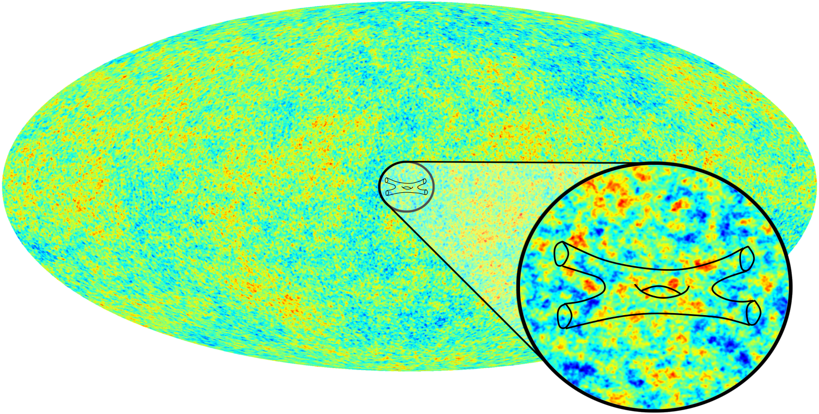
Large-Field Inflation and String Theory



Background Image: Planck Collaboration and ESA

Large-Field Inflation and String Theory

Arthur Hebecker (Heidelberg)

Outline

- Fundamentals of inflation
- The recent discussion triggered by BICEP
  and its impact on model building
- Why look for inflation in string theory
- Fundamental obstructions to large-field inflation
- Problems with large-field inflation in string theory
- Axion alignement and Axion monodromy:
  Early models and recent progress

Fundamentals of inflation

- Inflation 'resolves' the big bang singularity by introducing an early period in cosmology dominated by $\Lambda_{cosm.} = V(\varphi)$
- During this period, the universe expands (quasi-)exponentially: $a(t) \sim e^{Ht}$, where $H \sim \sqrt{\Lambda}/M_P$

Starobinsky '80
Guth '81
Linde '82

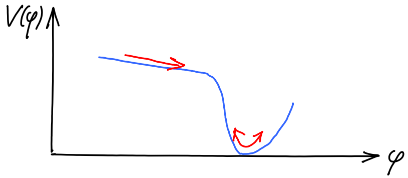
Note: from now on $M_P \equiv 1$

Fundamentals of inflation (continued)

- The simplest relevant action is

$$S = \int d^4x \sqrt{g} \left[\frac{1}{2} R[g_{\mu\nu}] + \frac{1}{2} (\partial\varphi)^2 - V(\varphi) \right]$$

- We can realise inflation if $V(\varphi)$ has a sufficiently flat region

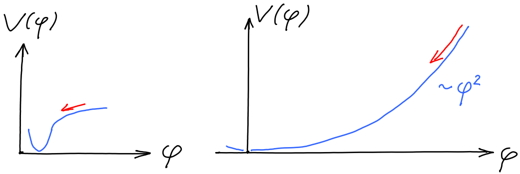


(More quantitatively, we need $V'/V \ll 1$ and $V''/V \ll 1$)

- In the end, φ oscillates and decays to SM particles
('reheating' $\equiv$ 'big bang')

Fundamentals of inflation (continued)

- If we allow ourselves to draw $V(\varphi)$ 'by hand', we can make some part of it very flat
- In this case, φ rolls very slowly, i.e. we get enough inflation (number of e-foldings) with $\Delta\varphi \ll 1$
- Such models are called 'small field' models



- Alternatively, one can use 'generic' potentials (e.g. $V(\varphi) \sim \varphi^2$)
- In such large field models, one needs $\Delta\varphi \gg 1$ (We will see that this may be a problem in quantum gravity)

(Trivial) technical comment:

- If $V \sim \varphi^n$, then $V'/V \sim 1/\varphi$. This is one way to see why 'generic potentials' require $\Delta\varphi \gg 1$.

- Stated in a positive way: If one can realize $\Delta\varphi \gg 1$, then no tuning of parameters is needed

Why look for inflation in UV-complete theories?

