String Landscape & Multiverse

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see also: lecture notes at 2008.10625 [hep-th]

and slides at www.thphys.uni-heidelberg.de/∼hebecker → 'Selection of recent talks'

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1 Why String Theory?

With minor exceptions, our world is perfectly described by

$$\mathcal{L} \;=\; \mathcal{L}_{\text{grav.}} + \mathcal{L}_{\text{SM}}.$$

Here

$$\mathcal{L}_{\text{grav}} \;=\; \tfrac{1}{2} M_P^2\, \mathcal{R}[g_{\mu\nu}]\;; \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Symbolically:

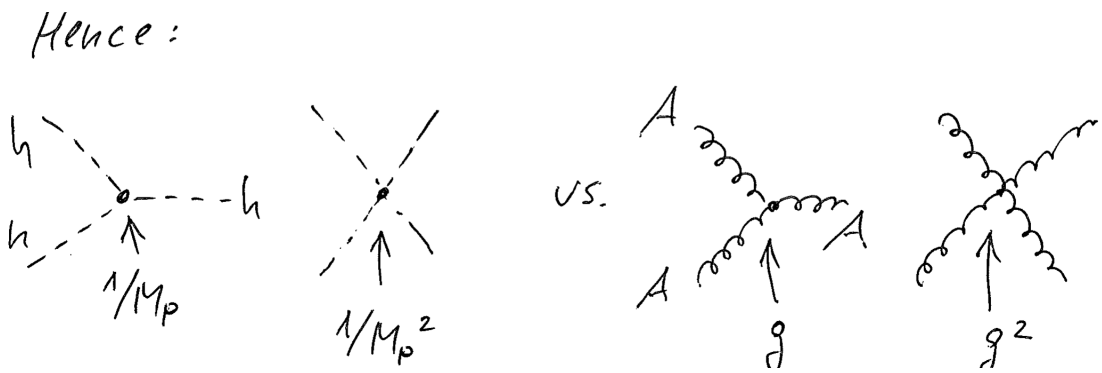
$$\mathcal{L}_{\text{grav}} \;=\; \tfrac{1}{2} M_P^2 \big[(\partial h)^2 + h(\partial h)^2 + h^2(\partial h)^2 + \cdots\big]$$

$$\text{Redefine}: \quad h_{\mu\nu} \longrightarrow h_{\mu\nu}/M_P$$

$$\Rightarrow \quad \mathcal{L}_{\text{grav}} \;=\; \frac{1}{2}(\partial h)^2 + \frac{1}{2M_P}\, h(\partial h)^2 + \frac{1}{2M_P^2}\, h^2(\partial h)^2 + \cdots$$

Compare this to $\mathcal{L}_{\text{QCD}} \subset \mathcal{L}_{\text{SM}}$:

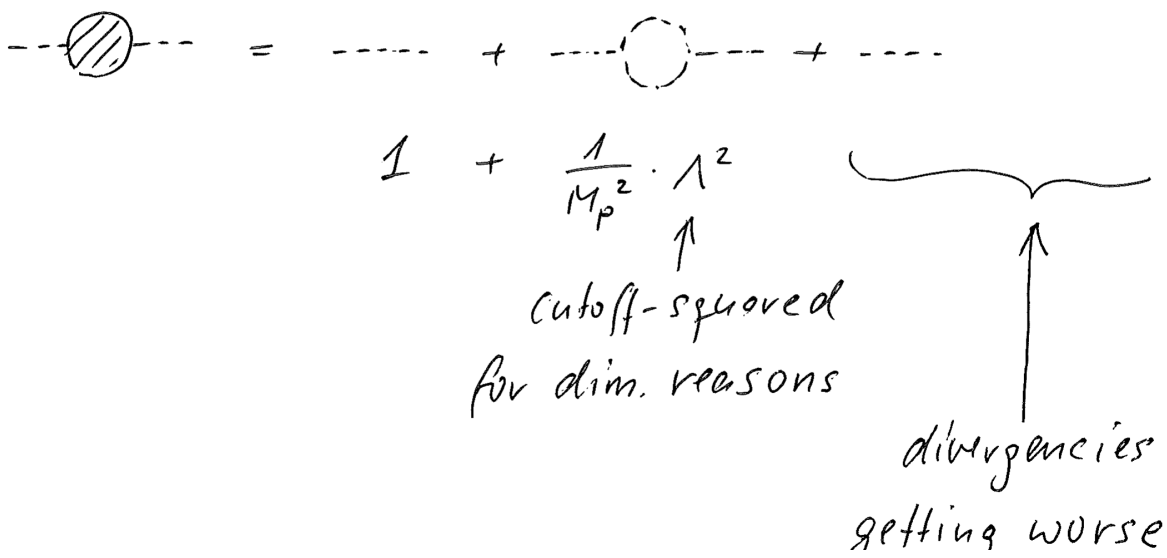
$$\mathcal{L}_{\text{QCD}} \;=\; \tfrac{1}{2}(\partial A)^2 + \, gA^2(\partial A) + \, g^2 A^4$$



• **Key difference for gravity:**

Coupling constant has negative mass-dimension in gravity.

$\Rightarrow$ not renormalizable by power-counting.



• **Even more catastrophic:**

Perturbative description completely breaks down at $E \sim \sqrt{s} \sim M_P$.

Indeed: For a renormalizable-QFT amplitude

$$\mathcal{A} \sim g^2 \quad \Rightarrow \quad \sigma \sim |\mathcal{A}|^2 \frac{1}{s}$$

for dimensional reasons.

(considering fixed-angle scattering)

By contrast, in gravity:

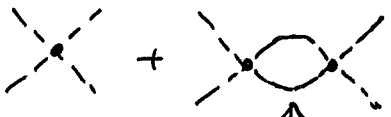
$$\mathcal{A} \sim \frac{1}{M_P^2} s \quad \Rightarrow \quad \sigma \sim |\mathcal{A}|^2 \frac{1}{s} \sim \frac{s}{M_P^4}$$

Unitarity violation!

⇒ Widely-held belief:

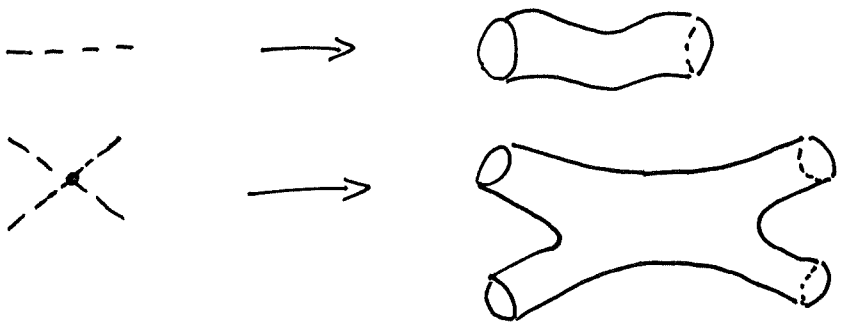
At $E \sim M_P$, QFT-logic breaks down in a very fundamental way.

- Best-developed option: <u>String theory</u>
- Motivation:
  - problems related to UV-divergences of QFT
  - these, in turn, are related to point-like particles or, better, <u>point-like vertices</u>.



; $\int_{x,y} [D(x-y)]^2 \longrightarrow \infty$ from $(x-y) \to 0$

- Stringy resolution:



all smooth, no vertex!

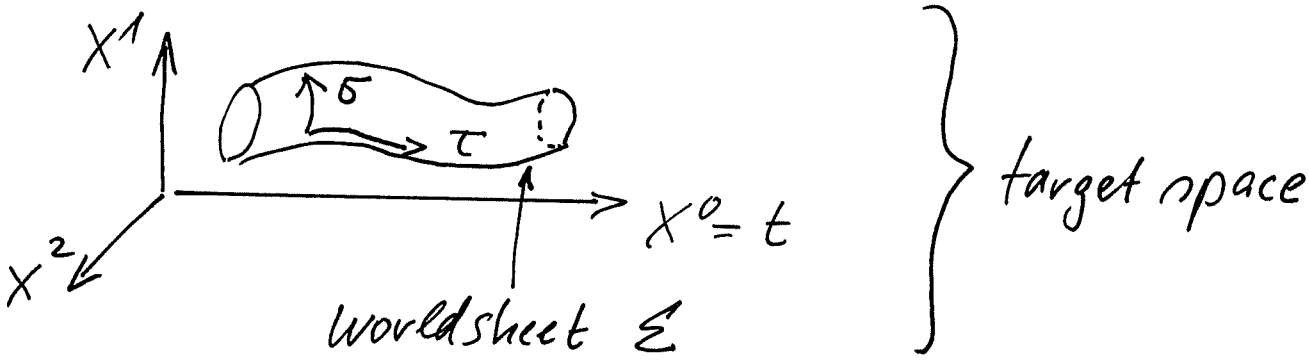
- Detailed study reveals:

This really only works for $D = 10$ and with SUSY (in the 10d theory).

⇒ Need to work hard to get back to our real world in $D = 4$ & ~~SUSY~~.

2 String Theory

2.1 Classical String



$(\tau, \sigma) \equiv (\xi^0, \xi^1) \equiv \xi^a$

- The most natural action is $S_{NG} = -T \int_\Sigma d\!f$.

  ↑ "Nambu-Goto"

- To make it explicit, we must quantify the embedding of the WS in the target space:

$$X^\mu = X^\mu(\xi) \quad , \quad \mu = 0, \dots, D-1$$

$$\Rightarrow S_{NG} = -T \int_\Sigma d^2\xi \sqrt{-G} \quad , \quad G \equiv \det(G_{ab})$$

↑ induced metric on Σ

- Explicitly: $G_{ab} = \eta_{\mu\nu} \left(\frac{\partial X^\mu}{\partial \xi^a}\right)\left(\frac{\partial X^\nu}{\partial \xi^b}\right)$

- Because of $\sqrt{\dots}$, this is very hard to quantize.

- Much better: $S_P = -\frac{T}{2}\int d^2\xi \sqrt{-h}\, h^{ab} G_{ab}$

  "Polyakov"

  $h^{ab}\eta_{\mu\nu}(\partial_a X^\mu)(\partial_b X^\nu)$

- Here h_{ab} is an independent "WS metric".

- For $\eta_{\mu\nu} = \mathrm{diag}(-1,1,\dots,1)$, S_P is just the action of a QFT with D free scalars X^μ.

- Crucially, it is classically equivalent to S_{NG}:

$$T_{ab} \sim \frac{\delta S_P}{\delta h^{ab}} \sim G_{ab} - \frac{1}{2} h_{ab}\,(G_{cd} h^{cd})$$

- EOM for h: $T_{ab} = 0 \Rightarrow h_{ab} = G_{ab} \cdot f(\xi)$

  any function

- Plugging $h_{ab} = f \cdot G_{ab}$ into S_P immediately recovers S_{NG}.

- Thus, from now on, our fundamental action is

$$S_P = -\frac{1}{4\pi\alpha'}\int d^2\xi \sqrt{-h}\,(\partial X)^2$$

"Regge slope"

$\alpha' \sim \ell_s^2$ ("string length")

$h^{ab}(\partial_a X^\mu)(\partial_b X^\nu)\eta_{\mu\nu}$

- S_p has three key symmetries:

  1) Diffeomorphisms: $\xi^a \to \xi'^a = \xi'^a(\xi^0, \xi^1)$

  2) Poincare: $X^\mu \to X'^\mu = \Lambda^\mu{}_\nu X^\nu + V^\mu$

  $(\Lambda \in SO(1, D-1))$

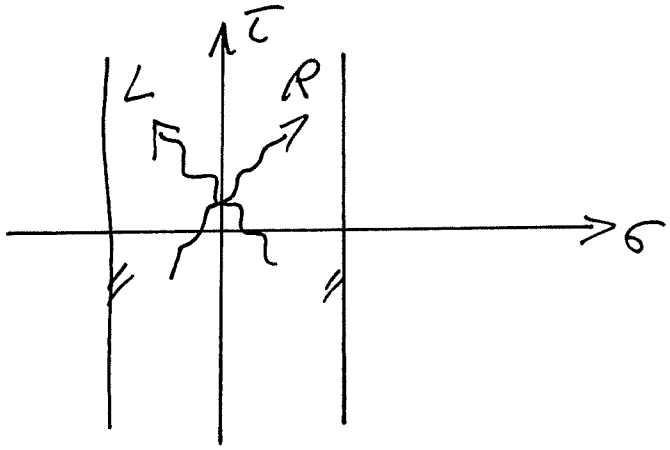
  [This is a global symmetry of our 2d QFT.]

  3) Weyl rescalings: $h_{ab}(\xi) \to h'_{ab}(\xi) = \varphi(\xi) h_{ab}(\xi)$

  [This key feature sets $d=2$ apart.]

- 1) & 3) are gauge symmetries.
  They may be used to choose a "flat gauge",
  i.e. $h_{ab} = \text{diag}(-1, 1)$

$\Rightarrow$ All we need to do is to quantize free scalars (X^μ's) on a flat cylinder:



2.2 Quantization

- Use light-cone coordinates: $\sigma^{\pm} \equiv \tau \pm \sigma$
- Then: $\Box X^{\mu} = 0 \iff \partial_+ \partial_- X^{\mu} = 0$
- General solution: $X^{\mu}(\sigma^+, \sigma^-) = X_L^{\mu}(\sigma^+) + X_R^{\mu}(\sigma^-)$

with

$$X_L^{\mu}(\sigma^+) = \frac{1}{2} x^{\mu} + \alpha' p^{\mu} \sigma^+ + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{\tilde{\alpha}_n^{\mu}}{n} e^{-2\pi i n \sigma^+}$$

(and analogously for X_R^{μ}, with $\tilde{\alpha} \to \alpha$)

- Canonical quantization proceeds as usual:

$$[p^{\mu}, x^{\nu}] = -i\eta^{\mu\nu} \,, \quad [\tilde{\alpha}_m^{\mu}, \tilde{\alpha}_n^{\nu}] = m \delta_{m+n} \eta^{\mu\nu}$$

same for α's.

$\tilde{\alpha}$: "left-movers" α: "right-movers"

$\delta_{m+n} \equiv \delta_{m+n,0}$

$(\alpha_m)^+ = \alpha_{-m}$, $m > 0 \Rightarrow$ "annihilators"

This part is special to string theory, because our space is compact: S^1.

x^{μ} – centre of mass, p^{μ} – momentum (of motion of $\langle X^{\mu} \rangle$)

- Fock space: All linear combinations of

$$\underbrace{\alpha_m^\mu \alpha_n^\nu \cdots \tilde{\alpha}_k^\rho \tilde{\alpha}_\ell^\sigma \cdots}_{\text{any number of creation operators of left- and right-moving excitations}} |0,p\rangle \;;\; m,n,k,\ell \ldots < 0$$

($|0,p\rangle$: any p; m,n,k,ℓ: creators!)

