Effective Potentials, Warping, and F-Term Uplifts

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recent work with: **Severin Lüst, Andreas Schachner, and Simon Schreyer**

- Motivations: Better control of 4d EFTs / Problems with dS.
- Goal: 10d derivation of the (warping corrected!) 4d off-shell scalar potential, using a $1/(Vol.)$ - expansion.

Plan:

- Integrate out KK-modes by solving EOMs ('off-shell!').
- Propose consistent general form of warped Kahler potential.
- Apply results to F-term uplift of:
  LVS (looks good) and KKLT (looks problematic).

Motivation 1: dS conjecture

- The dS-models of KKLT and LVS are the technical backbone of the (phenomenological) string landscape
- Following the proposal that stringy dS is impossible as a matter of principle ('is in the Swampland'),

  Danielsson/Van Riet; Obied/Ooguri/Spodyneiko/Vafa '18

  (see also Bena, Grana, Sethi, Dvali, ...)

  these constructions have been subjected to intense scrutiny.

  Bena/Grana/Van Riet, Van Riet, Moritz/Retolaza/Westphal, Gautason/Van Hemelryck/Van Riet, Hamada/AH/Shiu/Soler, Kallosh, Bena/Dudas/Grana/Lüst, Lüst/Randall, ...
- In particular, the **'Singular Bulk Problem'** of KKLT and the **'Tadpole Constraint'** of the LVS remain unresolved.

  Carta/Moritz/Westphal, Gao/AH/Junghans, Junghans, AH/Gao/Schreyer/.. Venken,... (see however: Carta/Moritz, McAllister/Moritz/Nally/Schachner,..)

Motivation 1 (continued):

- The $\overline{\mathrm{D3}}$ uplift is central in the problems just mentioned. Thus, looking for alternatives is mandatory.

- In particular, the (complex-structure) F-term uplift has a high potential for full explicitness. But: Must control corrections!

  Saltman/Silverstein, Denef/Douglas ... Gallego/Marsh/Vercnocke/Wrase ....AH/Leonhardt, Krippendorf/Schachner, Lanza/Westphal

  [Of course, other uplifts and uplift-free models are also worth pursuing, see e.g. Westphal '06, Cremades et al. '07, Cicoli/Quevedo/Valandro '15 Antoniadis/Chen/Leontaris '18, ...]

Motivation 2: Better control of string compactificiations

Quite generally, we should try to make progress in controlling the scalar potential, including in particular warping effects.

Basic starting point (GKP) :

- Warped metric ansatz:

$$ds_{10}^2 = \mathrm{e}^{2A(y)}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + \mathrm{e}^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n$$

- 3-form-flux and F_5:

$$G_3 = F_3 - \tau H_3 \qquad , \qquad \tilde{F}_5 = (1+\star)\, d\alpha \wedge d(Vol)_4$$

- Convenient ansatz: [Baumann/.../McAllister '08/'10]

$$\Phi_\pm = \mathrm{e}^{4A} \pm \alpha \quad , \quad G_\pm = (\star_6 \pm \mathrm{i})G_3 \quad , \quad \Lambda = \Phi_+ G_- + \Phi_- G_+$$

Much further work should be cited as a basis for what follows:

Gukov/Vafa/Witten, Dasgupta/Rajesh/Sethi, Grana/Polchinski, Grimm/Louis, DeWolfe/Giddings, Giddings/Maharana, Burgess et al., Frey/Maharana, Douglas/Shelton/Torroba, Körber/Martucci, Marchesano/McGuirk/Shiu, Grimm et al., Lüst/Randall, ..., Frey/Mahanta/Agarwal/Underwood '25

Equations of motion (only symbolically):

- Warp factor: e^{4A} / 5-form: α

$$\tilde{\nabla}^2 \Phi_\pm \;\sim\; G_\pm \tilde{\cdot} \bar{G}_\pm \;+\; |\tilde{\partial}\Phi_\pm|^2 \;+\; \tilde{\mathcal{R}}_4 \;+\; \rho_{loc.} \qquad (*)$$

- Internal Einstein:

$$\tilde{R}_{mn} \;\sim\; \partial_{(m}\tau\partial_{n)}\bar{\tau} + 2\partial_{(m}\Phi_+\partial_{n)}\Phi_- + \left(G_{+(m}{}^{\widetilde{pq}}\bar{G}_{-n)pq} + [+ \leftrightarrow -]\right)$$

$$+\tilde{g}_{mn}\left(\text{equation } (*) \text{ w/o } \tilde{\mathcal{R}}_4\right) + \text{local}$$

- 3-forms / axio-dilaton

$$d\Lambda \;\sim\; d\tau \wedge (\Lambda + \overline{\Lambda}) \qquad , \qquad \tilde{\nabla}^2\tau \;\sim\; G_+ \tilde{\cdot} G_- + \cdots$$

Famously, this is solved by ISD-flux ($G_- = 0$) and a warp factor derived from a simple Poisson equation (GKP).

