkowski, ...)

Outline

- Large-field inflation: Generalities
- Large-field inflation: Issues in quantum gravity / strings

  In particular: Weak Gravity Conjecture;
  Gravitational instantons

Slow-roll inflation and perturbations

Starobinsky '80; Guth '81
Mukhanov/Chibisov '81; Linde '82

- The simplest relevant action is

$$S = \int d^4x\sqrt{g}\left[\frac{1}{2}R[g_{\mu\nu}] + \frac{1}{2}(\partial\varphi)^2 - V(\varphi)\right] .$$

(We use $M_P \equiv 1$ here and below.)

- (Slow-roll) inflation requires

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \quad \text{and} \quad |\eta| = \left|\frac{V''}{V}\right| \ll 1 \, .$$

- To gain some intuition, assume that

$$V \quad \sim \quad \varphi^n \quad \text{or} \quad \ln(\varphi) \qquad \text{(or some combination thereof).}$$

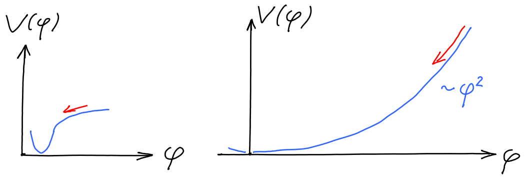
- This implies

$$\epsilon \sim \eta \sim 1/\varphi^2 \,,$$

such that inflation is generic if $\varphi \gg 1$.

- As a result, one can roughly distinguish

Small- and Large-Field Models



- Small field: $V(\varphi)$ has some tuned very flat region.

- Large field: 'Generic' potentials.

  But: $\Delta\varphi \gg 1$ may lead to problems with quantum gravity.

Recently, the focus has been on large-field models
for two reasons....

1) Observations

- The tensor-to-scalar ratio ('primordial gravity waves') is related to the field-range:

Lyth '96

$$r \equiv \frac{\Delta_T^2}{\Delta_R^2} = 16\epsilon \quad \Leftrightarrow \quad \Delta\varphi \simeq 20\sqrt{r}$$

- Even though the 'BICEP hype' went away, the combined Planck/BICEP analysis still sees a ($\sim 1.8\sigma$) hint for $r \simeq 0.05$.
- More importantly: Much better values are expected soon.

...reasons for interest in large-field models...

2) Fundamental

- On the one hand, large-field models are more 'robust'
- On the other hand, there are generic arguments against large-field models in consistent quantum gravity theories

  see e.g. Arkani-Hamed/Motl/Nicolis/Vafa '06 .... Conlon '12
  ....
  Kaloper/Kleban/Lawrence/Sloth '15

- This goes hand in hand with persistent problems in constructing large-field models in string theory.

- However, triggered by BICEP and bulding on earlier proposals

  Kim, Nilles, Peloso '07
  McAllister, Silverstein, Westphal '08

  new promising classes of stringy large-field models have been constructed (e.g. F-term axion monodromy)

  Marchesano, Shiu, Uranga '14
  Blumenhagen, Plauschinn '14
  AH, Kraus, Witkowski '14

- At the same time, there are ongoing efforts to sharpen the 'no-go arguments' as well as to refute them

  Rudelius '14...'15
  Ibanez, Montero, Uranga, Valenzuela '15
  Brown, Cottrell, Shiu, Soler '15
  AH, Mangat, Rompineve, Witkowski '15
  ...

- I will try to explain some aspects of this debate....

Natural (axionic) inflation in string theory

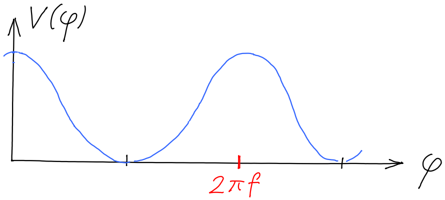
Freese/Frieman/Olinto '90; Banks/Dine/Fox/Gorbatov '03

- The ubiquitous axionic (pseudo-)scalars $(C_0, C_1, \ldots, B_2$ etc.$)$ appear to provide excellent inflaton candidates.

$$\mathcal{L} \supset -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{32\pi^2}\left(\frac{\varphi}{f}\right) \mathrm{tr}(F\tilde{F})\,.$$

- The shift symmetry is generically broken from $\mathbb{R}$ to $\mathbb{Z}$, but only non-perturbatively

$$V_{eff} \sim \cos(\varphi/f)\ , \qquad \varphi \equiv \varphi + 2\pi f\,.$$



- **Problem:** $f \ll 1$ in perturbatively controlled regimes.

