The 'Geometric Weak Gravity Conjecture'

(plus: The WGC and Axion Monodromy)

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based on work with F. Rompineve and A. Westphal

Outline

- Personal motivation
- The WGC without (useful) extremal objects
- The use of dualitites
- The geometric WGC

if time permits:

- Constraining monodromy by the WGC for domain walls

The WGC is interesting as ...

Arkani-Hamed/Motl/Nicolis/Vafa '06

1) A possible fundamental feature of quantum gravity

- It quantifies the non-existence of global symmetries

  (If $g \to 0$ is impossible, we need to know g_{min}.
  The WGC states $g_{min} = m$.)

- It may define a non-trivial boundary between landscape and swampland.

- Since it's always respected by string theory, it may teach us about the string's interplay with 'generic' quantum gravity.

- It may relate very directly to phenomenology....

The WGC is interesting because ...

2) The inflationary tensor-to-scalar ratio is...

$$r \equiv \frac{\Delta_T^2}{\Delta_R^2} = 16\epsilon \simeq 8 \left| \frac{d\varphi}{dN} \right|^2 \quad \Rightarrow \quad \Delta\varphi \simeq 20\sqrt{r}\,,$$

assuming $N \simeq 60$. (This is known as the Lyth bound).

- Thus, even though the BICEP 'discovery' of $r \simeq 0.15$ went away, the need to consider large-field models may return.

- Note: The Planck/BICEP analysis still sees a ($\sim 1.8\sigma$) hint for $r \simeq 0.05$. Much better values/bounds are expected soon.

  Cheung/Remmen; de la Fuente/Saraswat/Sundrum . . . '14
  Rudelius; Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler; Bachlechner/Long/McAllister; AH/Mangat/Rompineve/Witkowski; Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala; Harlow; AH/Rompineve/Westphal; . . . '15; Conlon/Krippendorf . . . '16

The WGC is interesting because ...

3) It has the potential to constrain Relaxion models...

Ibanez/Montero/Uranga/Valenzuela '15

- Such models fine-tune the Higgs mass-squared dynamically (during inflation), but require a large field range of an axionic scalar to do so...

Graham/Kaplan/Rajendran '15

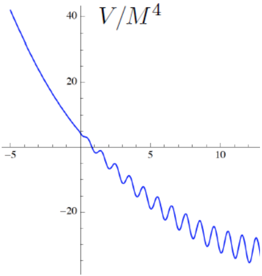


Fig. from Jaeckel/Metha/Witkowski '15

Crucially, one needs to constrain monodromy models

Brown/Cottrell/Shiu/Soler,
Ibanez/Montero/Uranga/Valenzuela '15
cf. also this and A. Westphal's talk

The (generalized) weak gravity conjecture

- The basic underlying lagrangian is (for p-dim. objects in d dims.; with $\overline{M}_P \equiv 1$)

$$S \;\sim\; \frac{1}{g^2}\int (F_{p+1})^2 \;+\; T\int_{p-dim.} dV \;+\; \int_{p-dim.} A_p$$

with

$$F_{p+1} = dA_p \,.$$

- To avoid stable extremal black branes, one requires charged objects with sub-extremal mass (tension):

$$q/T \geq \gamma_{p,d}^{1/2}\,, \quad \text{where} \quad \gamma_{p,d} = \frac{p(d-p-2)}{d-2}\,.$$

- As one clearly sees, this fails for instantons and objects with codimension 1 & 2 (domain walls and cosmic 'strings').

Heidenreich/Reece/Rudelius '15

Note:

- This failure outside the range $0 < p < d-2$ is not unexpected:
- Indeed, the argument that
  'the WGC protects us from too many stable objects'
  fails also intuitively outside this range.

  (E.g., strings and domain walls induce no long-range gravitational force.) see e.g. Susskind '95

However:

- The arguments that
  'the WGC protects us from the global-symmetry limit'
  and
  'string theory always obeys the WGC'
  support the conjecture even outside the above range.

- Arguments supporting/quantifying the WGC outside the 'canonical range' of $0 < p < d-2$ include
  - string dualities

    Brown/Cottrell/Shiu/Soler '15
  - consistency of generic KK-reductions
  - consideration of dilatonic black branes.