- Different types of questions have different sensitivity to the UV-completion / quantum gravity effects / string theory
- I want to argue that inflation is very sensitive to the UV
- Key point: In field-theory + quantum gravity we generically have higher-dimension operators $\sim \varphi^6/M_P^2 \equiv \varphi^6$ etc.
- Such effects may endanger the extreme flatness at $\varphi \ll 1$ or be completely fatal at $\varphi \gg 1$

An important warning / disclaimer:

- It is not impossible to ensure flatness (i.e. control higher -dimension operators) just in low-energy effective field theory
- The standard tools are shift symmetry ($\varphi \to \varphi + c$) and SUSY

  For an alternative approach, see
  Codello, Joergensen, Nielsen, Sannino, Svendsen '14...'15
- Nevertheless, one relies on assumptions about tree-level values of and (gravitational) corrections to operator coefficients....

$$\mathcal{L} \supset \alpha_6 \varphi^6 + \alpha_8 \varphi^8 + \cdots$$

- By contrast, in string theory such corrections are calculable

I will now focus on large-field models for two reasons....

1) Observations

- The amount of primordial gravity waves is measured by the tensor-to-scalar ratio:

$$r = \frac{\Delta_T^2}{\Delta_R^2} \simeq 8 \left| \frac{d\varphi}{dN} \right|^2 \quad \Rightarrow \quad \Delta\varphi \simeq 20\sqrt{r}$$

- Thus, even though the BICEP 'discovery' of $r \simeq 0.15$ went away, the need to consider large-field models may return

- Note: The new Planck/BICEP analysis still sees a ($\sim 1.8\sigma$) hint for $r \simeq 0.05$

- Much better values/bounds are expected soon

...reasons for interest in large-field models...

2) Fundamental

- On the one hand, large-field models are more 'robust'
- On the other hand, there are generic arguments against large-field models in consistent quantum gravity theories

  see e.g. Arkani-Hamed/Motl/Nicolis/Vafa '06 .... Conlon '12

- This goes hand in hand with certain problems in constructing large-field models in (the known part of) the string theory landscape

‘Fundamental reasoning’ continued...

- However, triggered by BICEP, new promising classes of stringy large-field have been constructed

- Example: ‘F-term axion monodromy’ (to be explained....)

  Marchesano/Shiu/Uranga ’14
  Blumenhagen/Plauschinn ’14
  AH/Kraus/Witkowski ’14

- At the same time, there are ongoing efforts to sharpen the ‘no-go arguments’ as well as to refute them

  Rudelius ’14...’15
  Montero, Uranga, Valenzuela ’15
  Brown, Cottrell, Shiu, Soler ’15
  AH/Mangat/Rompineve/Witkowski ’15
  ...

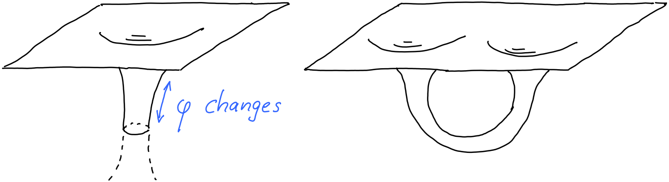
- I will try to explain some aspects of this debate....

No-go argument I: (Gravitational) instantons

- In Euclidean Einstein gravity, supplemented with an axionic scalar φ $\;(\varphi \equiv \varphi + f)$, instantonic solutions exist:

Giddings/Strominger '88

...



- The 'throat' is supported by the kinetic energy of φ, hence the large field range is essential

Caveats:

- a) Euclidean quantum gravity has its own fundamental problems
- b) It is not completely clear 'where the throat should connect' (our world, another world, 'crunch', 'baby universe' ......)
- Hence the interpetation of these instanton solutions still has issues...