- The proper Hilbert space is much smaller, analogously to the familiar case of gauge theory: $(\partial_\mu A^\mu)_{\text{annih.-part}} |\psi\rangle = 0$

- Here: $(T_{ab})_{\text{annih.-part}} |\psi\rangle = 0$

↑ Fourier modes of this ≡ "Virasoro generators" L_m, $\tilde{L}_m$.

- In particular:

$$(L_0 - 1)|\psi\rangle = 0 \;;\; L_0 = \frac{p^2}{4}\alpha' + \underbrace{\sum_{n=1}^{\infty} \alpha_{-n}\alpha_n}_{\text{"level" } N}$$

[and same for $\tilde{L}_0$]

⇒ Must always excite left- & right-movers together ("level matching")

$\Rightarrow$ Can calculate the mass-squared ($= -p^2$) for any state.

Note: In developing this framework, one must carefully think about ordering-ambiguity in L_0 and other operators.

The "parameter" D plays a crucial role.

One discovers that $SO(1, D-1)$ is anomalous unless $D = 26$.

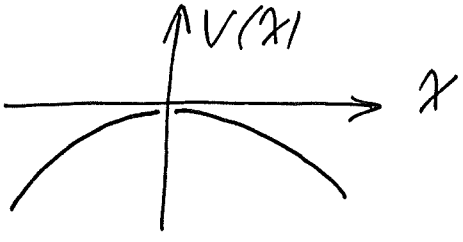
Alternatively, one may quantize in an $SO(1,D-1)$-covariant way. But then one discovers that 2d-Weyl-invariance is broken.

We focus here on "critical string", $D = 26$

2.3 String states = 26d particles or fields

- Vacuum: $|0, p\rangle$; $m^2 = -4/\alpha'$

This is a "tachyon", i.e. this particle is an excitation of 26d-QFT with potential:



$\Rightarrow$ System unstable, but some short-time questions can be answered.

$\Rightarrow$ Bosonic string is just a toy model.

- 1st excited state: $\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0,p\rangle\,\underbrace{\varepsilon(p)_{\mu\nu}}_{\text{polarization tensor}}$; $m^2=0$

1) traceless-symm. part: graviton ; i.e. $\delta g_{\mu\nu}$ in 26d.

2) antisymm. part: "Kalb-Ramond potential" $B_{\mu\nu}$

[Analogue of "A_μ", i.e.

$H_{\mu\nu\sigma} \sim \partial_{[\mu} B_{\nu\sigma]}$, or better: $H = dB$,

just as $F_{\mu\nu} \sim \partial_{[\mu} A_{\nu]}$ or $F = dA$.

Such a B-field was expected since:

$$\int A_\mu dx^\mu \quad \longleftrightarrow \quad \int B_{\mu\nu}\, dx^\mu dx^\nu .$$

Electron-WL ; String-WS]

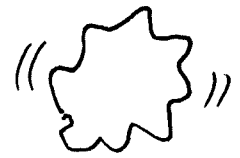
3) trace of $\epsilon_{\mu\nu}$: "dilaton" ϕ - massless scalar

- 2nd excited state: $\left(\alpha_{-1}^{\mu}\alpha_{-1}^{\nu}\tilde{\alpha}_{-1}^{\rho}\tilde{\alpha}_{-1}^{\sigma} + \alpha_{-2}^{\mu}\tilde{\alpha}_{-2}^{\nu}\right)|0,p\rangle$ etc.

$\vdots$

all heavy

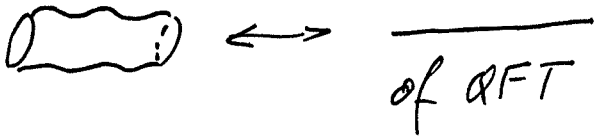
Intuitively, these are (highly) excited string states:



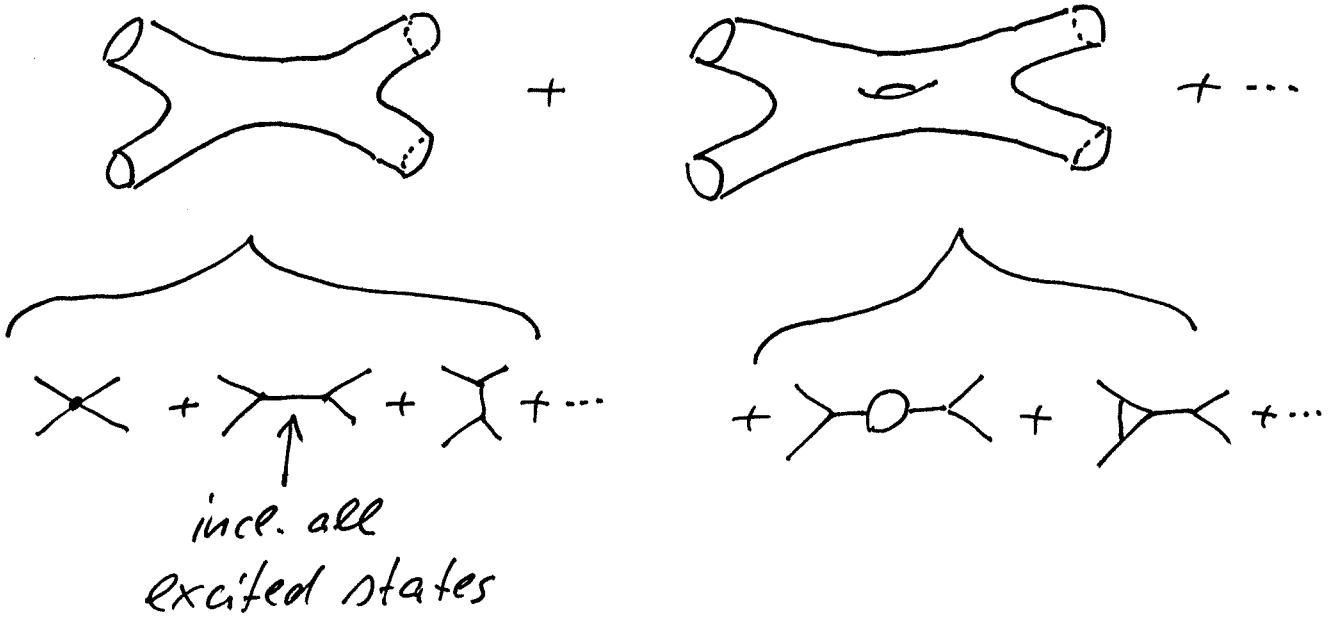
They are crucial for the consistency of the theory, but not important for us.

2.4 Amplitudes & 26d-QFT

- We now understand:



- We need also:



- Method:

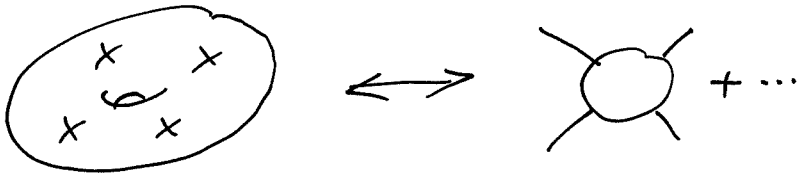
$$\rightarrow \quad \sim \int DX^\mu Dh_{ab}\, e^{-S_p[X,h]} \, O_1(p_1) \cdots O_4(p_4)$$

- The operators $O_i(p_i)$ on S^2 characterize incoming/outgoing string states: "Vertex operators".
- Result of a long analysis:

  Such scattering amplitudes can be calculated and display the expected "soft" UV behaviour: $\mathcal{A}(s,t) \longrightarrow 0$

  as $s \to \infty$ at fixed $\frac{s}{t}$.

- Loops are finite:



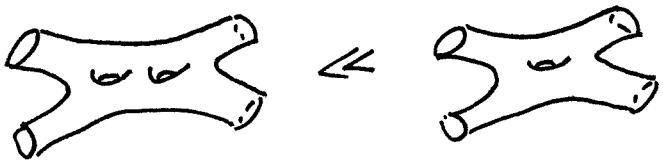
- At low s,t (i.e. $\ll 1/\alpha' \triangleq M_s^2$), these amplitudes coincide with the amplitudes of a 26d QFT:

$$S = \frac{1}{\kappa^2}\int d^{26}x\sqrt{-g}\, e^{-2\phi}\left[R[g] - \frac{1}{12}H_{\mu\nu s}H^{\mu\nu s} + 4(\partial\phi)^2\right]$$

$\kappa^2 \to (\alpha')^{-12} \to (M_{P,26})^{24}$

$\Rightarrow \langle\phi\rangle$ governs the ratio $M_s/M_{P,26}$.

If $\phi \to -\infty$, string is "weakly coupled" in the sense that:



One writes: $g_s \equiv e^{\phi}$

Important final comment:

One can study this theory in backgrounds where $g_{\mu\nu} \neq \eta_{\mu\nu}$; $B_{\mu\nu} \neq 0$; $\phi \neq 0$ (as just noted).

The background may change:

$$g_{\mu\nu} \to g_{\mu\nu} + \underbrace{\delta g_{\mu\nu}}$$

corresponds to "condensate" of elementary quanta of $\delta g_{\mu\nu}$, i.e. of massless string modes (just like $\langle \vec{E} \rangle$ is coherent state of photons).

2.5 Superstring

$$S \sim \int_{\Sigma} (\partial_a X^\mu)(\partial^a X^\nu)\eta_{\mu\nu} + i\bar{\psi}^\mu \not{\partial} \psi_\mu \equiv S_B + S_F$$

ψ^μ is fermionic partner of WS scalar X^μ

- <u>More precisely</u>: We have a 2d supergravity theory on the WS, with $h_{ab} \leftrightarrow$ graviton
  $X^\mu \leftrightarrow \psi^\mu$

- Can choose gauge $h_{ab} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, gravitino $= 0$.

- $\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix} \begin{matrix} \leftarrow \\ \leftarrow \end{matrix}$ both real (Maj.-Weyl-spinors)

- Use light-cone coordinates on WS: $\sigma^\pm \equiv \tau \pm \sigma$

$$\Rightarrow S_F \sim \int d^2\sigma \left(\psi_-^\mu \partial_+ \psi_{-\mu} + \psi_+^\mu \partial_- \psi_{+\mu} \right)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\partial_+ \psi_- = 0 \qquad \partial_- \psi_+ = 0$$

(right-movers) (left-movers)

- <u>Novelty</u>: Can choose different boundary conditions for ψ's on S^1:

$\psi_+(\sigma+\ell) = +\psi_+(\sigma)$; $\psi_-(\sigma+\ell) = +\psi_-(\sigma)$ R-R

$\psi_+(\sigma+\ell) = +\psi_+(\sigma)$; $\psi_-(\sigma+\ell) = -\psi_-(\sigma)$ R-NS

NS-R

NS-NS

$\vdots$

(Ramond, Neveu-Schwarz)

- Quantization: $[\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n}\eta^{\mu\nu}$

$$\{\psi_r^\mu, \psi_s^\nu\} = \delta_{r+s}\eta^{\mu\nu} \begin{cases} r,s \in \mathbb{Z} & (R) \\ r,s \in \mathbb{Z}+\frac{1}{2} & (NS) \end{cases}$$

- $D=26 \xrightarrow{\text{susy}} \underline{\underline{D=10}}$ (for consistency)

- As before: create states by applying

$$\alpha_m^\mu, \psi_r^\mu \quad (m,r<0) \text{ to } |0,p\rangle$$

- Derive masses from $(T_{ab})_{\text{annih.-part}}|\Psi\rangle = 0$

NS-NS (r./l.)

$|0,p\rangle$ tachyon

$\epsilon_{\mu\nu}\psi_{-1/2}^\mu \tilde{\psi}_{-1/2}^\nu |0,p\rangle$ massless $\rightarrow$ graviton, $B_{\mu\nu}$, ϕ

all higher states massive

R-NS (r./l.)

$|0,p\rangle \xrightarrow[R]{} |0,p,\alpha\rangle$ ← spinor index

(This is necessary since vacuum must carry repres. of ψ_0^μ's:

$\{\psi_r^\mu, \psi_s^\nu\} = \delta_{r+s}\eta^{\mu\nu}$, Cliff.-alg. for $r=s=0$.)

By level matching, this must be combined with the 1st excited level of L-moving sector:

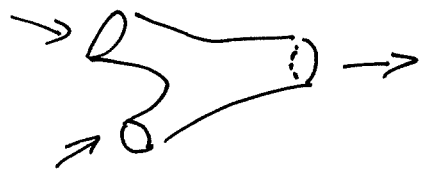
$$\epsilon_\mu \psi^\mu_{-1/2} |0,p\rangle \quad ; \quad \text{massless} \rightarrow \text{gravitini (etc.)}$$

Analogously: NS-R, R-R
(10d fermions) (10d bosons)

- For each (r & l) subsector we can also define a projector: $P = \frac{1}{2}\left(1 + (-1)^F\right)$
  ⇓ splitting in even/odd subspace of P
  F ↑ fermion number
- In total, this gives 10 consistent pairings

  (NS−, NS−)
  (NS+, NS+)
  (NS+, R−)
  (R−, NS+)
  ⋮

  } 2^{10} possibilities to choose a subset of 10
- Not all choices consistent with interactions:
- Need selection of sectors or "GSO projection"



⇒ Two consistent choices w/o tachyons exist:

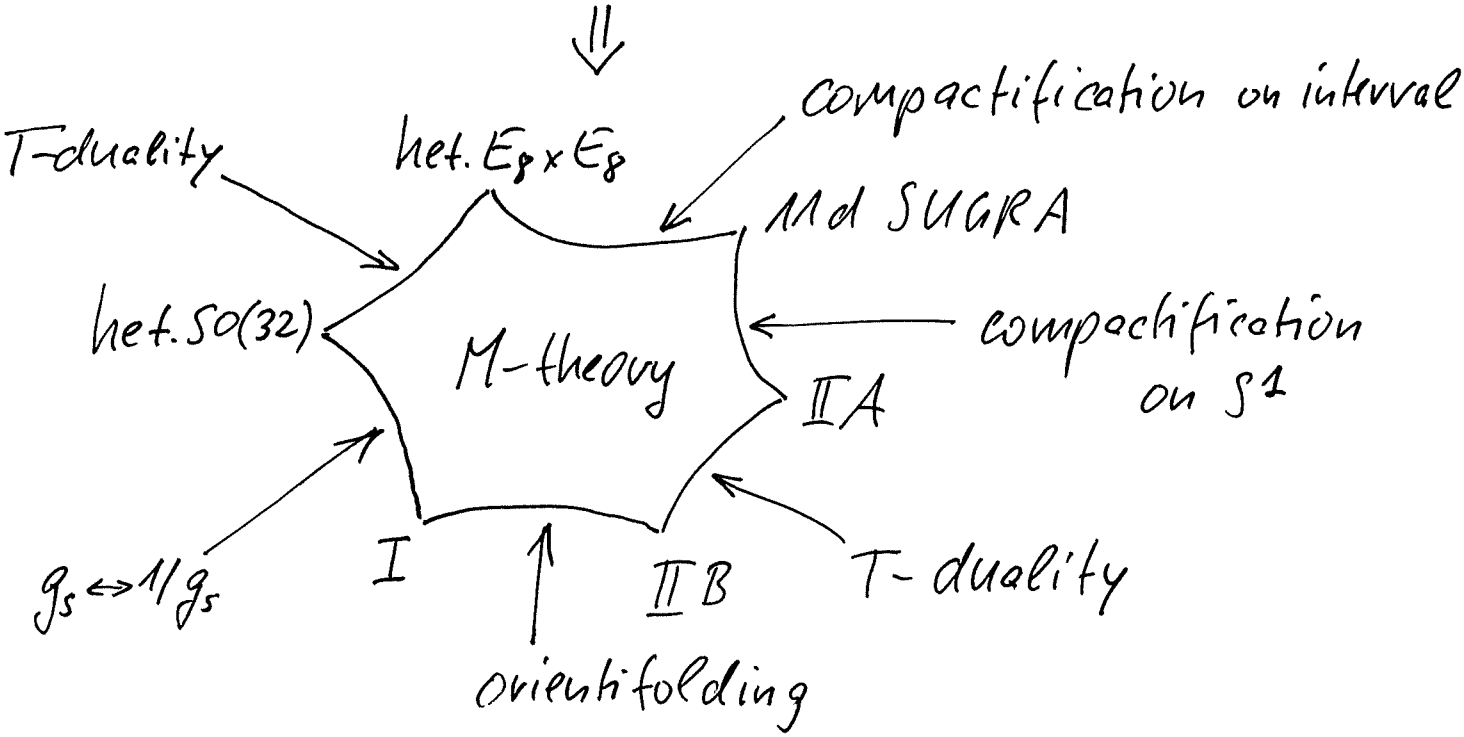
type IIA ; type IIB

(Here "II" means: $\mathcal{N}=2$ SUSY in 10d, i.e. $g_{\mu\nu} \leftrightarrow \lambda^{I\mu} \leftrightarrow \lambda'^{I\mu}$.)