    Heidenreich/Reece/Rudelius '15
- <u>Example for duality argument:</u>

  IIB on $\mathrm{CY}_3 \times \mathbb{R}^3 \times S^1$ — brane wrapped to give instantons

  $\Leftarrow$ T-duality $\Rightarrow$

  IIA on $\mathrm{CY}_3 \times \mathbb{R}^3 \times S^{1'}$ — brane wrapped to give particle (at large R' and large coupling; hence M-theory in 5d; WGC applicable to 5d Reissner-Nordstroem-BHs).

  $\Rightarrow$ WGC is carried back to IIB axion decay constant.

- In fact, the key is not in the dualities, but rather in the same CY underlying both the M-theory and the IIB model.
- Hence, there ought to be a

  <u>Geometric WGC</u>

- Consider a IIA-CY X with D2-branes wrapped on 2-cycles.
- Let w_i be a basis of $H^2(X, \mathbb{Z})$.

  The metric on X induces a metric for 2-forms,

$$K_{ij} \equiv \int_X w_i \wedge \star w_j \, ,$$

  and on the (dual) space of 2-cycles, K^{ij} .
- We make the standard ansatz

$$C_3 \;=\; A_1^i(x) \wedge w_i(y) \, .$$

- Focus on 4d particles coming from D2s on a particular cycle Σ
- The relevant 4d action reads ($l_s = 1$)

$$S_4 \sim (V_X/g_s^2) \int \sqrt{g}\, R \;+\; \int K_{ij}\, F_2^i \wedge \star F_2^j \;+\; q_i^\Sigma \int_{\text{world-line}} A_1^i$$

with the charges

$$q_i^\Sigma = \int_\Sigma w^i \,.$$

- Note that only a particular combination of A_1^i's is sourced by particles 'from Σ':

$$A_1^i \;\equiv\; A_1\, K^{ij} q_j^\Sigma \qquad \text{(this defines } A_1) \,.$$

- Thus, one arrives at the standard action

$$S_4 \supset \frac{1}{2e^2} \int F_2 \wedge \star F_2 + \int_{\text{world-line}} A_1 \,,$$

with e^2 given by.....

- With e^2 given by

$$e^2 = 2\pi\,|q^\Sigma|^2 \qquad \text{with} \qquad |q^\Sigma|^2 \equiv K^{ij} q_i^\Sigma q_j^\Sigma\,.$$

Here we reinstanted $\mathcal{O}(1)$ factors.

Note: Metric on X $\rightarrow$ natural norm on p-form space
$\rightarrow$ natural norm $|q^\Sigma|$ on p-cycle space.

- Finally, use $\overline{M}_P^2 = V_X/\kappa_{10}^2 g_s^2$ together with $M_\Sigma = (\mu_2/g_s)V_\Sigma$ and impose the WGC:

$$\frac{e\overline{M}_P}{M_\Sigma} \geq \frac{1}{\sqrt{2}} \qquad \Rightarrow \qquad \frac{|q^\Sigma|\,V_X^{1/2}}{V_\Sigma} \geq \frac{1}{2}\,.$$

Thus a particular, purely geometric (rescaling- and g_s-independent) quantity characterizing X is constrained.

- Crucially, the same function appears in WGC constraints on other objects obtained from other branes wrapped on 2-cycles.

- For example, D4s give domain walls with

$$\frac{e_{DW}\overline{M}_P}{T_{DW}} = \frac{(2V_X)^{1/2}|q^\Sigma|}{V_\Sigma}\,.$$

- Thus, using the 'particle-WGC', we constrain $V_X^{1/2}|q^\Sigma|/V_\Sigma$, obtaining a precise 'domain-wall-WGC':

$$\frac{e_{DW}\overline{M}_P}{T_{DW}} \geq \frac{1}{2}\,.$$

- This goes through for any dimension of the cycle Σ and any dimension of the brane. Hence, any object in 4d is constrained by the imposition of the WGC for particles.