Gravitational instantons (continued)

- Their Euclidean action is

$$S \sim n/f \qquad \text{(with } n \text{ the instanton number)}$$

- Their maximal curvature scale is f/n, which should not exceed the UV cutoff:

$$f/n < \Lambda$$

- This fixes the lowest n that we can trust and hence the minimal size of the instanton correction to the potential:

$$\boxed{\delta V \sim e^{-S} \sim e^{-n/f} \sim e^{-1/\Lambda}}$$

Gravitational instantons (continued)

- For gravitational instantons not to prevent inflation, the relative correction must remain small:

$$\frac{\delta V}{V} \sim \frac{e^{-1/\Lambda}}{H^2} \ll 1$$

- For a Planck-scale cutoff, $\Lambda \sim 1$, this is never possible
- However, the UV cutoff can in principle be as low as H (maybe just slightly above, for calculational control).
- Then, if also $H \ll 1$, everything might be fine....

$$\frac{\delta V}{V} \sim \frac{e^{-1/H}}{H^2}$$

Gravitational instantons (continued)

- Now, most string models of inflation do indeed have a low cutoff (e.g. compactification scale)
- However, it may be too naive to assume that 'uncalculable' gravitational instantons can simply be ignored
- They may find their 'continuation' in the gauge or D-brane instantons of the concrete string model
- Whether this is generically the case and whether such effects are always strong enough to spoil inflation is under debate ....

No-go argument II: Weak gravity conjecture

Arkani-Hamed/Motl/Nicolis/Vafa '06

- Roughly speaking: 'Gravity is always the weakest force.'
- More concretely (mild form):
  For any U(1) gauge theory there exits a charged particle with
  $$\boxed{q/m > 1\,.}$$
- Strong form:
  The above relation holds for the lightest charged particle.

Weak gravity conjecture (continued)

- One supporting argument:

  Quantum gravity forbids global symmetries. We should not be able to take the limit of small gauge couplings. The WGC quantifies this.

- Another supporting argument:

  In the absence of sufficiently light, charged particles, extremal BHs are stable

  Such remnants have the potential of violating the holographic entropy bound

  ..., Bousso '99, ...

Generalizations of the weak gravity conjecture

- The basic lagrangian underlying the above is

$$S \;\sim\; \int (F_2)^2 \,+\, m \int_{1-dim.} d\ell \,+\, q \int_{1-dim.} A_1$$

- This generalizes to charged strings, domain walls etc. Crucially, the degree of the corresponding form-field (gauge-field) changes:

$$S \;\sim\; \int (F_{p+1})^2 \,+\, m \int_{p-dim.} dV \,+\, q \int_{p-dim.} A_p$$

with

$$F_{p+1} = dA_p$$

Generalizations to instantons

- The supporting arguments based on remnants can still be made for strings, domain walls and other ‘branes’.

- This is less clear if one goes in the opposite direction, i.e. if one decreases the dimension of the charged object:

$$S \sim \int (d\varphi)^2 \; + m + q\,\varphi(x_{inst.})$$

- One easily recognizes that this is just a somewhat general way of talking about instantons and axions:

$$q\,\varphi(x_{inst.}) \;\Leftrightarrow\; \frac{1}{f}\int \varphi F\tilde{F} \qquad , \qquad m \;\Leftrightarrow\; S_{inst.}$$

WGC for instantons and inflation

- Now let us assume that the WGC, including the instanton case, will eventually be established
- The consequences for inflation are easy to derive
- First, recall that the instantons induce a potential (after the redefinition $\varphi \to \varphi/f$ to normalize the kin. term)

$$V(\varphi) \;\sim\; e^{-m} \cos(\varphi/f)$$

- Next, note that we are only in theoretical control (dilute instanton gas) if $m > 1$
- Since $q/m > 1$ now translates to $\boxed{\; m\,f < 1}$,

  this implies $f < 1$ and hence large-field 'natural' inflation can never work in the controlled (weakly-coupled) regime

Loopholes

- One obvious loophole is to go to the regime $m < 1$ in models where one knows the UV completion and can calculate

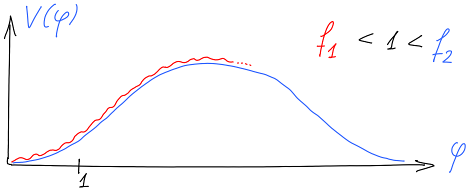
  de la Fuente, Saraswat, Sundrum '14

- However, so far the suggested scenarios still run into problems with the WGC for higher forms....