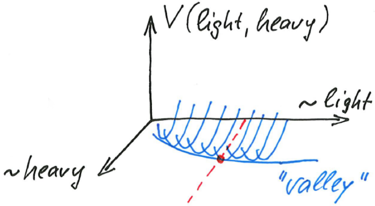
Goal: Off shell effective potential

- But: We are not interested in 'GKP' SUSY solutions.
- We want the (off-shell) low-energy EFT action, in particular the off-shell scalar potential.
- Its fields include the 4d metric ($\tilde{g}_{\mu\nu}$), Kahler moduli (T^A), complex structure (z^i), axio-dilaton (τ).

Strategy:

- Integrate out heavy fields (KK-modes).
- To do so, solve the EOMs w.r.t. the heavy fields while keeping the light fields at arbitrary fixed values. (Allowing in particular non-SUSY values of z^i and $G_- \neq 0$.)

Illustration:



- Note: The EOMs of the light fields are not solved.

Technical Implementation:

- Recursive solution of EOMs for heavy fields using an inverse volume expansion.

- More precisely: Use the inverse 4-cycle volume $1/c$, where

$$2\Phi_+^{-1} = \mathrm{e}^{-4A(y)} = c + \{y\text{-dependent}\}\,.$$

Inverse volume expansion (ansatz):

$$
\begin{aligned}
\Phi_\pm &= \frac{1}{c}\,\Phi_\pm^{(0)} + \frac{1}{c^2}\,\Phi_\pm^{(1)} + \cdots \\
\tau &= \tau^{(0)} + \frac{1}{c}\,\tau^{(1)} + \cdots \\
\tilde{g}_{mn} &= \tilde{g}_{mn}^{(0)} + \frac{1}{c}\,\tilde{g}_{mn}^{(1)} + \cdots \\
G_\pm &= G_\pm^{(0)} + \frac{1}{c}\,G_\pm^{(1)} + \cdots
\end{aligned}
$$

Note: For $\Phi_\pm$, we shifted the order of coefficients by one to account for $e^{-4A} \sim c$ at leading order.

Leading order:

$\Phi_\pm^{(0)} = 2/0$, i.e. $e^{-4A} = c$, $\tilde{F}_5 = 0$

$\tau^{(0)} =$ const. , $\tilde{g}^{(0)} =$ 'CY' , $G^{(0)}$ – harmonic, integral

Next-to-leading order (symbolically):

$$\tilde{\nabla}^2 \Phi_+^{(1)} \sim G_+^{(0)} \tilde{\cdot} \bar{G}_+^{(0)} + \rho_{loc.} + \tilde{\mathcal{R}}_4$$

$$\tilde{\nabla}^2 \Phi_-^{(1)} \sim G_-^{(0)} \tilde{\cdot} \bar{G}_-^{(0)} + \tilde{\mathcal{R}}_4 \qquad , \qquad \tilde{\nabla}^2 \tau^{(1)} \sim G_+^{(0)} \tilde{\cdot} G_-^{(0)}$$

$$\Delta \tilde{g}_{mn}^{(1)} \sim \Big(G_{+(m}{}^{\widetilde{pq}} \bar{G}_{-n)pq} + [+ \leftrightarrow -] \Big) + g_{mn}^{(0)} \Big(\nabla^2 \Phi_+^{(1)} + \cdots \Big)$$

(...plus eqs. for $G^{(1)}$, making fluxes non-harmonic)

Key technical point:

- As explained, the EOMs for the light fields are not solved.

  $\Rightarrow$ Interpret the equations above only as projections on the non-constant (non-kernel) part of the relevant field.