- **Illustration:** 5d $\rightarrow$ 4d compactification with $\varphi \sim \int_{S^1} A_5$

  One finds $f \sim 1/R$, such that perturbative control restricts one to sub-planckian f.

- Based on many stringy examples,
  this appears to be a generic result (cf. Banks et al.)

- Three ideas about how to enlarge the axionic field range without losing calculational control:

  (a) KNP   Kim/Nilles/Peloso '04

  (b) N-flation   Dimopoulos/Kachru/McGreevy/Wacker '05

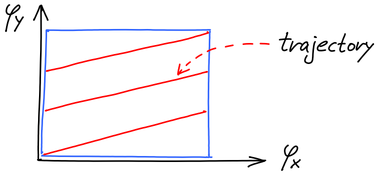
  (c) Axion-Monodromy   McAllister/Silverstein/Westphal '08

- The No-Go arguments alluded to earlier challenge these possibilities.

(a) KNP / Winding inflation

Kim/Nilles/Peloso '04; Berg/Pajer/Sjors '09; Ben-Dayan/Pedro/Westphal '14

- Consider a 'winding' trajectory on a 2d periodic field space:



- Clearly, such a trajectory can be much longer than the (naive) field range

- But: It is hard to realize the required potential in concrete string models

- Thus, even getting only an effective trans-planckian axion appears to be difficult. Is there a fundamental reason?

No-go argument I: Weak gravity conjecture

Arkani-Hamed/Motl/Nicolis/Vafa '06

- Some recent papers developing this in (more or less close) relation to large-field inflation:

  Cheung/Remmen; de la Fuente/Saraswat/Sundrum ...'14
  Rudelius; Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler; Bachlechner/Long/McAllister; AH/Mangat/Rompineve/Witkowski; Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala; Harlow; AH/Rompineve/Westphal; ...'15
  Conlon/Krippendorf ...'16

Weak gravity conjecture

- Roughly speaking: 'Gravity is always the weakest force.'
- More concretely (mild form):

  For any U(1) gauge theory there exists a charged particle with

  $$\boxed{q/m > 1\,.}$$

- Strong form:

  The above relation holds for the lightest charged particle.

Weak gravity conjecture (continued)

- One supporting argument:

  Quantum gravity forbids global symmetries. We should not be able to take the limit of small gauge couplings.

  The WGC quantifies this on the basis of stringy examples.

- Another supporting argument:

  In the absence of sufficiently light, charged particles, extremal BHs are stable. Such remnants are believed to cause inconsistencies.

  see e.g. Susskind '95

  The boundary of stability of extremal black holes is precisely $q/m = 1$ for the decay products.

Generalizations of the weak gravity conjecture

- The basic lagrangian underlying the above is

$$S \sim \int (F_2)^2 + m \int_{1-dim.} d\ell + q \int_{1-dim.} A_1 \,.$$

- This generalizes to charged strings, domain walls etc. Crucially, the degree of the corresponding form-field (gauge-field) changes:

$$S \sim \int (F_{p+1})^2 + m \int_{p-dim.} dV + q \int_{p-dim.} A_p$$

with

$$F_{p+1} = dA_p \,.$$

Generalizations to instantons

- One can also lower the dimension of the charged object, making it a point a in space-time:

$$S \sim \int (d\varphi)^2 \; + m + q\,\varphi(x_{inst.})\,.$$

- One easily recognizes that this is just a more general way of talking about instantons and axions:

$$m \quad \Leftrightarrow \quad S_{inst.} \qquad , \qquad q\,\varphi(x_{inst.}) \quad \Leftrightarrow \quad \frac{1}{f}\int \varphi F\tilde{F}\,.$$

WGC for instantons and inflation

- The consequences for inflation are easy to derive.

- First, recall that the instantons induce a potential

$$V(\varphi) \sim e^{-m} \cos(\varphi/f) .$$

- Since, for instantons, $q \equiv 1/f$, we have

$$q/m > 1 \qquad \Rightarrow \qquad \boxed{m f < 1} .$$

- Theoretical control (dilute instanton gas) requires $m > 1$.