- Another loophole arises if one supposes that only the mild form of the WGC holds

  Rudelius '15

- In this case, one can have one 'sub-planckian' instanton maintaining the WGC, together with a lighter 'super-planckian' instanton realizing inflation:



No-go arguments vs. string constructions

- One can confront the above (apparently very general) no-go arguments with explicit constructions in a well-defined and calculable model of quantum gravity (string theory)

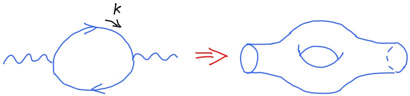
- This is an opportunity to sharpen our understanding of quantum gravity in general and string theory in particular

- Moreover, this fundamental endeavour has a rather direct relation to (at least possible, future) data

- This is rare in quantum gravity research and hence exciting!

String theory: 'to know is to love'

- String theory UV-completes QFT (including perturbative quantum gravity) in 10 dimensions:



- All we care about here is the (essentially unique) effective field theory arising in 10 dims. but at low energy ($E \ll 1/\ell_{string}$):

$$\mathcal{L} = R[g_{\mu\nu}] + F_{\mu\nu\rho}F^{\mu\nu\rho} + H_{\mu\nu\rho}H^{\mu\nu\rho} + \cdots$$

- Crucially, this theory also includes branes of various dimensions

- 4d models arise from compactifications and the 4d fields relevant for us are moduli of the compact space (e.g. T^6).

Compactifications

- To go from 10d to 4d, i.e. we need 6d compact spaces solving the vacuum Einstein's equations ($R_{\mu\nu} = 0$)

- Such geometries are called 'Calabi-Yau spaces' and $\sim 10^4$ of them are known (finiteness is conjectured but not established)

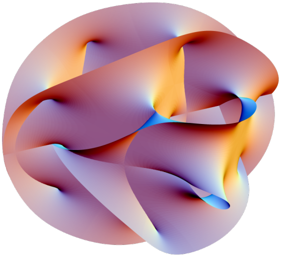
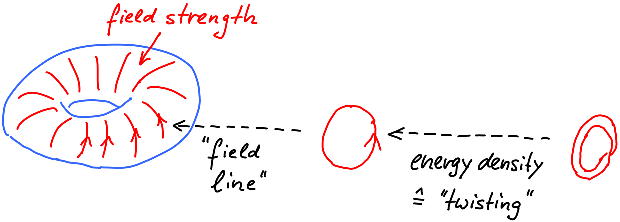


Image by J.F. Colonna

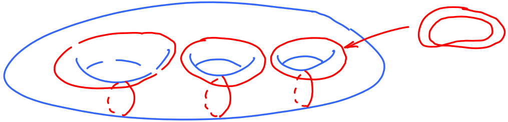
Next crucial ingredient: Fluxes

- Fluxes are field strengths of (higher-dimensional analogues) of gauge fields, such as $F_{\mu\nu\rho}\,,\;\; H_{\mu\nu\rho}$
- They are crucial for the landscape since they stabilize the geometry and lead to $\sim 10^{500}$ possibilites
- Simplest version of an explanation:

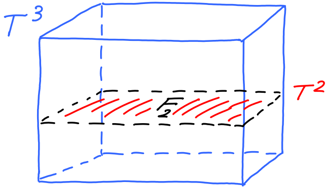


- This illustrates a flux wrapped on a 1-cycle of the torus

- Quite generally, fluxes ‘live’ on cycles of the compact space
- Example: several 1-cycles in 2d space



- Crucial: Higher-dimensional cycles (with fluxes) exist in higher-dimensional spaces
- Example: a 2-cycle in T^3



The string theory landscape

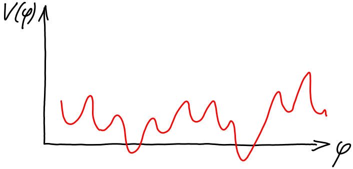
- Typcial CYs have $\mathcal{O}(300)$ 3-cycles
- Each can carry some integer number of flux of $F_{\mu\nu\rho}$, $H_{\mu\nu\rho}$
- With, for example, $N_{flux} \in \{-10, \ldots, 10\}$ on gets

$$(2 \times 20)^{300} \;\sim\; 10^{500} \text{ possibilities}$$

- This is the string theory landscape!
- To appreciate the complexity, recall that there are only $\sim 10^{80}$ atoms in our universe