- Thus, allowing also for multiple gauge fields,

  Cheung/Remmen '14; Rudelius '14/'15,
  Brown/Cottrell/Shiu/Soler, Bachlechner/Long/McAllister '15

  we find in full generality:

  Geometric conjecture:

  The convex hull spanned by the vectors $(V_X^{1/2}/V_\Sigma)\, q^\Sigma$ (with $\Sigma \in H^p(X,\mathbb{Z})$) contains the ball of radius $1/2$.

  Implication for $(q+1)$-dimensional objects in 4d:

  The convex hull spanned by the vectors $(\overline{M}_P/T_q)\, \tilde{q}^\Sigma$ contains the ball of radius $1/\sqrt{2}$.

- Note: We did not use SUSY, the CY-condition, or the existence of a SUSY-brane on Σ. So this may be much stronger then the 'not too surprising' BPS-like result.

Constraining axion monodromy with the WGC

Disclaimer:

Only brief summary; for deeper analysis and relation to earlier work...

Kaloper/Lawrence/Sorbo '08..'11 (see also Dvali '05)

Brown/Cottrell/Shiu/Soler; Ibanez/Montero/Uranga/Valenzuela '15

see talks by I. Valenzuela and A. Westphal.

- Let's assume, based on the above, that all 4d objects, in particular DWs, are constrained.
- Note: the 'light' stringy objects fulfilling the WGC above are nevertheless always heavier than the KK-scale $M_{KK} = \Lambda$.
- Thus, one might conjecture that the magnetic WGC

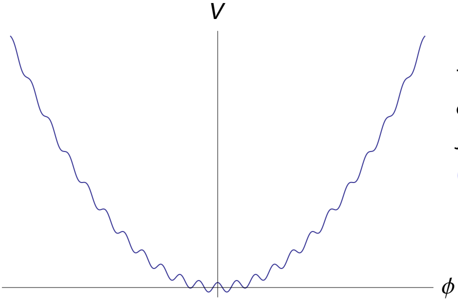
$$\Lambda^3 \lesssim e_2 \overline{M}_P$$

always holds.

- Start from the 'standard' monodromy potential (with 'instantonic wiggles')

AH/Rompineve/Westphal '15

$$\mathcal{L} = (\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \alpha\cos(\varphi/f)\,.$$



The low-energy effective theory of this model has no scalar but just a set of discrete vacua (as in the Bousso-Polchinski landscape).

(Effective) domain walls are automatically present, but are too light to give any useful WGC constraint.

(In fact, one may argue that they make the electric WGC useless.)

- Nevertheless, the effective action

$$S \sim \int \frac{1}{2(e_2)^2} F_4^2 + \int_{DW} A_3$$

is there and, using the quantization $F_4 = n\, e_2^2$, allows for matching the discrete effective potential

$$V(F_4)_{eff} = \frac{1}{2}(e_2)^2 n^2$$

to the previous effective potential

$$V(\varphi)_{eff} = \frac{1}{2} m^2 (2\pi n f)^2 \,.$$

- This implies $e_2 = 2\pi m f$ and hence

$$\Lambda^3 \lesssim e_2 \overline{M}_P = 2\pi m f \overline{M}_P \,.$$

- In the context of inflation, one has

$$H \sim m\,\varphi_{max} \lesssim \Lambda$$

and hence

$$\Lambda^3 \sim m\,f\,\overline{M}_P \qquad \Rightarrow \qquad \frac{\varphi_{max}}{\overline{M}_P} \lesssim \left(\frac{\overline{M}_P}{m}\right)^{2/3} \left(\frac{2\pi f}{\overline{M}_P}\right)^{1/3} .$$

To be better explained in A. Westphal's talk....

Summary

- Let's assume that string compactifications with form-fields / wrapped objects always obey the particle WGC.
- Then a geometric WGC follows.
- From this, one obtains a generalized WGC including axions, cosmic strings and DWs etc.
- The KK scale is always so low that also the generalized magnetic WGC is holds.
  Let's accept this latter form also more generally.
- The magnetic WGC for DWs provides for a very direct way of constraining axion-monodromy-type scalar potentials.