- A small set of further consistent constructions exist:
  - unoriented string ("type I")
  - supersymmetrize only l. or only r.-movers (heterotic $E_8 \times E_8$ & heterotic $SO(32)$)
  - 11d SUGRA with 3d-objects (M2-branes) as fundamental degrees of freedom.

- They are all related by <u>dualities</u> (different descriptions of the same physical theory)

[Example: $F = dA$ electric, $*F = \tilde{F} = d\tilde{A}$ magnetic > formulation of electrodynamics]



3 Type IIB supergravity

3.1 10d action

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left\{e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}H_3^2\right]\right.$$

$$\left. - \frac{1}{2}F_1^2 - \frac{1}{2\cdot 3!}\tilde{F}_3^2 - \frac{1}{4\cdot 5!}\tilde{F}_5^2\right\} + S_{CS} + S_{loc.} + S_F$$

$$2\kappa_{10}^2 = (2\pi)^7\alpha'^4$$

- The key novelty w.r.t. the bosonic string are

  p-form-fields: $F_p = dC_{p-1}$

  (just like $H_3 = dB_2$)

  $H_{\mu\nu s}$ ↑ ↑ $B_{\mu\nu}$

- Explicitly:

$$\tilde{F}_3 = F_3 - C_0 \wedge H_3 \quad ; \quad \tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$$

$$\sim (C_2)_{[\mu_1\mu_2}(H_3)_{\mu_3\mu_4\mu_5]}$$

- This implies special gauge trfs., e.g. $C_2 \to C_2 + d\lambda_1$ & $C_4 \to C_4 + \frac{1}{2}\lambda_1 \wedge H_3$

- Note: The above action and its IIA-cousin are the only existing maximally supersymm. actions in $D = 10$.

$$S_{CS} = \frac{1}{4\kappa_{10}^2} \int \underbrace{C_4 \wedge H_3 \wedge F_3}$$

10-form, can be integrated over 10d spacetime without $\sqrt{-g}$.

- Think of the above as "$A_\mu j^\mu$"

  ↑ gauge field C_4 ↑ source $H_3 \wedge F_3$

  (This structure will be crucial below.)

- S_F – fermionic terms ("gravitini/dilatini")

- Note: NS-NS → $g_{\mu\nu}, B_{\mu\nu}, \phi$

  R-R → C_p

  R-NS → fermions

3.2 D-branes and their action

Recall basic electrodynamics:

A_μ or $A_1 \equiv (A_1)_\mu dx^\mu$ ⇒ demands existence of e^- with:

$$S_{e^-} = \int_{WL} A - m \int_{WL} d\tau$$

worldline ↑ $\triangleq A_\mu dx^\mu$ ↑ eigentime

- Similar logic applies to the <u>bosonic string</u>:

B_2 demands existence of a 2d object with

$$S = \int_{ws} B_2 - T \int_{ws} d\Sigma$$

($d\Sigma$: surface element)

(of course, we already know this object. It is the "<u>fundamental string</u>")

- For the <u>10d superstring</u> we found:

gauge fields C_p — p even for IIB
\ p odd for IIA

⇒ demands existence of "$D(p-1)$-branes"

($p-1$: counts only spatial dimensions, s.t.

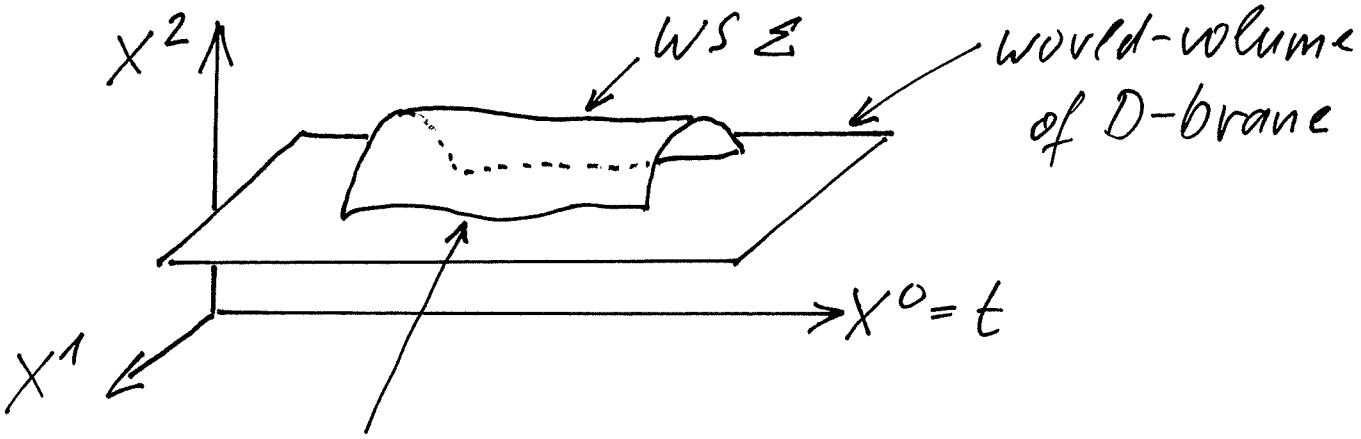
D0 = particle, D1 = "string", D2 = membrane etc.

- The actions are "natural", as above. E.g.:

$$S_{loc} \supset S_{D3} = \frac{1}{2\pi^3 \alpha'^2} \int_{D3} C_4 - \int_{D3} d^4\xi \sqrt{-g}\, T_3 \quad , \quad T_3 = \frac{e^{-\phi}}{(2\pi)^3 \alpha'^2}$$

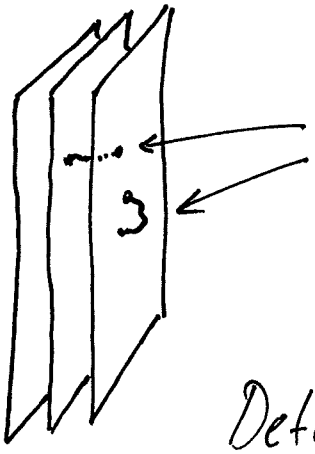
- Thus, Dp-branes couple to C_{p+1}-forms.

- "D" stands for "Dirichlet" since they can also be understood as defining boundary conditions for open strings:



$$X^2(\partial\Sigma) = X^2_{\text{D-brane}} = \text{fixed.}$$

- One can "stack" D-branes:



strings can end on the same or on different branes

$\Downarrow$

Detailed analysis of this type of open superstrings reveals:

A stack of N Dp branes realizes an $SU(N)$ $(p+1)$-dimensional super-Yang-Mills-theory

4 Compactifications

To describe the real world, consider target space $\mathcal{M}_{10} = \mathbb{R}^{1,3} \times X_6$, X_6 – compact.

4.1 5d → 4d toy model

$\mathcal{M} = \mathbb{R}^{1,3} \times S^1$

$$S = \int_{\mathcal{M}} d^5x \; \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) \quad , \quad \mu \in \{0,1,2,3,5\}$$

$x^\mu = \{x^0, \dots, x^3\} \in \mathbb{R}^{1,3}$, $x^5 \equiv y \in (0, 2\pi R)$

- Ansatz: $\phi(x,y) = \sum_{n=0}^{\infty} \phi_n^c(x) \cos\left(\frac{ny}{R}\right) + \sum_{n=1}^{\infty} \phi_n^5(x) \sin\left(\frac{ny}{R}\right)$

- Performing $\int dy$, one finds a 4d action with one massless and many massive scalars ("Kaluza-Klein modes")

$$S = 2\pi R \int d^4x \left[\frac{1}{2} (\partial \phi_0^c)^2 + \frac{1}{4} \sum_{n=1}^{\infty} \left\{ (\partial \phi_n^c)^2 + m_n^2 (\phi_n^c)^2 + (\partial \phi_n^5)^2 + m_n^2 (\partial \phi_n^5)^2 \right\} \right]$$

- ϕ_0^c has no potential at all – can take any value ("it is a modulus")

$m_n = \frac{n}{R}$ $\Rightarrow$ KK-tower is irrelevant for low-energy ($E \ll 1/R$) observer

4.2 The "historical" Kaluza-Klein theory

$\mathcal{M} = \mathbb{R}^{1,3} \times S^1$, $\quad S = \frac{M_{P,5}^3}{2} \int d^4x\, dy\, \sqrt{-g}\, \mathcal{R}_5$

Ansatz:

$$(g_5)_{MN} = \left(\begin{array}{c|c} g_{\mu\nu} + (2/M_p^2)\,\phi^2 A_\mu A_\nu & (\sqrt{2}/M_p)\,\phi^2 A_\mu \\ \hline (\sqrt{2}/M_p)\,\phi^2 A_\nu & \phi^2 \end{array}\right)$$

$M, N, \ldots \in \{0, \ldots, 3, 5\}$; $\mu, \nu, \ldots \in \{0, \ldots, 3\}$

Let $\langle (g_5)_{MN} \rangle = \left(\begin{array}{c|c} \eta_{\mu\nu} & 0 \\ \hline 0 & \phi^2 \end{array}\right)$, $M_p^2 = 2\pi R \phi M_{P,5}^3$

$\Downarrow$ 4d

$$S = \int d^4x \sqrt{-g}\, \phi \left(\frac{M_p^2}{2} \mathcal{R} - \frac{1}{4} \phi^2 F_{\mu\nu} F^{\mu\nu} + \frac{M_p^2}{3} \frac{(\partial\phi)^2}{\phi^2} \right)$$

[Can be brought to Einstein-form by $g_{\mu\nu} \to g_{\mu\nu}/\phi$.]

Lessons: • $U(1)$-symm. of $\mathcal{M}$ $\Rightarrow$ $U(1)$ gauge-symm. in 4d

• The "modulus" ϕ determines

$$\mathrm{Vol}(S^1) = 2\pi R \phi .$$

4.3 T-duality

A very deep and important feature of string theory emerges upon compactification on S^1:

IIA on $\mathbb{R}^{1,8} \times S^1$ → Same theory in 9d, including all massive states & interactions!

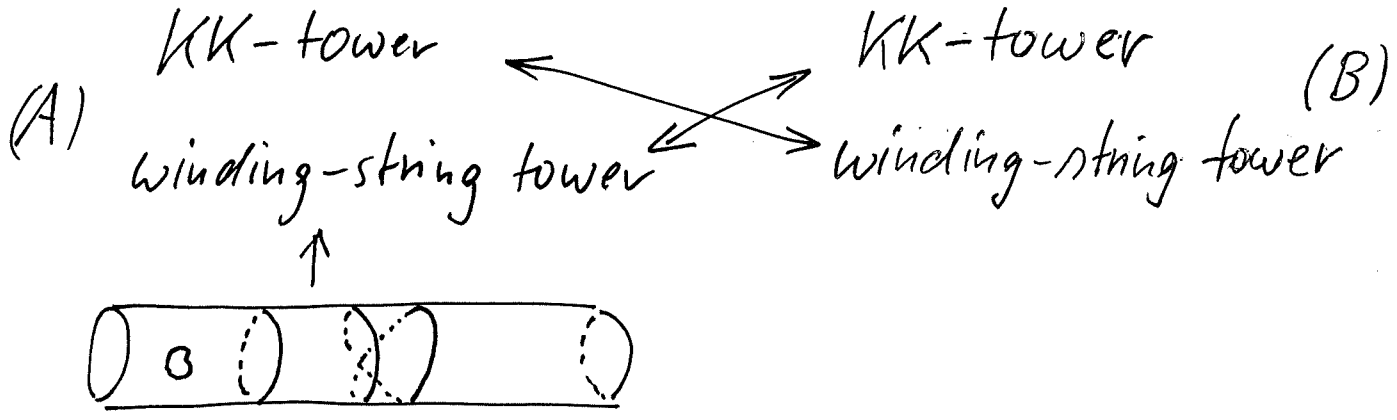
IIB on $\mathbb{R}^{1,8} \times S^1$ ↗

But: This identification is non-trivial.

It requires IIA on S^1 with R

IIB on S^1 with $R' = \frac{\alpha'}{R}$

the matching works as follows:



Implication: String theory does not allow distances $< l_s$. Making an S^1 small implies that a large S^1 "opens up" in the dual description.