The string theory landscape (continued)

- Each of these geometries corresponds to a solution ('vacuum') of the same, unique fundamental theory
- Each solution has a different vacuum energy



Here φ corresponds to $\{\varphi_1, \ldots, \varphi_n\}$, parametrizing the shape of the CY

Weinberg '87
Bousso/Polchinski '00
Giddings/Kachru/Polchinski '01 (GKP)
Kachru/Kallosh/Linde/Trivedi '03 (KKLT)
Denef/Douglas '04

Technical interlude

- Scalar potentials are derived in 4d supergravity:

$$V = e^K(|DW|^2 - 3|W|^2)$$

- Here the Kahler potential K defines the metric $g_{i\bar{\jmath}} \equiv \partial_i \partial_{\bar{\jmath}} K$ on the (complex) field space.

- The superpotential W is a function on this space (more precisely bundle section over it)

- **In string theory:** K encodes the geometry of the CY
  W encodes the fluxes

- Let us focus on 'complex structure moduli' $z \equiv \{z^i\}$

- The relevant Kahler potential reads

$$K = -\ln\left[\Pi_\alpha(z)\overline{\Pi}^\alpha(\bar{z})\right] \simeq -\ln f(z - \bar{z})$$

Technical interlude (continued)

- The 'periods' $\Pi_\alpha(z)$ measure the relative size and orientation of 3-cycles (cf. shape-modulus τ of T^2)
- More explicitly:

$$\Pi_\alpha = \begin{pmatrix} 1 \\ z^I \\ \frac{1}{2}\kappa_{IJK} z^J z^K + \sum_p A_{Ip} e^{-\sum_J a_{pJ} z^J} \\ -\frac{1}{3!}\kappa_{IJK} z^I z^J z^K + \sum_p B_p e^{-\sum_J b_{pJ} z^J} \end{pmatrix}$$

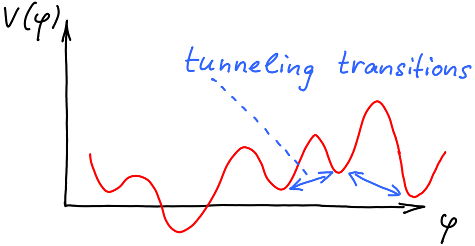
- Finally, (a representative part of) the superpotential reads

$$W \supset N^\alpha \Pi_\alpha(z)$$

where N^α is the number of flux units of F_3 on the 3-cycle labelled by α

Populating the landscape

- Any vacuum with $\Lambda > 0$ gives classically an eternally expanding (de Sitter) universe
- However, by a quantum fluctuation, a bubble of a different vacuum can form, which then also expands
- .... just like bubble nucleation in first order phase transitions



Bubbles within bubbles within bubbles ....

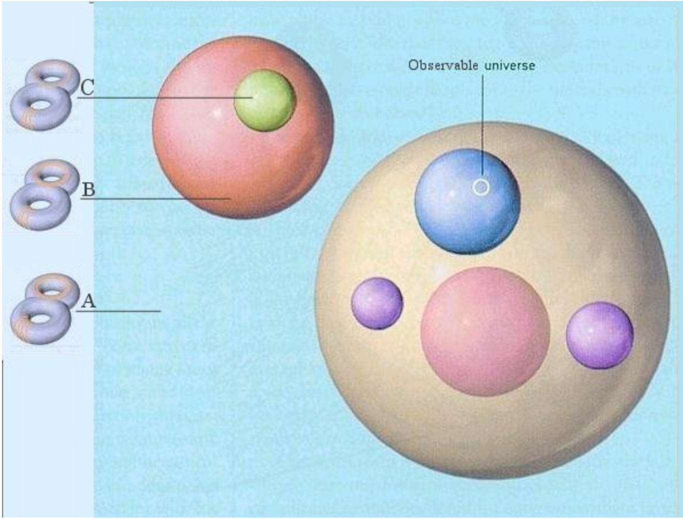
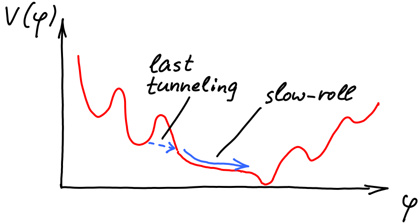


image from "universe-review.ca"