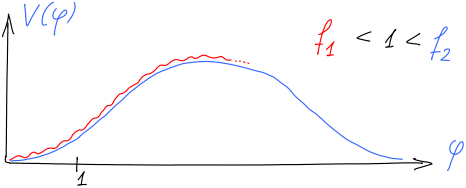
- This implies $f < 1$ and hence
  large-field 'natural' inflation is in trouble.

A Loophole

Rudelius '15

- Suppose that only the mild form of the WGC holds.

- In this case, we can have one 'sub-planckian' instanton maintaining the WGC, together with a lighter 'super-planckian' instanton realizing inflation:



For other arguments and loopholes see e.g.
de la Fuente, Saraswat, Sundrum '14
Bachlechner, Long, McAllister '15.

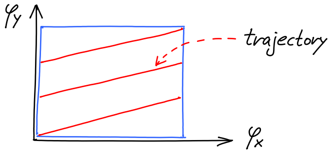
String theory appears to realize this loophole...

AH/Mangat/Rompineve/Witkowski '15

- The fields φ_x and φ_y are two 'string theory axions', both with $f < 1$ (obeying the WGC).
- They are also moduli. Hence, fluxes (e.g. $\langle F_3 \rangle \neq 0$ on the compact space) can be used to stabilize them.
- A judicious flux choice allows for stabilizing just one linear combination, forcing the remaining light field on the winding trajectory:

$$V \supset (\varphi_x - N\varphi_y)^2 + e^{-M}\cos(\varphi_x/f) + e^{-m}\cos(\varphi_y/F)$$

with $N \gg 1$.



Concrete realization at (partially) large complex stucture

- Let $z_1, \cdots, z_n, u, v$ be complex structure moduli of a type-IIB orientifold, let $\mathrm{Im}(u) \gg \mathrm{Im}(v) \gg 1$.

$$K = -\log\left(\mathcal{A}(z,\bar{z},u-\bar{u},v-\bar{v}) + \mathcal{B}(z,\bar{z},v-\bar{v})e^{2\pi i v} + \mathrm{c.c.}\right)$$

$$W = w(z) + f(z)(u - Nv) + g(z)e^{2\pi i v}$$

- Without exponential terms, it is clear that W leaves one of the originally shift-symmetric directions $\mathrm{Re}(u)$ and $\mathrm{Re}(v)$ flat
- If $N \gg 1$, this direction is closely aligned with $\mathrm{Re}(u)$
- The exponential terms induce a long-range cosine potential for this light field φ:

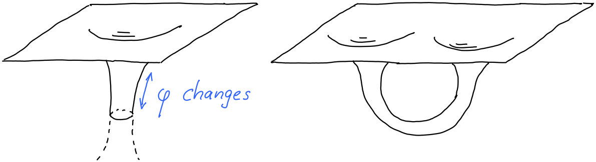
$$e^{2\pi i v} \quad \to \quad \cos(2\pi\varphi/N)$$

No-go argument II: (Gravitational) instantons

- In Euclidean Einstein gravity, supplemented with an axionic scalar φ , instantonic solutions exist:

Giddings/Strominger '88

...



- The 'throat' is supported by the kinetic energy of $\varphi = \varphi(r)$, with r the radial coordinate of the throat/instanton.

- The relevance for inflation arises through the induced instanton-potential for the originally shift-symmetric field φ.

Montero/Uranga/Valenzuela '15

Gravitational instantons (continued)

- The underlying lagrangian is simply

$$\mathcal{L} \sim \mathcal{R} + f^2|d\varphi|^2\,, \quad \text{now with} \quad \varphi \equiv \varphi + 2\pi\,.$$

- This can be dualized $(dB_2 \equiv f^2 * d\varphi)$ to give

$$\mathcal{L} \sim \mathcal{R} + \frac{1}{f^2}|dB_2|^2\,.$$

- The 'throat' exists due the compensation of these two terms. Reinstating M_P, allowing n units of flux (of $H_3 = dB_2$) on the transverse S^3, and calling the typical radius R, we have

$$M_P^2\, R^{-2} \sim \frac{n^2}{f^2} R^{-6} \quad \Rightarrow \quad M_P\, R^2 \sim \frac{n}{f}\,.$$

Gravitational instantons (continued)