4.4 Calabi-Yaus — basic facts

- We want $\mathbb{R}^{1,3} \times X_6$ to solve Einstein equs., s.t. our 4d world is not immediately unstable.
- This needs $R_{MN}(g_{6d}) = 0$, i.e. "Ricci-flatness". (Note that $R_{MNP}{}^{Q} \neq 0$ is OK.)
- Many examples of such manifolds are "Calabi-Yau". (In some sense most.)
- More formally, CYs are defined by a number of features:

1) They are complex (3d) manifolds, i.e. the transition maps are $z'^i = z'^i(\underbrace{z^1, z^2, z^3}_{\text{no } \bar{z}\text{'s here!}})$

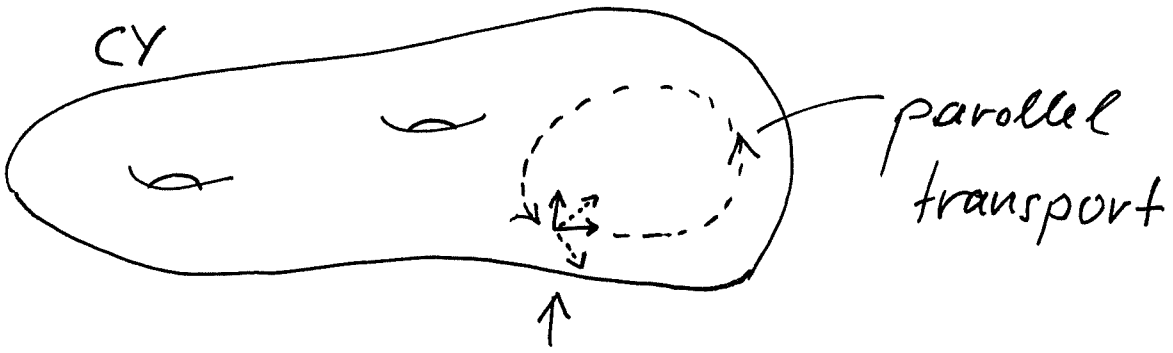
2) They are "Kahler", i.e.

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial z^i \partial \bar{z}^{\bar{j}}} \longleftarrow \text{Kahler potential}$$

$1,2,3 \quad \bar{1},\bar{2},\bar{3}$

$(\& \; g_{ij} = g_{\bar{i}\bar{j}} = 0)$

3) CY's have <u>SU(3) holonomy</u>, i.e.



Transformation in tangent space is in

$$SU(3) \subset SO(6)$$

↑ CY! ↑ generic case for 6d manif.

One can show:

This is equivalent to the existence of a covariantly constant spinor and to unbroken ($\mathcal{N}=2$) SUSY after KK reduction.

- CYs can be deformed in many ways without losing their crucial feature $R_{mn}=0$.

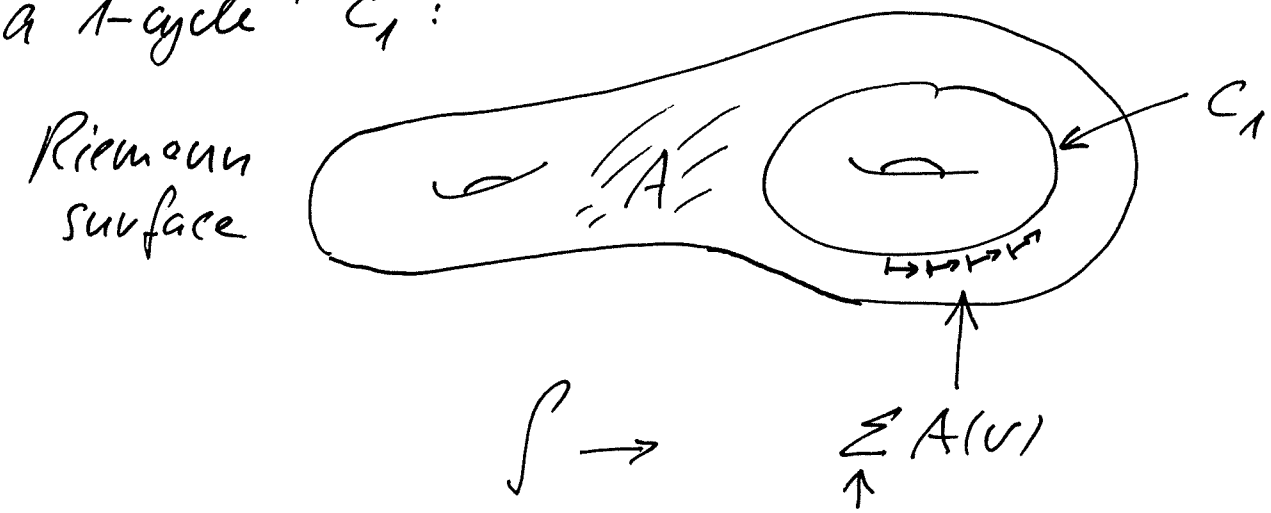
  $\Rightarrow$ "Moduli space" of a CY.

  (Morally this corresponds to the field space of $\phi \sim \mathrm{Vol}(S^1)$ of our S^1 toy model)

- "Moduli" are 4d scalars and the moduli space is the space in which they take their values. (cf. "sigma model", where this space is $SU(2)$.)

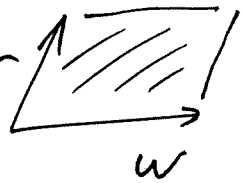
4.5 Differential forms / homology / cohomology

- Let $A = A_\mu dx^\mu$ be a 1-form.
- If "naturally" acts on a vector $v = v^\mu \partial_\mu$ by: $A(v) = A_\mu v^\nu \underbrace{dx^\mu(\partial_\nu)}_{\delta^\mu{}_\nu} = A_\mu v^\mu$
- This allows us to integrate A along a "1-cycle" C_1:



- Similarly, a 2-form $B_2 = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu$ acts on two vectors, "measuring" the area of the corresponding parallelepiped:

$B(v,w) = B_{\mu\nu} v^\mu w^\nu$



$\Rightarrow \int_{C_2} B_2 \equiv \sum_{\substack{\text{infinites.} \\ \text{parallelepipeds}}} \text{"areas"}$ (no metric needed!)

- We mainly care about "closed" forms, i.e. with

$$dB_2 = \frac{1}{2}\,\partial_\mu(B_{\nu\rho})\,dx^\mu \wedge dx^\nu \wedge dx^\rho = 0$$

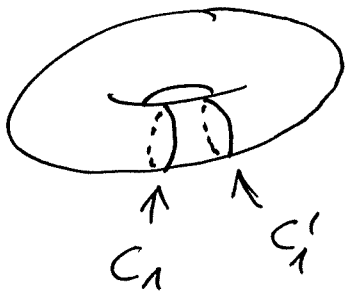
- We identify forms differing by an "exact" form (i.e. by a "gauge transformation" $d\Lambda$):

$$B_2 \sim B_2' \quad \text{if} \quad B_2 - B_2' = dA_1$$

$$A_1 \sim A_1' \quad \text{if} \quad A_1 - A_1' = d\Lambda_0 \quad \text{etc.}$$

- This reduced space of forms is called "cohomology" and has finite dimension for a compact manifold.

- It is dual to the space of cycles, identified if they can be deformed into each other:



$C_1 \sim C_1'$

- The vector-space-duality works by integration:

$$C_1(A_1) = \int_{C_1} A_1 \quad \longleftrightarrow \quad \begin{matrix} X^\mu, Y_\mu \\ \uparrow \quad\quad \uparrow \\ V \quad\quad V^* \end{matrix}$$

4.6 CY moduli space

For a CY, there are two key forms:

1) The Kähler form

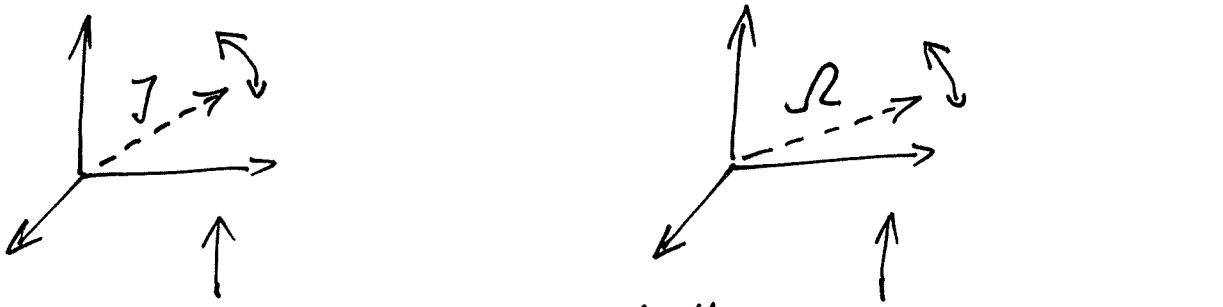
$$J = i\, g_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}} \quad \rightarrow \text{"metric information"}$$

2) The holomorphic 3-form

$$\Omega = \frac{1}{3!}\,\Omega_{ijk}(z^1, z^2, z^3)\, dz^i \wedge dz^j \wedge dz^k \sim \bar{\psi}\gamma^i\gamma^j\gamma^k\psi \cdots$$

$\rightarrow$ "complex-structure information"

covariantly constant spinor



The dimensions of the spaces are fixed by the numbers of 2-cycles and 3-cycles of our CY.

- Coordinates for explicit parametrization:

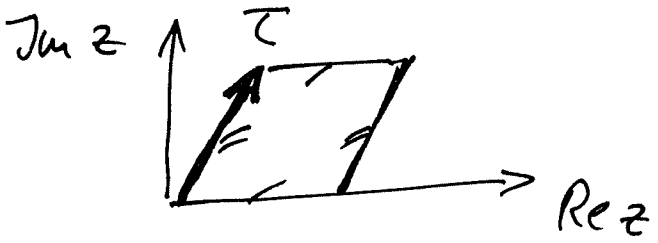
$$\int_{C_2^\alpha} J \sim t^\alpha \quad \text{(2-cycle volumes)}$$

$$\int_{C_3^a} \Omega \sim \Pi^a(z) \quad \text{(3-cycle volumes} \sim \text{"periods")}$$

4.7 Torus toy model

T^2 is a "CY 1-fold": $z = x + \tau y$

$x, y \in [0, 1]$



$$\Omega = \alpha\, dz \longrightarrow \Pi_1 = \int_{y=\text{const.}} \Omega = \alpha$$

$$\longrightarrow \Pi_2 = \int_{x=\text{const.}} \Omega = \alpha\tau$$

$$J = t\, dx \wedge dy \longrightarrow \text{Vol}(T^2) = \int_{T^2} J = t$$

$$g_{ab} = \frac{t}{\text{Im}(\Pi_2/\Pi_1)} \begin{pmatrix} 1 & \text{Re}(\Pi_2/\Pi_1) \\ \text{Re}(\Pi_2/\Pi_1) & |\Pi_2/\Pi_1|^2 \end{pmatrix}$$

Remark: The moduli space is the space of non-equivalent metrics. It can be described in terms of (the components of) J & Ω.

4.8 Explicit CY moduli spaces

Use $\mathcal{N}=1$ SUGRA language, s.t.

$$\mathcal{L} = K_{I\bar{J}}\,(\partial\varphi^I)(\partial\bar{\varphi}^{\bar{J}}) + \text{gauge} + \text{fermions} + \cdots$$

↑ Derivatives of 4d Kähler potential
(not to be confused with Kähler pot. of CY metric)

① $J = t^\alpha \omega_\alpha$, $\alpha = 1, \cdots, h^{1,1}$

↑ basis 2-forms

CY volume: $V = \frac{1}{6}\int_X J\wedge J\wedge J = \frac{1}{6} K_{\alpha\beta\gamma}\, t^\alpha t^\beta t^\gamma$

↑ topolog. data

A useful (for us) change of variables:

$$t^\alpha \longrightarrow \tau_\alpha = \frac{1}{2}\int_{C_{(4)}^\alpha} J\wedge J = \frac{\partial V}{\partial t^\alpha}$$

$$\Rightarrow K_K = -2\ln V(T_\alpha, \bar{T}_\alpha)$$

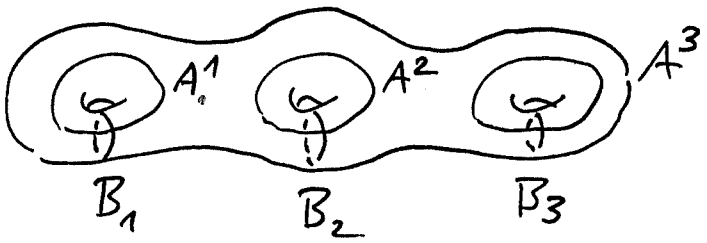
↑ "Kähler"

$T_\alpha = \tau_\alpha + i c_\alpha$; $c_\alpha = \int_{C_{(4)}^\alpha} C_4$

---

② $\Omega \to \cdots$ periods: $z^a = \int_{A_{(3)}^a} \Omega$; $\mathcal{G}_b(z) = \int_{B_{(3)b}} \Omega$

A's & B's : symplectic basis of 3-cycles, like...



$$K_{cs} = -\ln\left(i\int_X \Omega\wedge\bar{\Omega}\right) = -\ln\left(-i\bar{z}^a \mathcal{G}_a(z) + c.c.\right)$$

↑ "compl. structure"

↑ calculated by solving PDEs for each CY "by hand."

Finally:

$$K = K_K(T^\alpha, \bar{T}^{\bar{\alpha}}) + K_{cs}(z^a, \bar{z}^{\bar{a}}) - \ln(-i(S-\bar{S}))$$

$$S \equiv C_0 + \frac{i}{g_s} \leftarrow \text{"axio-dilaton"}.$$

$M_{P,4} = 1$; All CY-related volumes are measured in units of $\ell_s = 2\pi\sqrt{\alpha'}$.

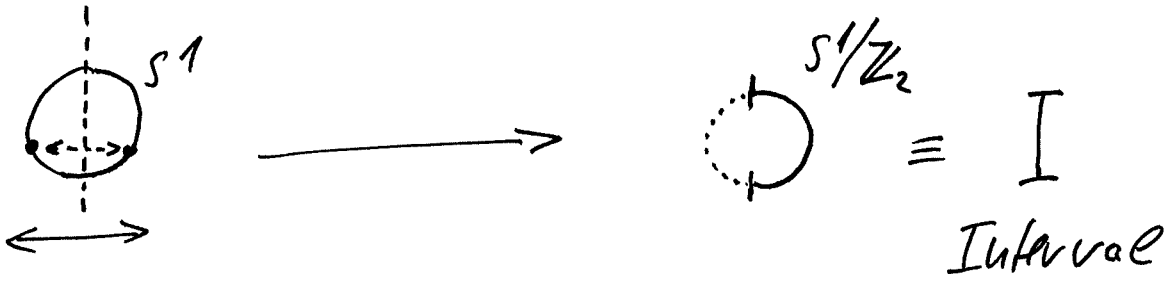
5 The flux landscape

(Bousso/Polchinski ; Giddings/Kachru/Polchinski)

"GKP"

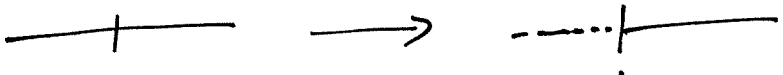
5.1 Orbifolds

A useful way of constructing new compact spaces is by "modding out" a discrete symmetry:



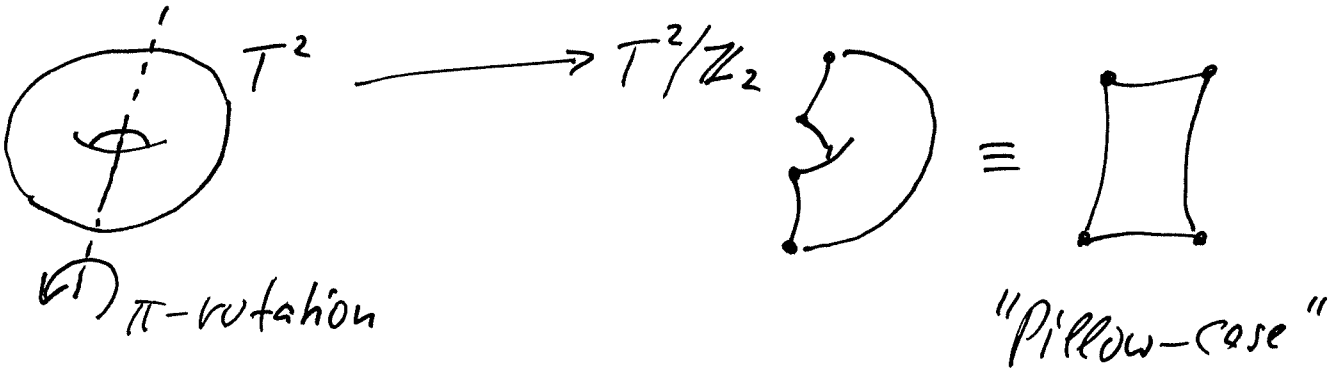
locally, this is just

$$\mathbb{R} \longrightarrow \mathbb{R}_+ = \mathbb{R}/\mathbb{Z}_2$$



↑ a singularity, in this case a boundary, is created

<u>A more interesting, 2d example:</u>



[Singularities of this type are OK for the stringy UV completion as long as some SUSY is preserved.]