Slow-roll inflation in the landscape

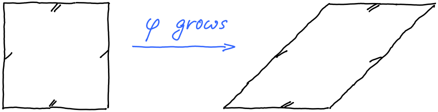
- To make our universe flat, we need a period of slow-roll inflation after the last tunneling event



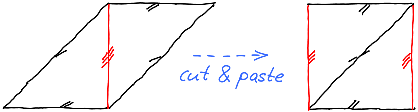
- This last period of slow-roll inflation is what we observe on the CMB-sky

Why is large-field inflation ($\Delta\varphi \gg 1$) problematic?

- The field φ generically corresponds to some geometric feature of the CY, e.g. the shape of a torus



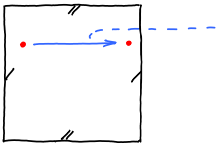
- However, after the angle of a torus has grown to 45°, it is secretly the same torus



- The problem is that this applies (more or less) to all 4d fields of a string compactification

- Another, even more obvious example arises if φ is a brane position. Clearly, this field is also periodic and the field space is hence limited:

Dvali/Tye '98



$\varphi \triangleq$ brane position

$\varphi \equiv \varphi + 1$

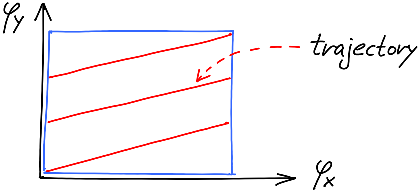
- <u>Note:</u> Thus, we naturally get the axionic scalars discussed earlier. But their periodicity is always too short.

One needs ideas!

(I) Winding inflation / KNP

Kim/Nilles/Peloso '04; Berg/Pajer/Sjors '09; Ben-Dayan/Pedro/Westphal '14

- One such idea is to realize a 'winding' trajectory on a 2d periodic field space:



- Clearly, such a trajectory can be much longer than the (naive) field range

- The technical challenge is the realization of the required potential in concrete string models

Winding inflation (continued)

- The fields φ_x and φ_y are two 'string theory axions', both with $f < 1$ (obeying the WGC)

- They are also moduli.
  Hence, fluxes can be used to stabilize them

- A judicious choice of fluxes allows for stabilizing just one linear combination, forcing the remaining light field on the winding trajectory:

$$V \supset (\varphi_x - N\varphi_y)^2 + e^{-M}\cos(\varphi_x/f) + e^{-m}\cos(\varphi_y/F)$$

  with $\qquad N \gg 1$

- This realizes inflation and avoids the WGC!

AH/Mangat/Rompineve/Witkowski '15

Winding inflation (continued)

- To be more precise, let's change variables:

$$\varphi \equiv \varphi_x \,, \qquad \psi \equiv \varphi_x - N\varphi_y$$

- While ψ is 'frozen', our inflaton φ 'sees' both the instantons belonging to φ_x as well as those belonging to φ_y:

$$V \supset \psi^2 + e^{-M} \cos(\varphi/f) + e^{-m} \cos[(\varphi - \psi)/NF]$$

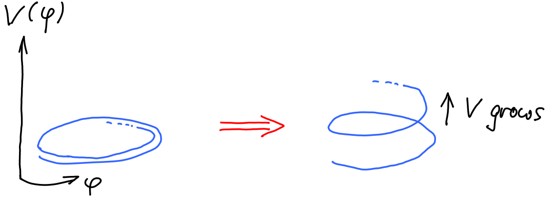
- Crucially, in our proposal the quantities M and m are precisely the type of variables that can be tuned in the landscape (like the vacuum energy)
  ....thus, getting a largish M is not a problem

- Getting a sufficiently large N may be a problem due to tadpole constraints....