- For example, the axio-dilaton equation

$$\tilde{\nabla}^2 \tau^{(1)} \sim G_+^{(0)} \tilde{\cdot} G_-^{(0)}$$

  should be read as

$$\tilde{\nabla}^2 \tau^{(1)} \sim G_+^{(0)} \tilde{\cdot} G_-^{(0)} - \int G_+^{(0)} \tilde{\cdot} G_-^{(0)}$$

- Similarly, the Kahler/CS-moduli and $\tilde{g}_{\mu\nu}$ (and hence $\tilde{\mathcal{R}}_4$) remain undetermined.

Moreover: The r.h. side of metric equation is in the kernel, such that $\Delta \tilde{g}_{mn}^{(1)} = 0$ and the metric remains 'conformal CY' at NLO.

Result: Scalar potential

(obtained directly from 10d action)

$$V_{\text{eff}} = \frac{1}{c^3}\left[V^{(1)} + \frac{1}{c}V^{(2)} + \ldots\right]$$

where

$$V^{(1)} \sim \int_6 |\tilde{G}_-^{(0)}|^2 \qquad\qquad \text{('GKP appendix')}$$

and (simplified presentation)

$$V^{(2)} \sim \int_6 \left[\Phi_+^{(1)}|\tilde{G}_-^{(0)}|^2 + \text{Re}\, G_-^{(0)} \tilde{\cdot}\, \bar{G}_-^{(1)} + |\tilde{\partial}\tau^{(1)}|^2\right].$$

- Parametric result with $G_+^{(0)} \sim \sqrt{N}$ and $G_-^{(0)} \sim \epsilon$:

$$V = V_{\text{flux}} + V_{\text{warp}} = \frac{g_s \epsilon^2}{c^3} + \frac{g_s^2 N \epsilon^2}{c^4}.$$

Key features of warping correction:

- Suppression by 4-cycle volume c .
- No term linear in 'small F-term parameter' $\epsilon \sim G_-$.

4d $\mathcal{N} = 1$ SUGRA interpretation

Inspired by but going beyond Martucci '14/'16.

(see also Frey/Torroba/Underwood/Douglas)

$$K_k(T^A + \overline{T}^A) \;\rightarrow\; K_k\big(T^A + \overline{T}^A + f^A(\phi^I, \overline{\phi}^{\bar{I}})\big)$$

with $\phi^I = (z^i, \tau)$.

- This conjecture implies a potential

$$V = e^K K^{I\bar{J}} (D_I^{(0)} W) \overline{(D_J^{(0)} W)}$$

where $D_I^{(0)} W$ (with $I = (A, i, \tau)$) remains uncorrected.

(see also Grimm/Louis, Burgess/Cicoli/Ciupke/Krippendorf/Quevedo)

We may now ask: What does this imply for uplifting?

- We assume moduli stabilization based on

$$V = \mathrm{e}^{K} \left(K^{M\overline{N}} D_M W \bar{D}_{\overline{N}} \overline{W} - 3|W|^2 \right)$$

with $\quad K = K_{\mathrm{cl}} + \delta K \quad$ and $\quad W = W_{\mathrm{GVW}} + \delta W\,.$

- The relevant corrections are

δW – Non-perturbative: $\sim \exp(-T)$

δK – α'-corrections (BBHL): $1/\mathcal{V} g_s^{3/2}$

– loop-corrections: $g_s t/\mathcal{V}$ (local) and $1/\mathcal{V}\tau$ (generic)

Gersdorff/AH , Berg/Haack/Körs, ..., Cicoli/Conlon/Quevedo, Gao/AH/Schreyer/Venken

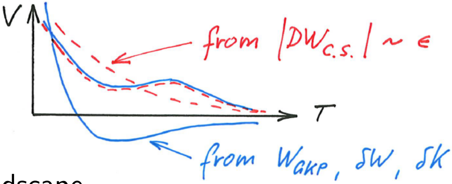
(see also 'α' review' in Cicoli/Quevedo/Savelli/Schachner/Valandro)

Final step:

F-term uplifting

Saltman/Silverstein, Denef/Douglas, ... Gallego/Marsh/Vercnocke/Wrase
Honma/Otsuka, AH/Leonhardt, Krippendorf/Schachner, Lanza/Westphal

- Basic idea:



- Assuming unlimited
  tuning power of the landscape,
  this should work both for LVS and KKLT (etc....)