<u>Even better:</u> $CY \longrightarrow CY/\mathbb{Z}_2$

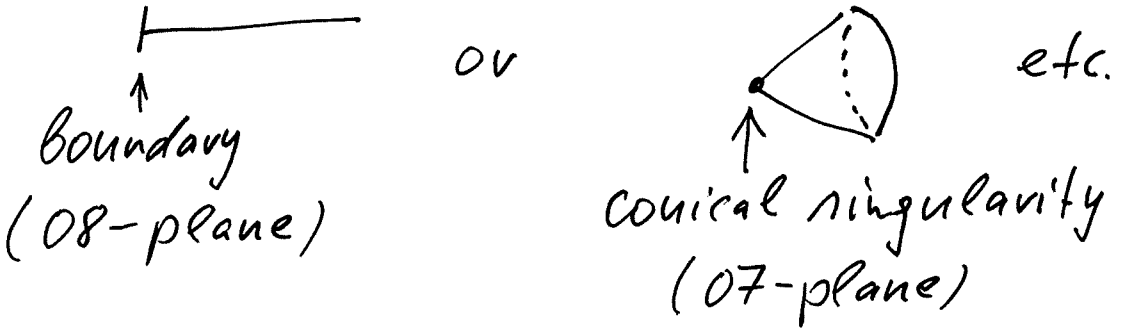
with $\mathbb{Z}_2$ locally e.g. $(z^1, z^2, z^3) \longrightarrow (-z^1, -z^2, -z^3)$

5.2 Orientifolds

- Replace $\mathbb{Z}_2$-action by <u>the same</u> geometric action, e.g. $y \to -y$, combined with an orientation change of the string:

$$(\tau, \sigma) \to (\tau, -\sigma).$$

- The 10d geometry is then modified in the same way as for orbifolds, e.g.

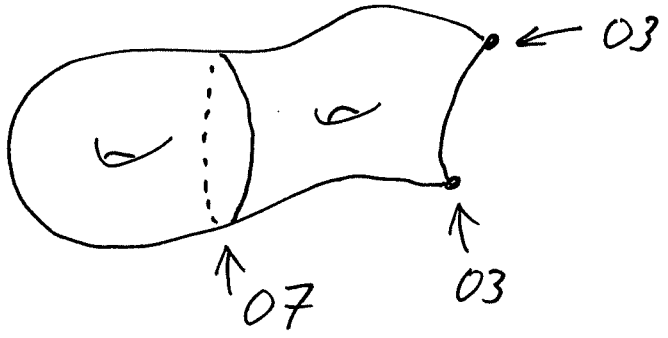


- But, <u>crucially</u>, the singular locus acquires non-trivial charges/couplings:

> The induced Op-planes carry opposite charge w.r.t. the corresponding Dp-branes, e.g.

> An O7 plane needs 4 D7 branes to be placed "on top" to be neutral w.r.t. the gauge potential C_8.

- Most relevant for us: O7, O3 (with D7, D3 on top).
- Thus, we consider type IIB on $CY/\mathbb{Z}_2$ ("CY-orientifolds") with O3/O7-planes:



- Need to include D7/D3s to make the whole compact geometry charge-neutral and hence consistent (technical term: "Tadpole cancellation").
- <u>Crucially</u>: One may choose <u>not</u> to cancel the "D3 tadpole" by D3-branes <u>but instead</u> by <u>fluxes</u>:

> Background-values (quantized) of F_3, H_3: $\langle F_3 \rangle, \langle H_3 \rangle$

- To see this, recall:

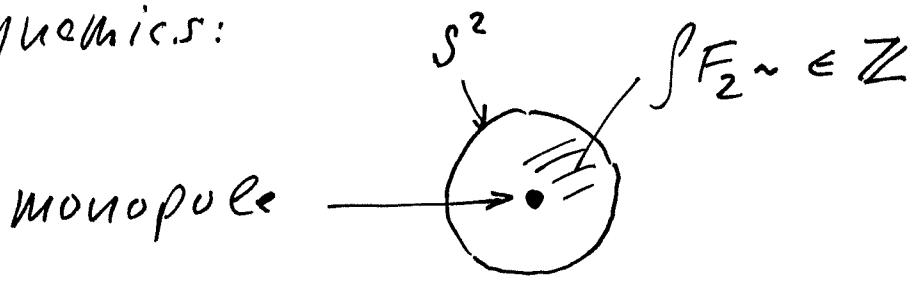
$$S_{IIB} \supset \int C_4 \wedge \underbrace{H_3 \wedge F_3}_{\text{"} j_{D3} \text{"}} + \int_{D3} C_4$$

$\Rightarrow \langle H_3 \wedge F_3 \rangle \triangleq$ D3's in CY

$\Rightarrow$ IIB orientifolds allow for the presence of H_3, F_3-flux with a specific, limited "tadpole", i.e. $\int H_3 \wedge F_3 \leq |Q_3| \in \mathbb{Z}$

5.3 Flux compactifications "at GKP level"

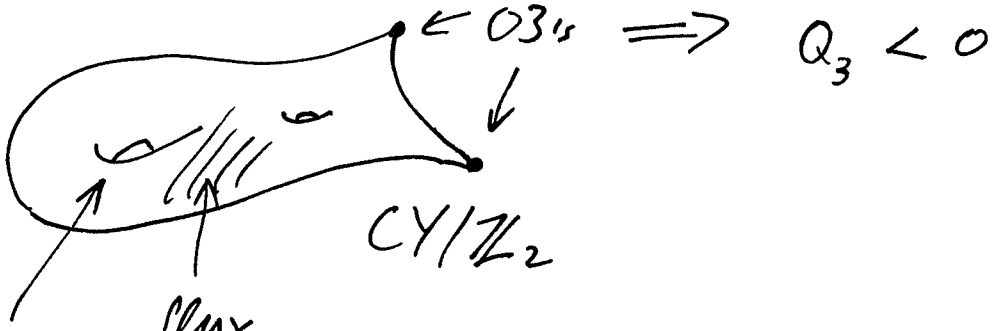
- Key fact: H_3, F_3 are quantized in the sense that $\int_{C_{(3)}} H_3 \,,\; \int_{C_{(3)}} F_3 \sim \in \mathbb{Z}$
- This is completely analogous to the magnetic charge quantization familiar from electrodynamics:



[Reason: $\psi_{\text{electron}}(\theta,\varphi)$ can otherwise not be a well-defined fct. on the whole S^2.]

- In our case: $\psi_{D1/F1}$ must be well-defined
  ↑ fund. string

- Thus, our model is specified by a CY-orientifold with a certain (negative) tadpole:



3-cycles & 2/4-cycles

fluxes ↖ quantified by integer-valued vectors $f, h \in (\mathbb{Z})^{h^3}$

- Must obey:

$$\int H_3 \wedge F_3 \sim f \cdot h = -Q_3 - N_{D3}$$

$f \cdot h$: symplectic metric

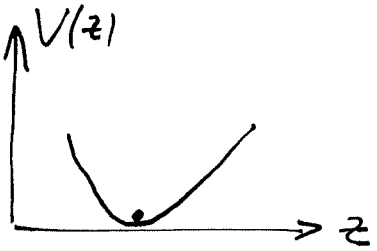
N_{D3}: (pos.) number of D3 branes

- Since $S \supset \int |F_3|^2 + |H_3|^2 e^{-2\phi}$,

(the underbrace: involves metric)

the presence of fluxes "stabilizes" 3-cycle moduli and axio-dilaton

"z^a" "$S = C_0 + \frac{i}{g_s}$"



- To quantify this, we employ 4d $N=1$ supergravity language:

$$\mathcal{L} = \frac{1}{2}R + K_{I\bar{J}}(\partial\varphi^I)(\partial\bar{\varphi}^{\bar{J}}) - V(\varphi,\bar{\varphi}) + \cdots$$

with

$$K = K_k(T^\alpha, \bar{T}^{\bar{\alpha}}) + K_{cs}(z^a, \bar{z}^{\bar{a}}) - \ln(-i(S-\bar{S}))$$

&

$$W = \int G_3 \wedge \Omega_3 = (2\pi)^2 (f - Sh)\cdot \Pi(z)$$

($G_3 \equiv F_3 - S H_3$; $\Pi(z)$: z^a, $G_b(z)$)

"Gukov-Vafa-Witten superpotential"

&

$$V = e^K \left(K^{I\bar{J}} (D_I W)(\overline{D_J W}) - 3|W|^2 \right)$$

with $I \to (\alpha, a, S)$

(α: Kähler; a: compl.-structure; S: axio-dilaton)

& $D_I W = \partial_I W + K_I W$

- The following crucial cancellation occurs:

$$V \supset e^K \left(K_k^{\alpha\bar{\beta}} (D_\alpha W)(\overline{D_\beta W}) - 3|W|^2 \right) \equiv 0$$

if: • W is independent of T^α

• $K_K = -2 \ln \underbrace{V(T^\alpha, \bar{T}^{\bar{\alpha}})}$

homogeneous fct. of degree $3/2$.

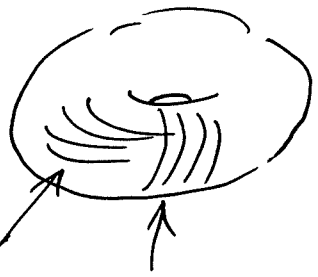
[This is the famous "no-scale structure" of this type of supergravity model.]

$\Rightarrow V = e^K K^{i\bar{j}} (D_i W)(\overline{D_j W})$

with $i \to (\alpha, S)$, i.e. just compl.-structure moduli & axio-dilaton

- This V is manifestly ≥ 0.
- Minima are at $D_i W = 0$, which generically stabilizes ("fixes") z^α & S.

$\Rightarrow$ Fluxes stabilize 3-cycle-ratios, as intuitively expected:



fluxes on two different 1-cycles of a torus.

$\Rightarrow V \equiv 0$ (independently of T^α)

$\Rightarrow T^\alpha$'s are not stabilized – their potential remains flat.

$\Rightarrow$ Volume (and all 4-cycle ratios) unfixed.

SUSY-scale can change freely

$\Rightarrow$ name "no-scale model".

Note: $D^i W \equiv F^i = 0 \Rightarrow$ no SUSY in compl.-structure sector

$D^\alpha W \equiv F^\alpha \neq 0 \Rightarrow$ SUSY in Kähler sector

$(|D^\alpha W|^2 = 3|W|^2)$

$\Rightarrow$ Loop-corrections will induce a potential also for the T^α's, but this is hard to calculate.

Summary: ("GKP")

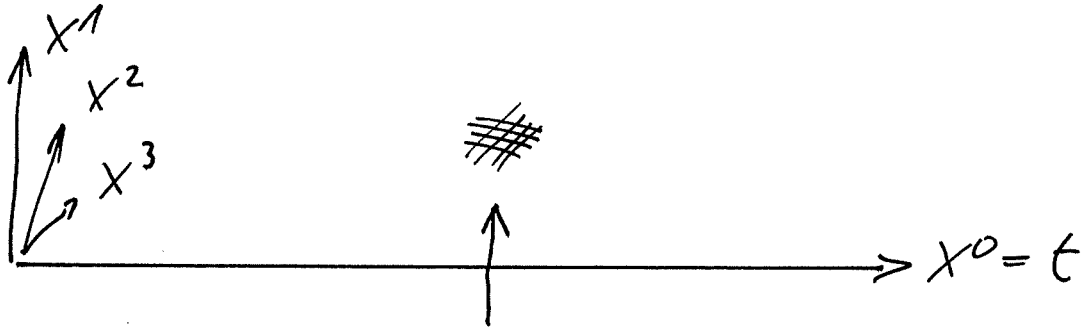
z^a, S fixed ; T^α free ; SUSY ;

$V \equiv 0$ (at LO) $\Rightarrow$ not bad!

6 Volume stabilization and the cosmological constant

6.1 Non-perturbative effects

A) Gauge instantons (e.g. in QCD)

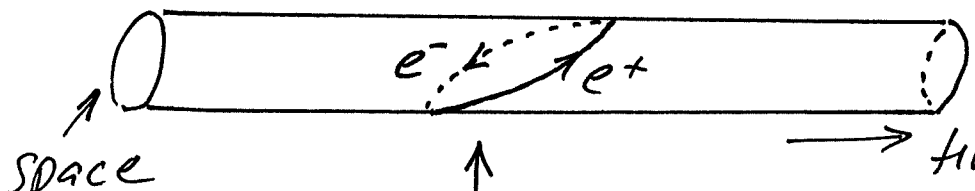


region with $F_{\mu\nu} \neq 0$

(specifically $\frac{1}{8\pi^2}\int \mathrm{tr}(F \wedge F) = 1$.)

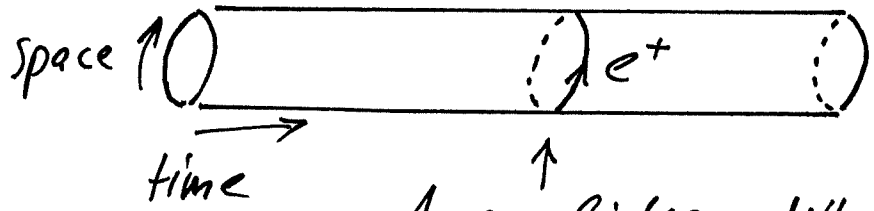
This is a localized tunneling process or "instanton". [Summing over all of them famously induces a potential for the axion field.]