- Returning to units with $M_P = 1$, their instanton action is

$$S \sim n/f \qquad \text{(with } n \text{ the instanton number).}$$

- Their maximal curvature scale is $\sqrt{f/n}$, which should not exceed the UV cutoff:

$$f/n < \Lambda^2$$

- This fixes the lowest n that we can trust and hence the minimal size of the instanton correction to the potential $V(\varphi)$:

$$\boxed{\delta V \sim e^{-S} \sim e^{-n/f} \sim e^{-1/\Lambda^2}}$$

Gravitational instantons (continued)

- For gravitational instantons not to prevent inflation, the relative correction must remain small:

$$\frac{\delta V}{V} \sim \frac{e^{-1/\Lambda^2}}{H^2} \ll 1$$

- For a Planck-scale cutoff, $\Lambda \sim 1$, this is never possible
- However, the UV cutoff can in principle be as low as H
- Then, if also $H \ll 1$, everything might be fine....

$$\frac{\delta V}{V} \sim \frac{e^{-1/H^2}}{H^2}$$

AH, Mangat, Rompineve, Witkowski '15

- Now, most stringy models do indeed have a low cutoff (e.g. moduli scale, KK scale)

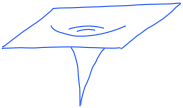
- With this in mind, can one obtain reasonably model-independent bounds from gravitational instantons?

AH/Mangat/Theisen/Witkowski '16

Note:

- Our analysis also includes the closely related issue of (singular) 'cored instantons', which have been brought up by

Heidenreich, Reece, Rudelius '15



Very rough summary of results

- Look at the case where we expect the strongest bound: A string model with $g_s = 1$ on T^6 at self-dual radius.
- Need to decide when to trust a wormhole / extremal instanton

(i.e., what is the smallest allowed S^3-radius r_c)

Note: For the instanton we demand that the action is dominated by the 'outside' of this S^3.

The following two choices appear 'natural':

$$2\pi^2 r_c^3 = \mathcal{V}_{self-dual}^{1/2}$$

$$2\pi r_c = \mathcal{V}_{self-dual}^{1/6}$$

One finds:

First case: $r_c M_P \simeq 1.3$

$$\text{Giddings-Strominger wormhole:} \qquad e^{-S} \simeq 10^{-68} \,,$$
$$\text{Extremal instantons:} \qquad e^{-S} \lesssim 10^{-15} \,.$$

Second case: $r_c M_P = \simeq 0.56$

$$\text{Giddings-Strominger wormhole:} \qquad e^{-S} \simeq 10^{-13} \,,$$
$$\text{Extremal instantons:} \qquad e^{-S} \lesssim 10^{-3} \,.$$

Thus: A model-independent bound appears out of reach, even in the most high-scale models available.

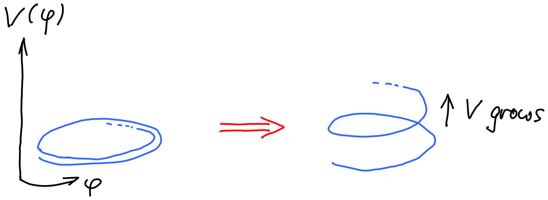
(Note that extremal instantons are UV-sensitive and choosing the more favorable 'second case' is not a priori justified.)

(c) Monodromy inflation

Silverstein/Westphal/McAllister '08

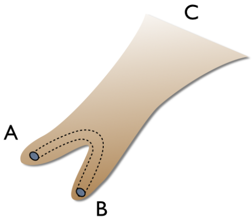
Very general but simple-minded definition:

- Start with a single, shift-symmetric, periodic inflaton φ
- Break the periodicity weakly by the scalar potential



The 'classical' model ...

$$S_{\rm NS5} \sim \int \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu} + C_{\mu\nu})}$$



Bifid throat with shared 2-cycle
(figure from Retolaza et al. '15)

... has issues with the explicit geometry and quantitative control.