- But, beyond LO, two types of corrections are dangerous:

(1) $\delta V_{\rm warp} \sim \epsilon^2/c^4$ (as discussed above)

(2) $\delta V_{\rm mix} \sim \big[\,(DW)_{\rm c.s.} \sim \epsilon\,\big] \times \big[\,\delta W$ or $\delta K\,\big]$

Potential problems with ‘warped’ and ‘mixed’ corrections

- The main issue is not that these corrections may become larger than standard LVS/KKLT potential terms.
- The main issue is the presence of an exceptionally light complex structure direction in all F-term-uplifted models.

Denef/Douglas, Gallego et al., ...

Reason:

- Recall: $V_{\text{flux}} \sim |G_-|^2 \sim |DW_{\text{c.s.}}|^2 \sim \epsilon^2$
- $DW_{\text{c.s.}}$ is a vector-valued function: $\mathbb{C}^n \to \mathbb{C}^n$, $\delta z \mapsto DW_{\text{c.s.}}$.
- This function can not be surjective at linear order near a minimum (or else one could lower the value of $|DW|_{\text{c.s.}}$).
- Thus, at linear order, $DW_{\text{c.s.}}$ can not be injective either.
- Hence one direction in δz space only gets a small mass$^2 \sim \epsilon$ at sub-leading order.

Specifically for the LVS:

- The most dangerous effects come from $\delta V_{\rm mix}$:

$$\delta V_{\rm mix} \sim \frac{\epsilon}{\mathcal{V}^2}\left(e^{-\tau_s} + \delta K_M W_0 + \overline{T}^{\overline{A}} \delta K_{\overline{A}i} W_0\right)$$

(symbolic, with M – all moduli; A – Kahler; i – complex-structure)

- The destabilization of the 'light direction' is avoided if

$$\frac{\partial^2_{\rm light}\,\delta V_{\rm mix}}{m^2_{\rm light}} \sim \left.\frac{1}{\mathcal{V}^{1/2}}\right|_{\delta W,\alpha',{\rm loops}} + \left.\frac{1}{\mathcal{V}^{1/6}}\right|_{\alpha'\,{\rm on\,D7}} \ll 1$$

(most critical $\mathcal{V}^{-1/6}$ term arises only of D7s wrapped on τ_b.)

- Same condition ensures $\delta V_{\rm mix}/V_{\rm flux} \ll 1$.

  $\Rightarrow$ F-term uplift of LVS can be parametrically controlled.

Specifically for KKLT:

- Achieving control is harder since the 'light' c.s.-modulus has masses:

$$\begin{aligned} m^2_{\rm light} &= \qquad\qquad + \mathcal{O}(\,|F|\,) + \mathcal{O}(\,|F|^2\,) \\ m^2_{\rm light'} &= 4|W_0|^2 - \mathcal{O}(\,|F|\,) + \mathcal{O}(\,|F|^2\,) \end{aligned}$$

- Unlike the LVS, now $|W_0| \ll |F|$ and the light partner state is destabilised unless one tunes the $\mathcal{O}(|F|)$-term to be negligible.

Denef/Douglas, Marsh/McAllister/Wrase, Vercnocke....

- Thus, we have $m^2_{\rm light(')} \sim |F|^2 \sim \epsilon^2$ and, including factors of g_s and $\mathcal{V}$, one finds:

$$\frac{\partial^2_{\rm light(')}\,\delta V_{\rm warp}}{m^2_{\rm light(')}} \sim \frac{g_s N}{\epsilon\,\mathcal{V}^{2/3}}$$

- The uplift needs $\epsilon \sim |W_0|$ and the volume is not very large, $\mathcal{V} \sim \ln(1/|W_0|) \Rightarrow$ Can not make this small or control its sign.
- A similar disastrous instability issue is induced by $\delta V_{\rm mix}$.

Summary / Conclusions

- Developing our understanding of moduli scalar potentials remains a key challenge of string phenomenology.
- I discussed a systematic procedure for calculating, in $1/\mathcal{V}$-expansion, the warped off-shell potential in type-IIB.
- Noteworthy: conformal-CY even at sub-sub-leading order!
- **Obvious application:** F-term uplift.
  Our findings: OK for LVS but uncontrolled for KKLT.
- Side remark: String Theory has a 'Photinoverse'. The (hidden) photino masses depend on the complex-structure F-term.
  $\Rightarrow$ Unique opportunity to test F-term vs. anti-brane uplift.

Coudarchet/AH/Jaeckel/Steiner

[In parallels: More on F-term uplift (Sifo) and Multiverse (Coudarchet)]