B) QED in $d=2$ compactified on S^1



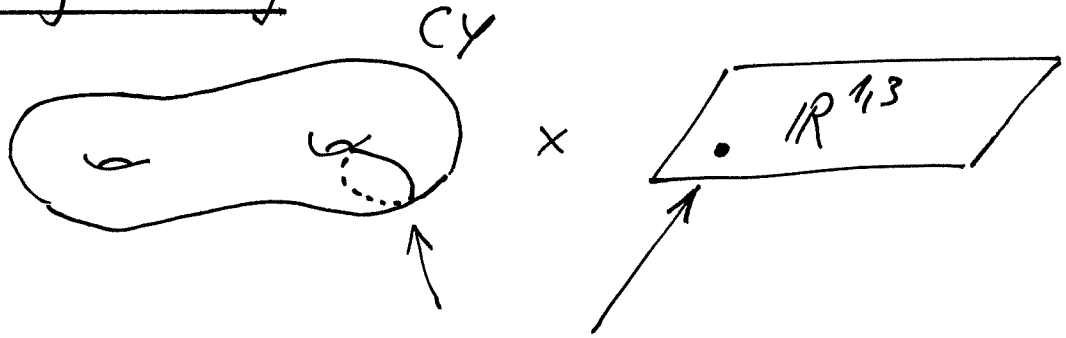
This type of tunneling process is the simplest "brane-instanton".

- A better way to think of this:



A euclidean WL of an electron is wrapped on a 1-cycle of the compact space at a certain locus in (non-compact) time.

c) String theory



Euclidean D3-brane ("E3-brane") wrapped on 4-cycle of CY at a certain locus in $\mathbb{R}^{1,3}$.

- In all cases the potential receives corrections $\sim e^{-S_{inst.}}$
- In the last case: $S_{inst.} \sim \mathrm{Vol}(4\text{-cycle})$.
- Concretely we know: $\mathrm{Vol}\left(C^{\alpha}_{(4)}\right) = \mathrm{Re}\, T^{\alpha}$
- In our case δV is induced by a superpotential-correction δW:

Fact: $$\delta W = \sum_{4\text{-cycles}} A(z)\, e^{-2\pi T^\alpha}$$

[Note: A similar δW-effect is induced by gaugino condensation in $SU(N)$ gauge theories realized by D7-brane stacks on 4-cycles.]

6.2 SUSY-KKLT

(Kachru/Kallosh/Linde/Trivedi)

- Start with a CY-orientifold with a single Kähler modulus:

$$K_K = -2\ln(T+\bar{T})^{3/2} = \underline{\underline{-3\ln(T+\bar{T})}}.$$

- $W = W(S,z)$ due to flux
- $K = K_K(T,\bar{T}) + K_{CS+AD}(S,\bar{S},z,\bar{z})$

$$z = \{z^1, \dots, z^n\}$$

- As explained above, the scalar potential (or equivalently the SUSY conditions $D_z W = D_S W = 0$) fix the moduli S, z.

$\Rightarrow$ Our low-energy EFT has:

$$K = -3\ln(T+\bar{T}) \quad ; \quad W = W_0 = \text{const.}$$

$\uparrow$ $\equiv W(S_0, z_0)$ as just explained

$$V = e^K \left(K^{T\bar{T}} |D_T W|^2 - 3|W|^2 \right) \equiv 0.$$

- Including instanton corrections, we have:

$$W = W_0 + \delta W = W_0 + A(S_0, z_0)\, e^{-2\pi T}$$

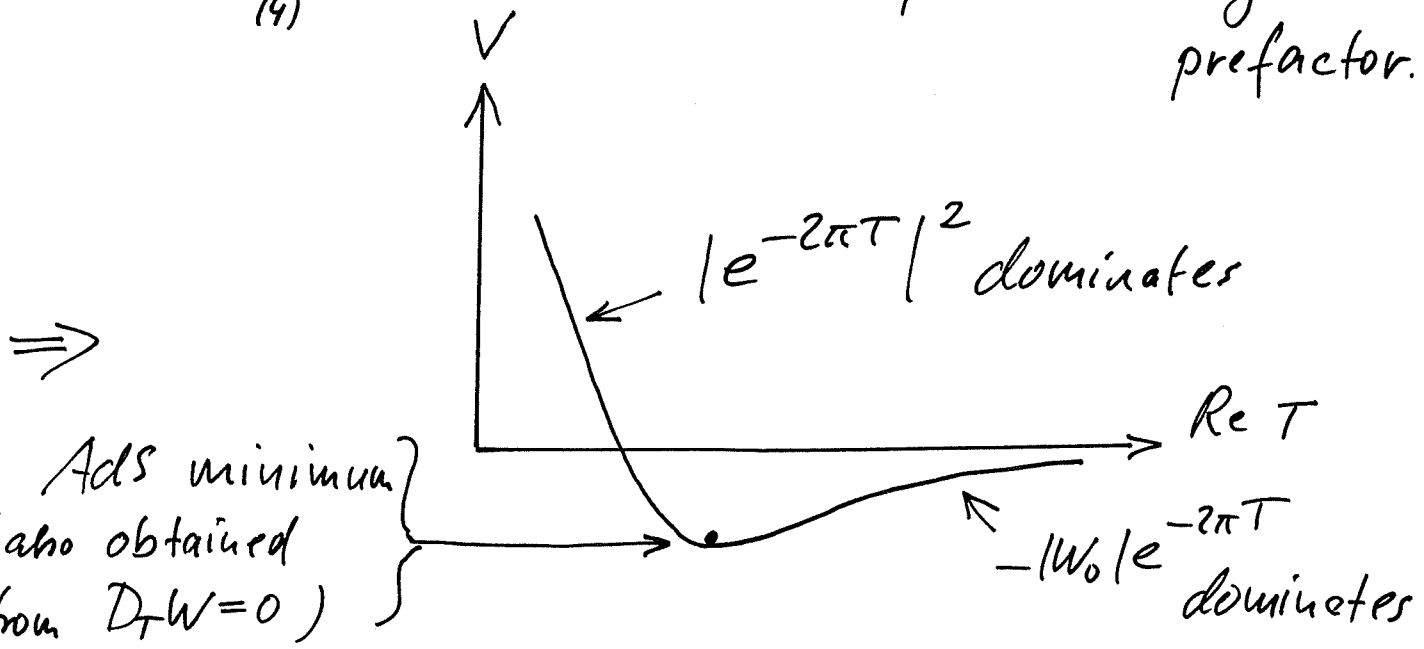
$$\Rightarrow V \sim e^K K^{T\bar{T}} \left(\left| A e^{-2\pi T} \right|^2 + \underbrace{A \bar{W}_0 e^{-2\pi T} + \text{h.c.}}_{} \right)$$

$T = \tau + ic$

$c \sim \int_{C_{(4)}} C_4$

The phase $\exp(-2\pi i c)$ adjusts such that this term acquires a negative prefactor.

$\Rightarrow$



- In this vacuum: $V \sim T^{3/2} \sim \ln^{3/2}(1/|W_0|)$

  $\Rightarrow$ Need $|W_0| \ll 1$ for "control" in 10d SUGRA.

- This is achievable by flux choice, which determines S_0, z_0 and hence W_0

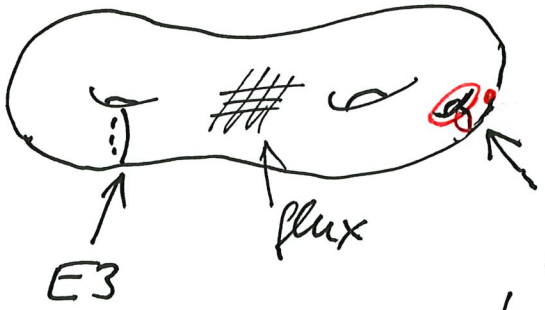
  $\Rightarrow$ Huge "landscape" of SUSY-AdS vacua with all moduli stabilized.

- Key open problems:
  - cosm. const. $\Lambda < 0$
  - SUSY unbroken

6.3 Uplift – general idea & toy model

- Need positive contribution to potential

- Idea:



some local arrangement of brane-stacks and/or singularities giving rise to light fields: "X".

E.g.:

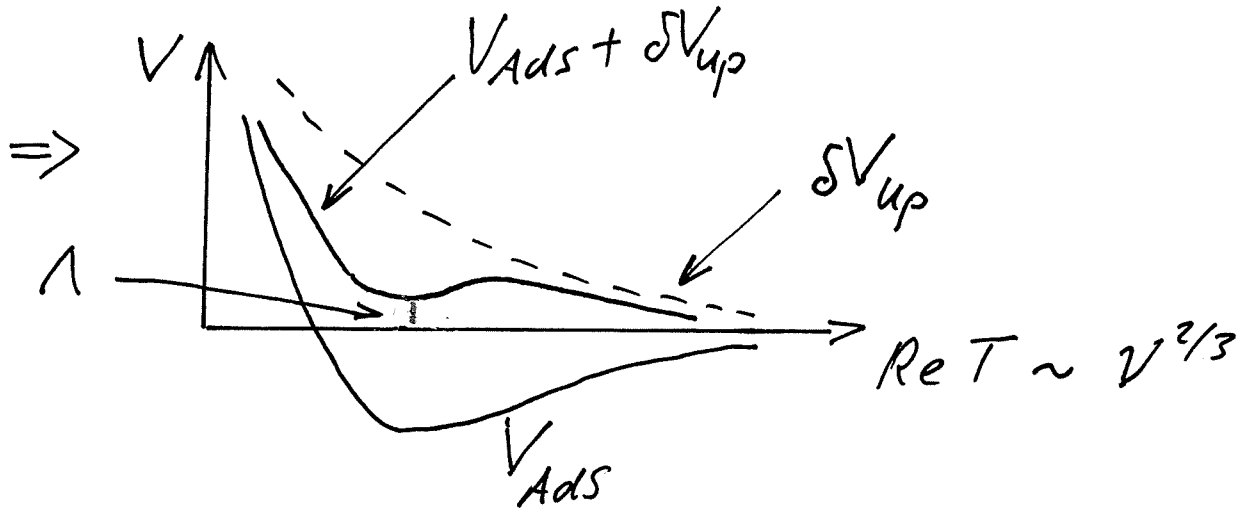
$$K = X\bar{X} - (X\bar{X})^2 \,, \quad W = \alpha X$$

$$\Rightarrow |D_X W| \neq 0 \text{ in vacuum.}$$

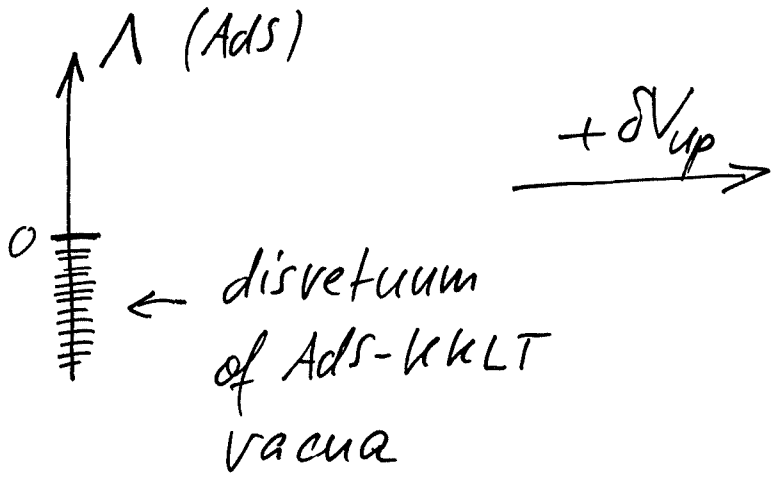
$$\Rightarrow \delta V = e^{K} K^{x\bar{x}} |D_x W|^2 > 0$$

$$\uparrow \sim 1/\mathcal{V}^2$$

(Potential in units of M_p^4 goes to zero because as $\mathcal{V} \to \infty$ because $M_p^2 \sim \mathcal{V}$.)



$$\Lambda = \delta V_{up} + V_{AdS} = \delta V_{up} - c\,|W_0|^2$$



$\leftarrow$ discretuum of AdS-KKLT vacua

(The homogeneity of this distribution has been argued for by Denef/Douglas.)

many vacua with very small $|\Lambda|$ are expected (depending on size of landscape)

6.4 Tuning the cosmological constant

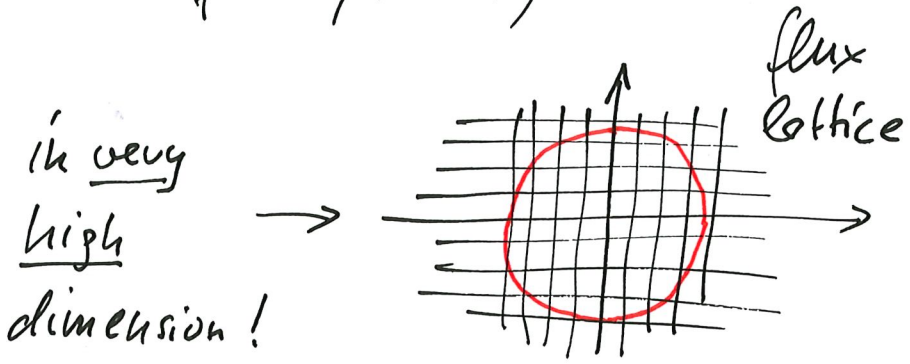
- The key object is the flux vector $\vec{n} = \begin{pmatrix} f \\ h \end{pmatrix}$
- It lives in a lattice of high dimension:

$$\vec{n} \in (\mathbb{Z})^{2h_3}$$

- Its length is constrained by the available negative tadpole: $f \cdot h \leq |Q_3|$

$$\Rightarrow N_{vac.} \sim |Q_3|^{h_3} / h_3!$$

(This corresponds to the scaling of the volume of a sphere.)



- The historical estimate was roughly

$$\underbrace{|Q_3| \sim 10^4}_{\text{By now much larger values known in non-perl. branch of type-IIB ("F-theory").}} ; \; h_3 \sim 300 \Rightarrow N_{vac.} \sim 10^{600}$$

- To appreciate size of landscape, note that there are $\sim 10^{80}$ atoms in visible universe.

- If our logic works, we can find vacua with $|\Lambda|$ as small as $10^{-600} M_p^4$ (i.e. much smaller than the observed value $10^{-120} M_p^4$).

Note: - This does not say anything about why we live in such a "very special" vacuum. (Cf. however Weinberg's prediction of Λ and "anthropic" arguments.)

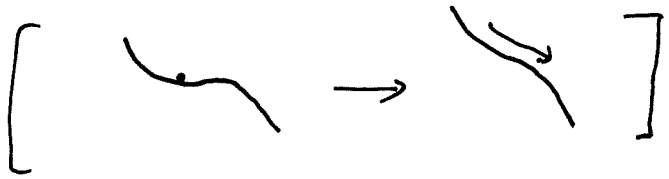
- Our arguments were only about the existence of such vacua in spite of them being strongly fine-tuned.