(II) Monodromy inflation

Silverstein/Westphal/McAllister '08

- We start with a single, periodic inflaton φ
- The periodicity is then weakly broken by the scalar potential



F-term axion monodromy

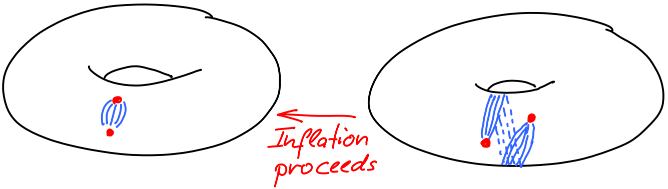
- Very recently, the first suggestions have emerged how this could be realized in a quantitatively controlled way

  (i.e. in a 4d supergravity description, with a stabilized compact space)

  Marchesano/Shiu/Uranga '14
  Blumenhagen/Plauschinn '14
  AH/Kraus/Witkowski '14

- In particular, in our suggestion inflation corresponds to brane-motion

- The monodromy arises from a flux sourced by the brane



F-term axion monodromy (continued)

- The strong point of these constructions is the manifest supergravity description (SUSY is broken only spontaneously, the basic geometry is still approximately Calabi-Yau, explicit calculations are feasible)

- The weak point is the required fine-tuning to make the monodromy-effect weak

- Implementing this fine tuning is subject of an ongoing debate

  Blumenhagen, Herschmann, Plauschinn '14
  AH, Mangat, Rompineve, Witkowski '14
  Blumenhagen et al. '15

- **Also:** It is not clear whether any of the no-go arguments discussed earlier applies to monodromy models....

F-term axion inflation (more technical level)

- The Kahler potential is shift-symmetric (and periodic):

$$K(z,\bar{z}) = K(z-\bar{z})$$

- This situation arises e.g. in the ‘large complex structure limit’
- The flux-induced superpotential breaks this symmetry (induces a monodromy):

$$W(z) = W_0 + a\,z$$

- The challenge is to ensure that a is sufficiently small

Summary

- ...large vs. small-field inflation, UV-sensitivity, BICEP etc....
- Quantum gravity (Instantons / Weak gravity conjecture) may be constraining large-field inflation at a very fundamental level
- ....the (flux-) landscape, eternal inflation and the multiverse....
- Concrete problems with large-field inflation in string theory reflect the fundamental 'issues' and may help to resolve them
- ....winding inflation / axion monodromy:
  Early models and recent progress...

'Conclusion'

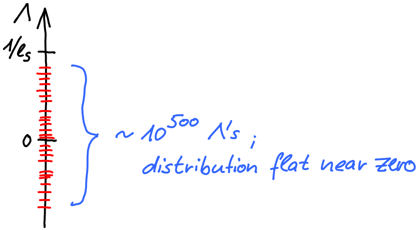
In primordial gravity waves / large-field inflation,
fundamental quantum gravity problems may meet reality!

Backup slides:

The cosmological constant in the landscape

- Crucially, at least for part of the landscape, the statistical distriution of $\Lambda = V(\varphi_{\text{min}})$ can be calculated.

  It is 'flat' in the region near $\Lambda = 0$

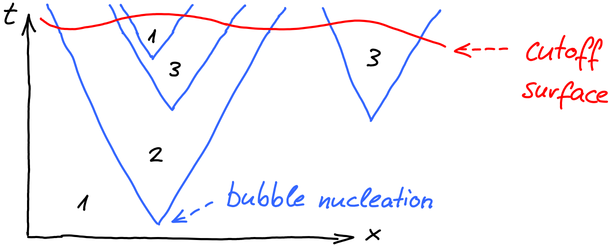


- Thus, while having $\Lambda \sim 10^{-120}$ (as is measured) is extremely unlikely, it is known that such vacua do exist

- One can appeal to anthropic arguments to explain why we find ourselves in such an 'rare' vacuum

Bubbles within bubbles within bubbles ....

- More scientific but less pretty: A cartoon of eternal inflation in 2 dimensions



- The arbitrariness of the 'cutoff surface' is one of the faces of the measure problem – we don't know how to count and thus how to make even just statistical predictions