For recent progress see e.g. McAllister/Silverstein/Westphal/Wrase '14
...
Retolaza/Uranga/Westphal '15

F-term axion monodromy

- Alternative suggestions have emerged how this could be realized in a quantitatively controlled way

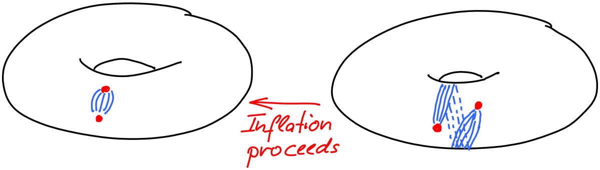
  (i.e. in a 4d supergravity description, with a stabilized compact space)

  Marchesano/Shiu/Uranga '14
  Blumenhagen/Plauschinn '14
  AH/Kraus/Witkowski '14

- One option is that inflation corresponds to brane-motion

  Dvali/Tye '98....Dasgupta et al. '02....Lüst et al. '11

- The monodromy arises from a flux sourced by the brane



Recent issues in F-term axion monodromy

- The difficulties of getting a small monodromy effect, especially moduli-backreaction were initially underestimated

$$\varphi = \mathrm{Re}(u) \ , \qquad K = K(z, \bar{z}, u - \bar{u}) \ , \qquad W = w(z) + f(z) u \ .$$

- Possible way's out include landscape tuning, appropriate hierarchical flux choice and high-scale non-geometric moduli-stabilization.

  Blumenhagen/Damian/Font/Fuchs/Herrschmann/Plauschinn/Sekiguchi/Sun/Wolf '14-15; Hassler/Lüst/Massai '14

  AH/Mangat/Rompineve/Witkowski '14

More precise but also constraining monodromy definition:

Dvali '05, Kaloper/Lawrence/Sorbo '08..'11

- Start with axion φ and 3-form C_3:
  (ignore all $\mathcal{O}(1)$ factors and couplings for now)

$$\mathcal{L} \sim |d\varphi|^2 + |dC_3|^2 \,.$$

- Note: Since $dC_3 = F_4 = *F_0$ is quantized, the 3-form theory corresponds to a discrete set of cosmolgical constants. The only dynamics is in the connecting domain walls
  (cf. 'Bousso-Polchinski landscape').

- Dualize by writing $d\varphi = *dB_2$.

- Gauge B_2 by C_3: $dB_2 \to dB_2 + C_3 \,.$

- Result: A potential $\sim \varphi^2$ is induced

- The possible nucleation of domain walls has been applied to bound monodromy models

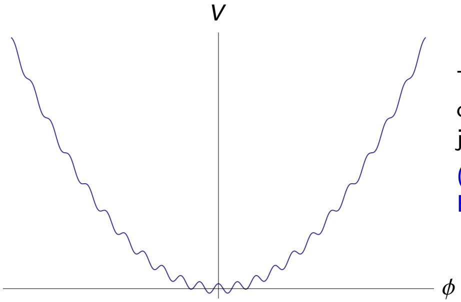
  (in particular in the context of the 'Relaxion')

  Ibanez/Montero/Uranga/Valenzuela '15

- More directly: Start from 'standard' monodromy potential (with 'instantonic wiggles')

AH/Rompineve/Westphal '15

$$\mathcal{L} = (\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \alpha\cos(\varphi/f)\,.$$



The low-energy effective theory of this model has no scalar but just a set of discrete vacua (as in the Bousso-Polchinski landscape).

(Effective) domain walls are automatically present, but are too light to give any useful WGC constraint.

(In fact, one may argue that they make the electric WGC useless.)

- Focus only on the effective action

$$S \sim \int \frac{1}{2g^2}\, F_4^2 + \int_{DW} A_3 \, ,$$

with the quantization $F_4 = n\, g^2$.

Match the discrete 4-form effective potential

$$V(F_4)_{eff} = \frac{1}{2} g^2 n^2$$

to the previous scalar effective potential

$$V(\varphi)_{eff} = \frac{1}{2} m^2 (2\pi n f)^2 \, .$$

- This implies $g = 2\pi m f$. Thus, we expressed the coupling of our 4-form theory in terms of scalar-potential parameters.

- A constraint can now be derived from the

Magnetic Weak Gravity Conjecture:

Arkani-Hamed/Motl/Nicolis/Vafa '06

- Consider an A_1/F_2 gauge theory with coupling g.

- The mass (field energy) of the smallest monopole is

$$M \sim \frac{1}{g^2} \cdot \frac{1}{R_{min}} \sim \frac{1}{g^2} \cdot \Lambda\,.$$

- For this monopole to exist, i.e. not to be a black hole, one needs

$$R_{min} \;>\; R_{BH}(M) \sim \frac{M}{M_P^2} \sim \frac{1}{R_{min}\, g^2 M_P^2}\,.$$

- Thus, at small g our theory must have a low cutoff: $\Lambda < g M_P$.