- Even if $1/10^{120}$ of the 10^{600} vacua have the right Λ, these are still 10^{480}.

$\Rightarrow$ Many further fine-tunings,
e.g. for $m_H \ll m_{susy}$, are possible.

6.5 Uplift – anti-D3s and beyond

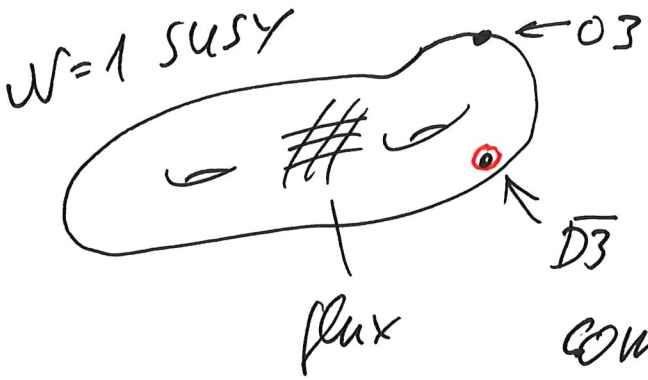
- Since $-V_{AdS} \sim |W_0|^2 \ll 1$, the uplift must be very small. Otherwise, no meta-stable minimum arises:



- So far, no explicit stringy version of our "QFT toy model uplift" has been built.
- The most explicit uplift is the anti-D3 (or "$\overline{D3}$") of KKLT / KPV:
- Starting point:

F_3 / H_3 & D3:
same D3-charge & preserve same $\mathcal{N}=1$ SUSY in our CY.

$\overline{D3}$:
opposite D3-charge and preserve opposite $\mathcal{N}=1$ SUSY of $\mathcal{N}=2$ of CY



$\overline{D3}$ breaks 4d SUSY completely ($N=0$)

↑

It also "uplifts" by $T_{D3} \sim M_s^4$, which is much too high

- Idea: look more carefully at CY geometries in presence of fluxes:

(cf. e.g. "GKP")

↑

Giddings / Kachru / Polchinski

$$ds^2 = h^{-1/2}(y)\, dx^2 + h^{1/2}(y)\, \tilde{g}_{mn}(y)\, dy^m dy^n$$

↑ $IR^{1,3}$; ↑ CY metric

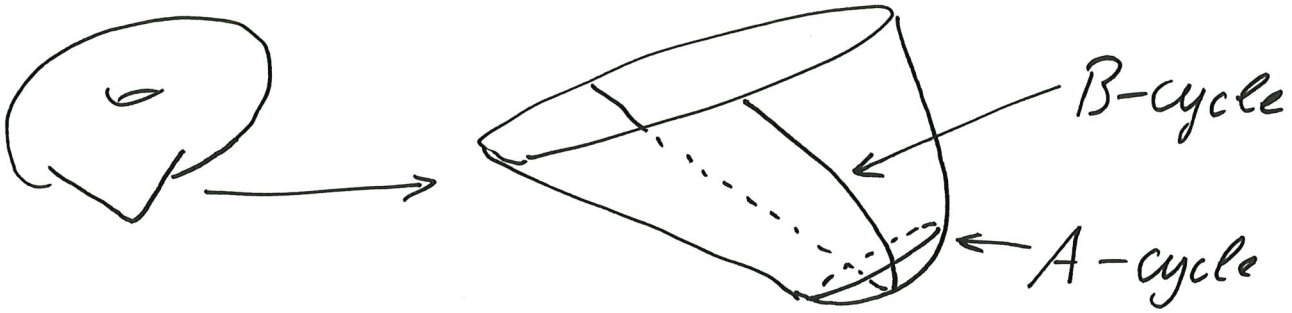
This is not $IR^{1,3} \times$ CY due to warp-factor $h(y)$.

Its EOM: $-\tilde{\nabla}^2 h(y) = g_s \rho_{D3}(y)$

↑

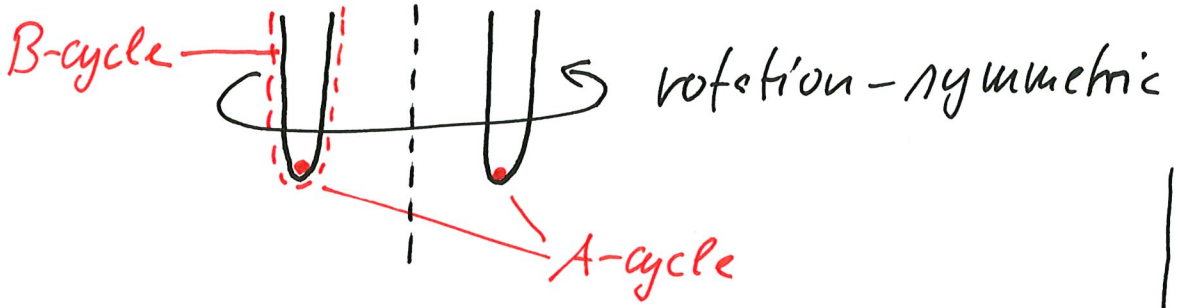
$\sim F_{3 \wedge} H_3 + \sum_i \delta^6(y - y^i_{D3}) + \cdots$

• Fact: CYs generically possess "conifold regions":



two 3-cycles (A, B) with one of them (A) potentially shrinking to zero size:

Illustration by a toy model:

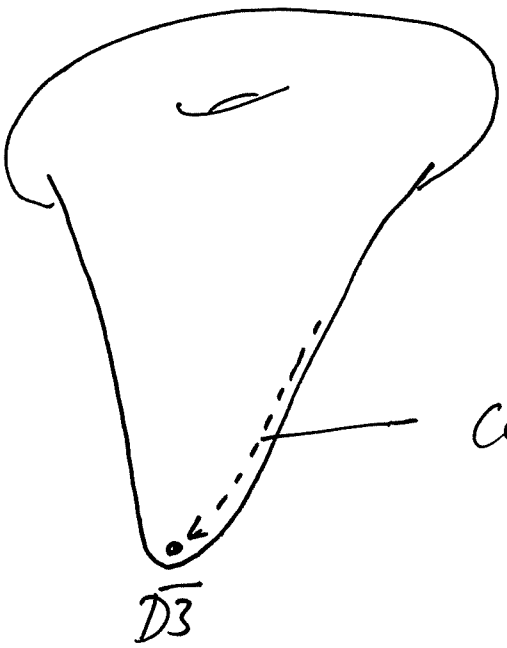


• Now place M units of F_3-flux and K units of H_3-flux on A & B cycle respechively. Let $K \gg M$.

$\Longrightarrow$

$h \gg 1$

- Key conceptual point: $h^{-1/2}(y)\,dx^2$ makes 4d effective mass scales small ("red shifted") in regions of CY where $h \gg 1$.



The $\overline{D3}$ "sinks" to the bottom and stays there, contributing a potential

$$\delta V_{up} \sim T_{D3}\, h^{-1}$$

$$\sim T_{D3} \exp\left(-\frac{8\pi K}{3 g_s M}\right)$$

- This (very explicit) uplifting mechanism, together with "AdS-KKLT" is a celebrated result and historically a cornerstone of the "landscape".

- Various concerns about the (meta-) stability of the $\overline{D3}$ were raised but never substantiated (→ Bena, Graña, Van Riet, Shiu, ...)

- More recently, in the "Swampland" context, these de Sitter constructions have been scrutinized again.
- It was in particular found that: (Carta/Moritz/Westphal; Gao/AH/Junghans)

  a) The constraints on the various parameters enforce $R_{throat} \gtrsim R_{CY}$

  ("strange geometry")

  b) This, in turn, induces such a strong warping (i.e. strongly varying h), that singular regions ($h < 1$) arise.

  ⇓

  "Singular bulk problem"

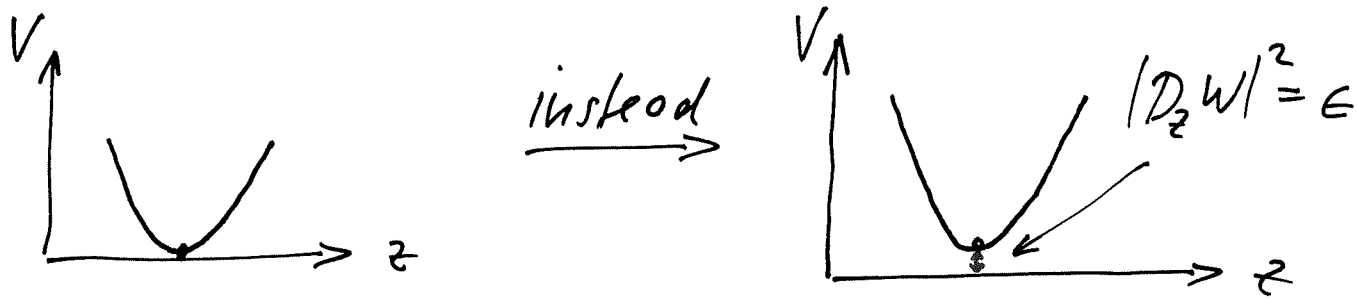
--- c.f. also subsequent work by McAllister, Moritz, ...

⇒ Maybe worth exploring alternative idea:

Complex-Structure F-term uplift

(Saltman / Silverstein)

- Here, one does not start with $|D_z W| = 0$.



- This relies on the existence of non-SUSY local minima of

$$V(z,\bar{z}) \sim |D_z W|^2 \equiv \epsilon \quad \text{with} \quad \epsilon \ll 1.$$

(numerical evidence exists).

Also: statistical analysis by Denef / Douglas

- Other uplift-ideas exist. More work needed!

6.6 Large Volume scenario

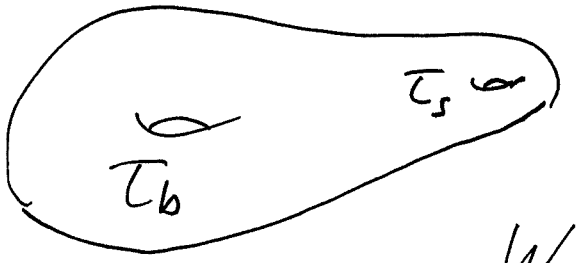
(Balasubramanian / Berglund / Conlon / Quevedo)

- Consider CYs with a minimally more complicated 4-cycle lattice:

$$K_K = -2\ln V = -2\ln\left(\tau_b^{3/2} - \tau_s^{3/2}\right)$$

with $\tau_b = (T_b + \bar{T}_b)/2$

$\tau_s = (T_s + \bar{T}_s)/2$



$$W = W_0 + A e^{-2\pi T_s}$$

(corresponding T_b-term irrelevant)

$$K = K_K + \delta K = -2\ln\left(V + \underbrace{\xi/g_s^{3/2}}\right)$$

"α'-correction"

[Follows from R^4-term in 10d lagrangian.
ξ explicitly known in terms of topological features of CY.]

- One can now explicitly minimize the 4-field scalar potential

$$V = e^K\left(|D_{T_i} W|^2 - 3|W|^2\right).$$

• Result:

$$\tau_s \sim \xi^{2/3}/g_s \quad ; \quad \mathcal{V} \sim \tau_b^{3/2} \sim |W_0| e^{2\pi\tau_s}$$

tunable to $g_s \ll 1$ in landscape

This is exponentially large in τ_s and hence $\mathcal{V} \gg \ell_s^6$ is easy to achieve.

But: Finer effects (like R^4-term) were needed in the analysis.

⇒ Warping-related "control issues" (like in KKLT) still exist.

Reliability unclear.

[→ Junghans; AH/Venken/Schreyer]

Summary: My bets are on LVS + "complex-structure F-term uplift." KKLT too fragile to warping corrections.

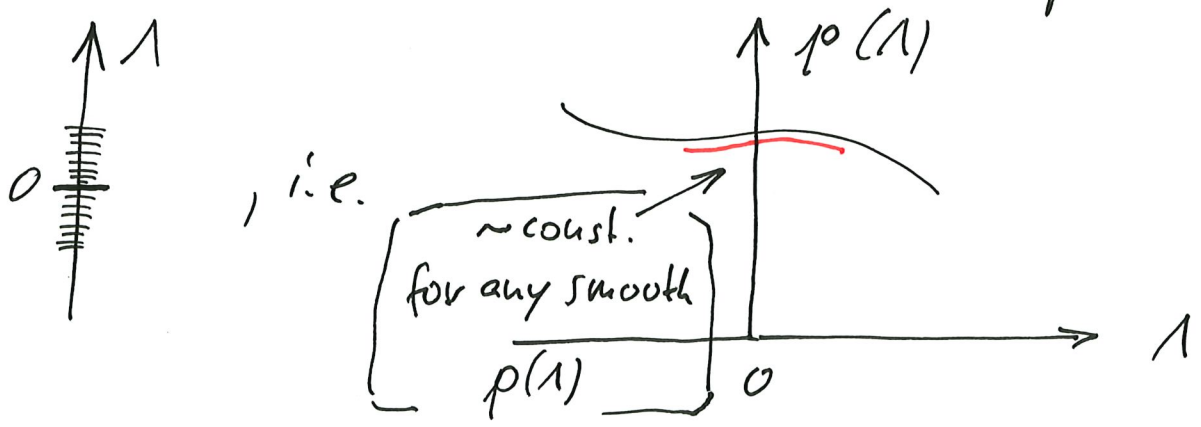
7 Multiverse

7.1 Predictions & measure problem

- So far, we only argued about the existence of certain solutions (e.g. $0 < \Lambda \ll 1$) in the landscape.
- No probabilities were assigned to values of parameters. Hence, no predictions were discussed.
- Famously, Weinberg made a prediction for Λ using "landscape/multiverse" logic.

  Rough idea: (in my reading)

  - $|\Lambda| < \Lambda_c$ experimentally
  - Assume many universes with different Λ (where $\Lambda = 0$ is not special).



$\Rightarrow$ Exploring the region $\Lambda < \Lambda_c$, we expect to find $|\Lambda_{obs}| \sim \Lambda_c$

(not $|\Lambda_{obs}| \ll \Lambda_c$)

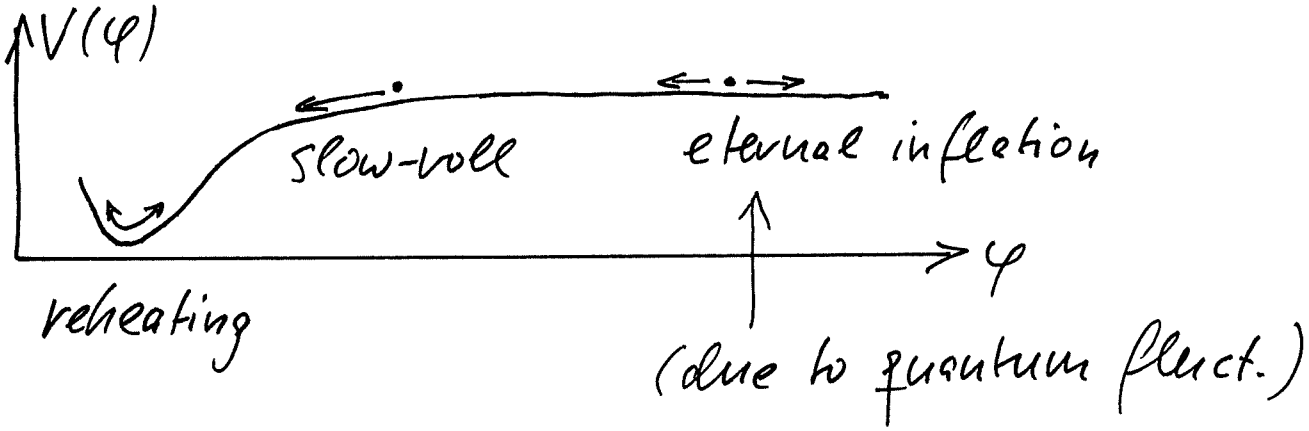
- However: This does not come for free. It uses a measure ($p(\Lambda)$ or the assumption that each vacuum is equally likely.)
- What is the basis for this (or any other) measure? ("Measure problem")

7.2 Eternal Inflation

- To quantify the problem, we must consider early cosmology and hence inflation.
- Reminder: $ds^2 = -dt^2 + a^2(t)\, g_{ij}\, dx^i dx^j$

↑ max. symmetric 3d space.

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$



- <u>Slow-roll:</u> $\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$; $\eta \equiv \frac{V''}{V} \ll 1$

  (EOM: $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{V}{3}$; $3H\dot{\varphi} = -V'$)

- <u>Eternal:</u> $\epsilon < V$

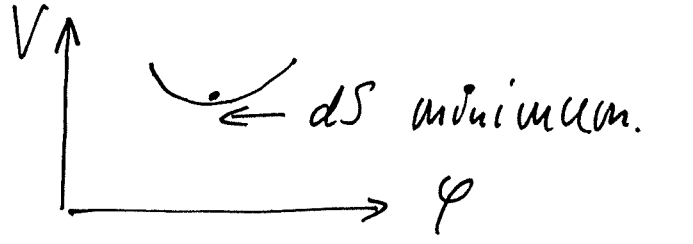
  (EOM: $H = \text{const.}$ ("de Sitter")

  or $a \sim \exp(Ht)$)

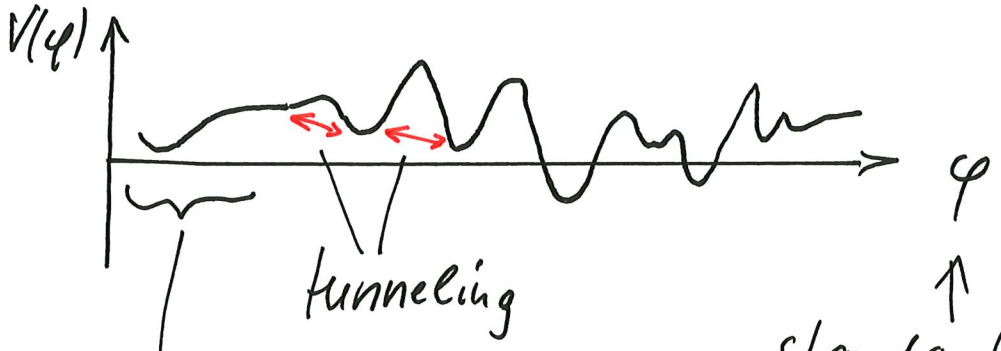
- But this is <u>not</u> the type of eternal inflation relevant for the string multiverse.

- In the string landscape, flat potentials at $V > 0$ are very rare (or non-existent).

- More probable:



- Or many such minima:

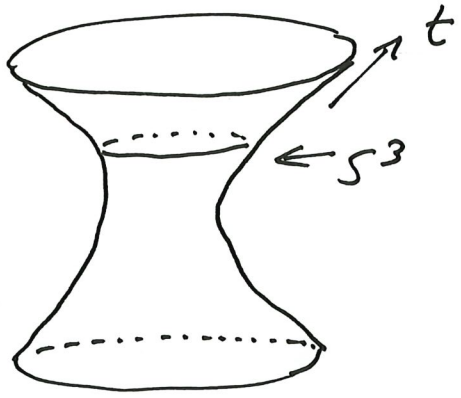


"Our" vacuum with its inflationary plateau

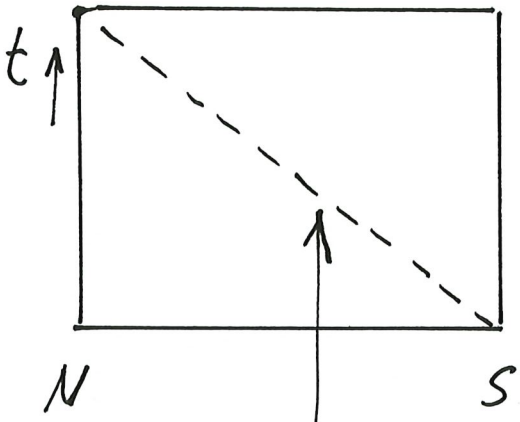
stands for many moduli:

$$\varphi = \{\varphi^1, \dots, \varphi^m\}$$

- If φ just sits in one of those minima with $\Lambda > 0$, the world is dS:

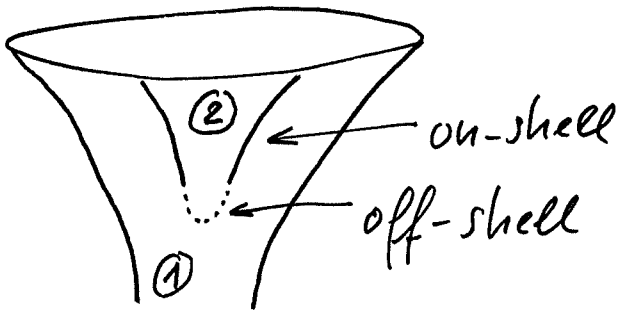


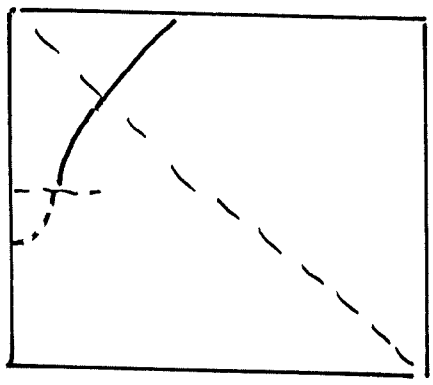
Penrose:



horizon of observer sitting at north pole

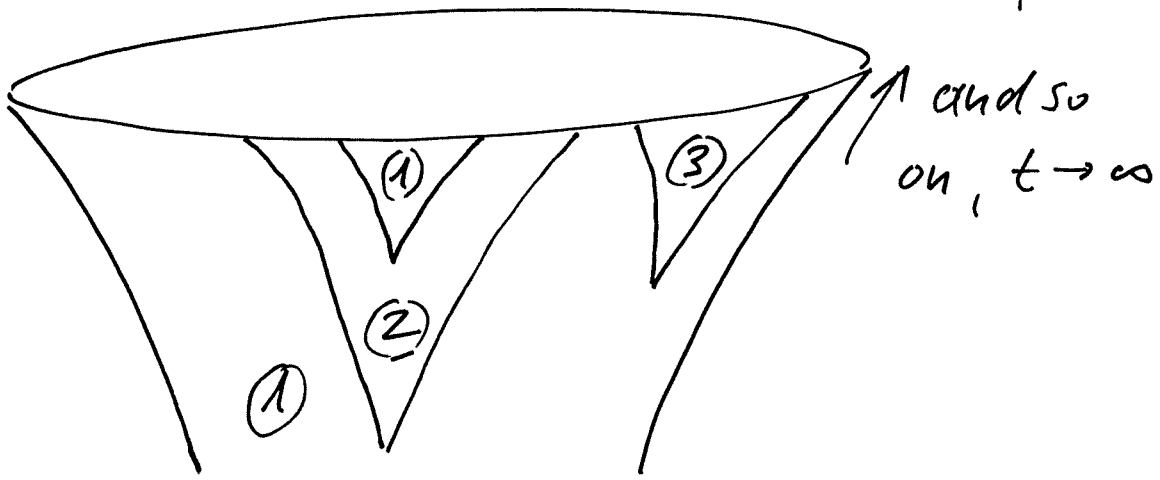
- <u>As in 1st order phase transition:</u> Spontaneous creation of a supercritical bubble of a new vacuum is possible.





→ Famous "Coleman-De Luccia" calculation of nucleation rate.

- This bubble nucleation can repeat itself:

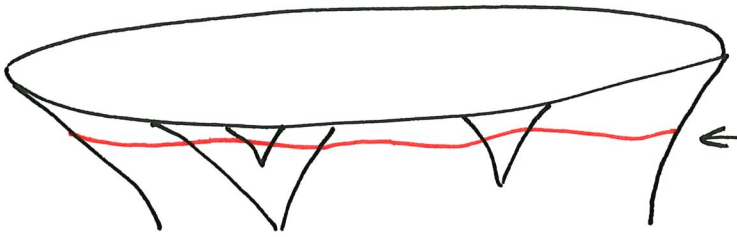


⇒ multiverse made of bubbles in bubbles in bubbles....

⇒ "Everything that can happen will happen an infinite number of times" (Guth)

- How to ascribe probabilities in this situation? This is the real, hard measure problem!

- Technical issue: <u>Choice of slicing</u>



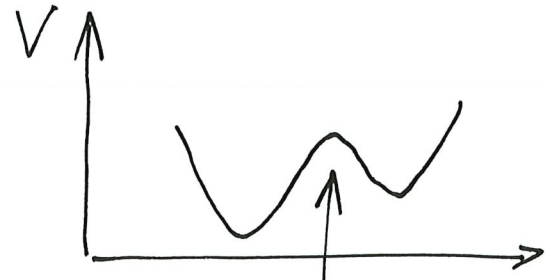
great freedom due to diffeomorphism-invariance.

- Two main choices:
  - Volume weighted measures
  - Local measures [do not reward volume growth]

(→ Linde, Vilenkin, Guth, Bousso, Freivogel, ...)

also: Hassfeld / AH / Westphal

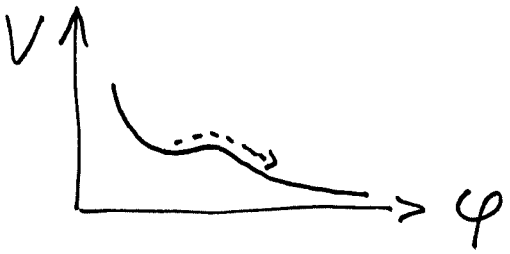
- An extra issue:



This picture of a potential barrier separating vacua is wrong!

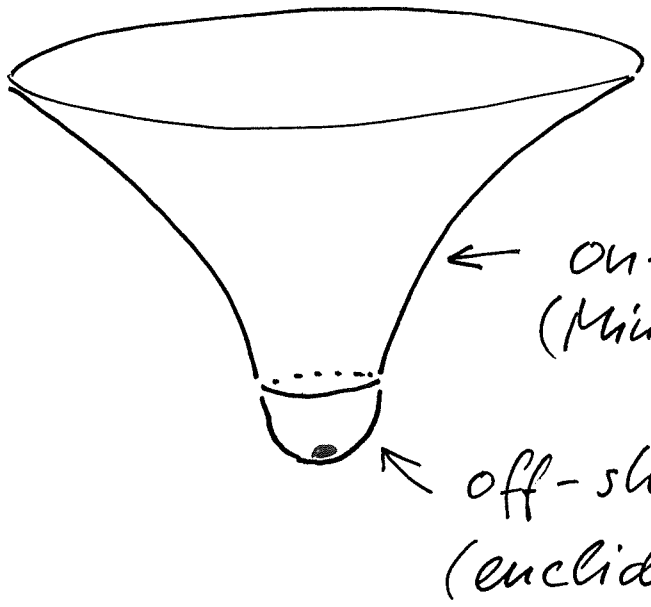
Correct picture: Singular domain walls, specifically D5 branes wrapped on 3-cycles of CY, separate vacua.

Problem: This wall may destabilize the (very fragile) LVS or WMLT vacua adjacent to it (→ Larfors/Johnson)



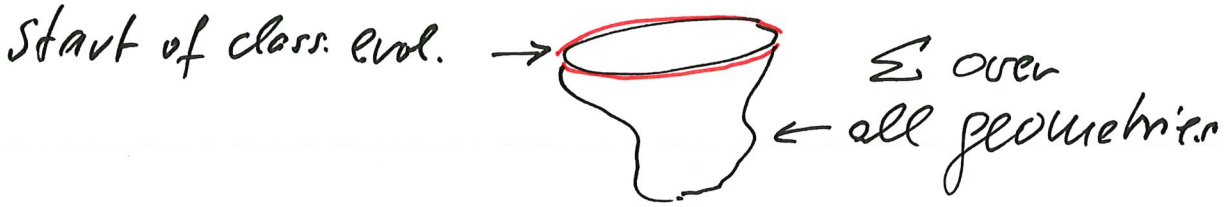
7.3 Creation from Nothing

An alternative (and at the moment very popular) picture is that of Hartle/Hawking or Linde/Vilenkin:



LV-logic: Small universe (•) "quantum fluctuates" out of nothing, tunnels to large size, goes on-shell and expands classically.

HH-Logic: Think of the above picture as defining initial conditions in a euclidean path integral approach to the "wave function of the universe"!



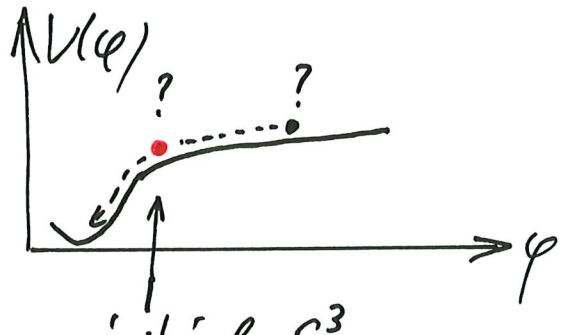
Difference LV/HH:

$$\text{prob.} \sim \exp\left(\mp S_{ds}\right)$$

$$\sim 1/V(\varphi)$$

Situation at the moment:

- Community prefers HH point of view.
- But: This gives a wrong "post-diction" even with just a single vacuum: The universe tends to start very low on the inflationary plateau:



⇒
- too small universe
- too much curvature

(cf. Maldacena '24)

- In principle, eternal inflation and creation from nothing can <u>both</u> be relevant <u>together</u>.
- One should work out all the details of this, combine with what we know about the string landscape, and make predictions.
- One key target: ~~SUSY~~ scale (and details of ~~SUSY~~)

  → e.g. many papers by Baer

<u>But</u>: • There is much technical uncertainty, both on the stringy and on the dS / cosmo side.

- Much more work is needed!