- Applied to domain walls, where ($[g] = (mass)^2$), this gives

$$\Lambda^3 < g\, M_P \sim m\, f\, M_P\,.$$

- In the context of inflation, one has

$$H \sim m\, \varphi_{max} \lesssim \Lambda$$

and hence

$$\Lambda^3 \lesssim m\, f\, M_P \qquad \Rightarrow \qquad \frac{\varphi_{max}}{M_P} \lesssim \left(\frac{M_P}{m}\right)^{2/3} \left(\frac{2\pi f}{M_P}\right)^{1/3}\,.$$

Summary/Conclusions

- Quantum gravity (Instantons / Weak gravity conjecture) may be constraining large-field inflation at a very fundamental level
- Concrete problems with large-field inflation in string theory reflect these fundamental 'issues'
- Progress is being made both in understanding the generic constraints as well as in constructing counterexamples (i.e. models)

In primordial gravity waves / large-field inflation, fundamental quantum gravity problems may meet reality!

Backup slides:

More precise but also constraining monodromy definition:

Kaloper/Lawrence/Sorbo '08..'11 (see also Dvali '05)

- Start with axion φ and 3-form C_3:
  (ignore all $\mathcal{O}(1)$ factors and couplings for now)

$$\mathcal{L} \sim |d\varphi|^2 + |dC_3|^2 \,.$$

- Note: Since $dC_3 = F_4 = *F_0$ is quantized, the 3-form theory corresponds to a discrete set of cosmolgical constants. The only dynamics is in the connecting domain walls
  (cf. 'Bousso-Polchinski landscape').

- Dualize by writing $d\varphi = *dB_2$, i.e.

$$\mathcal{L} \sim |dB_2|^2 + |dC_3|^2 \,.$$

- Finally, gauge B_2 by C_3: $\qquad dB_2 \to dB_2 + C_3 \,.$

Note: This gauging is the just the straightforward generalization of the familiar gauging of a U(1)-symmetry,

$$|\partial\Phi|^2 \rightarrow |(\partial + iA_1)\Phi|^2$$

or a corresponding scalar shift symmetry ($\varphi \equiv \arg(\Phi)$),

$$d\varphi \wedge *d\varphi \rightarrow (d\varphi + A_1) \wedge *(d\varphi + A_1).$$

- The result in our case is

$$\mathcal{L} \sim |dB_2 + C_3|^2 + |dC_3|^2$$

- In dualising back to φ, one now has to be very careful: One writes $dB_2 \equiv H_3$ and imposes the Bianchi identity through the lagrange multiplier φ:

$$\begin{aligned} \mathcal{L} &\sim |H_3 + C_3|^2 + \varphi\, dH_3 + |dC_3|^2 \\ &\sim |H_3|^2 + \varphi(dH_3 - dC_3) + |dC_3|^2 \end{aligned}$$

- After integrating out H_3 and writing $dC_3 = F_4$:

$$\mathcal{L} \sim |d\varphi|^2 - \varphi F_4 + |F_4|^2 \,.$$

- Finally, after also integrating out F_4 ,

$$\mathcal{L} \sim |d\varphi|^2 - \frac{1}{2}\varphi^2 \,.$$

one obtains the desired monodromy potential for φ.

- In summary: One can define axion monodromy as arising from the gauging of the dual 2-form by a 3-form.
- As an advantage, one can argue more systematically about protection by from higher-order potential terms
- Furthermore: The WGC can be applied to this construction...

Brown/Cottrell/Shiu/Soler; Ibanez/Montero/Uranga/Valenzuela '15

- Indeed, reinstating couplings, one has

$$\mathcal{L} \sim (\partial\varphi)^2 - \frac{g^2}{2}\varphi^2 ,$$

  where g is the coupling of C_3 to the domain walls.

- By the domain-wall WGC (if such a thing exists...), the domain walls become light if $g \ll 1$.

- Now, fast nucleation of these walls lowers the cosmological constant, which is equivalent to tunneling to $\varphi = 0$.

- This has been applied to bound monodromy models, in particular in the context of the 'Relaxion'

Ibanez/Montero/Uranga/Valenzuela '15