String Landscape & Multiverse

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see also: lecture notes at 2008.10625 [hep-th]
and slides at www.thphys.uni-heidelberg.de/∼hebecker → ‘Selection of recent talks’

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1 Why String Theory?

With minor exceptions, our world is perfectly described by

$$\mathcal{L} \;=\; \mathcal{L}_{\text{grav.}} + \mathcal{L}_{\text{SM}}.$$

Here

$$\mathcal{L}_{\text{grav}} \;=\; \tfrac{1}{2}M_P^2\,\mathcal{R}[g_{\mu\nu}]\;; \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Symbolically:

$$\mathcal{L}_{\text{grav}} \;=\; \tfrac{1}{2}M_P^2\big[(\partial h)^2 + h(\partial h)^2 + h^2(\partial h)^2 + \cdots\big]$$

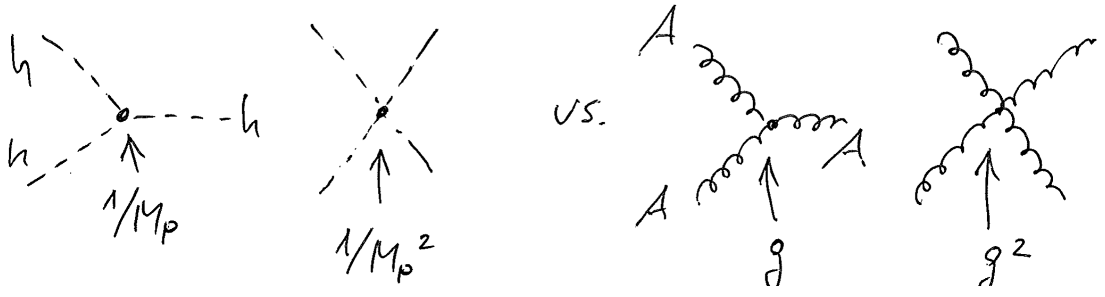
$$\text{Redefine}: \quad h_{\mu\nu} \;\longrightarrow\; h_{\mu\nu}/M_P$$

$$\Rightarrow \quad \mathcal{L}_{\text{grav}} \;=\; \frac{1}{2}(\partial h)^2 + \frac{1}{2M_P}\,h(\partial h)^2 + \frac{1}{2M_P^2}\,h^2(\partial h)^2 + \cdots$$

Compare this to $\mathcal{L}_{\text{QCD}} \subset \mathcal{L}_{\text{SM}}$:

$$\mathcal{L}_{\text{QCD}} \;=\; \tfrac{1}{2}(\partial A)^2 + \,gA^2(\partial A) + \,g^2A^4$$

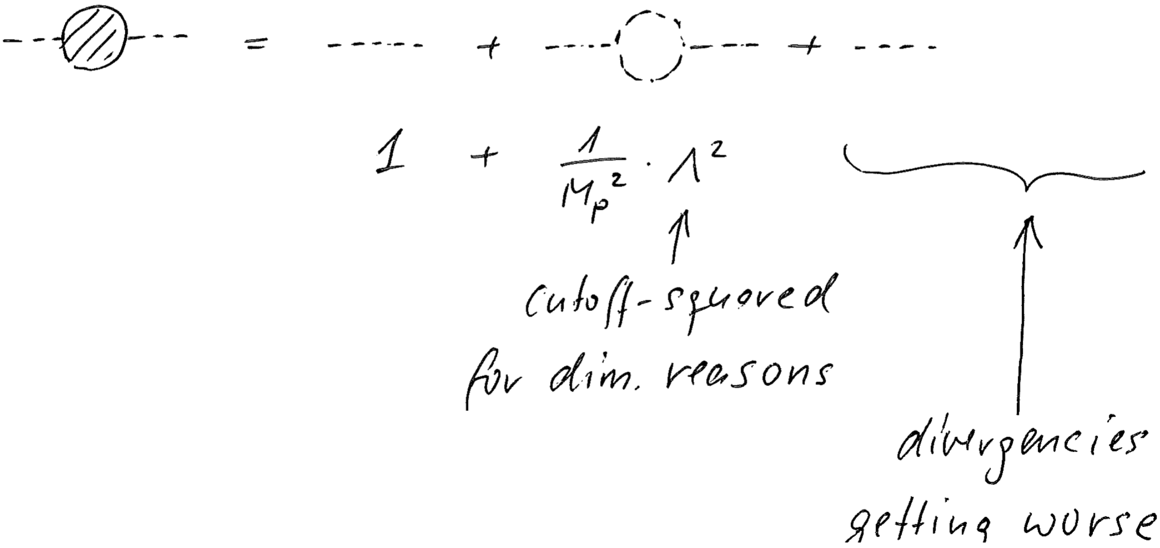
Hence:



• Key difference for gravity:

Coupling constant has negative mass-dimension in gravity.

⇒ not renormalizable by power-counting.



• Even more catastrophic:

Perturbative description completely breaks down at $E \sim \sqrt{s} \sim M_P$.

Indeed: For a renormalizable-QFT amplitude

$$\mathcal{A} \sim g^2 \quad \Rightarrow \quad \sigma \sim |\mathcal{A}|^2 \frac{1}{s}$$

for dimensional reasons.

(considering fixed-angle scattering)

By contrast, in gravity:

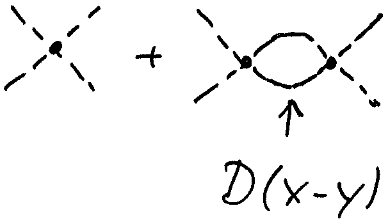
$$\mathcal{A} \sim \frac{1}{M_P^2} s \quad \Rightarrow \quad \sigma \sim |\mathcal{A}|^2 \frac{1}{s} \sim \frac{s}{M_P^4}$$

Unitarity violation!

$\Rightarrow$ Widely-held belief:

At $E \sim M_P$, QFT-logic breaks down in a very fundamental way.

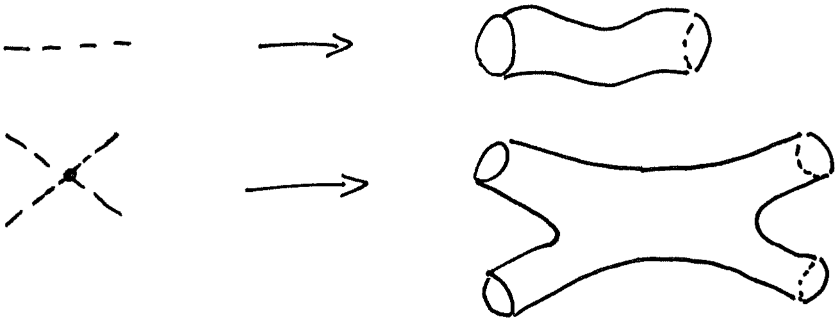
- Best-developed option: String theory
- Motivation:
  - problems related to UV-divergences of QFT
  - these, in turn, are related to point-like particles or, better, point-like vertices.



; $\int_{x,y} [D(x-y)]^2 \longrightarrow \infty$

from $(x-y) \to 0$

- Stringy resolution:



all smooth, no vertex!

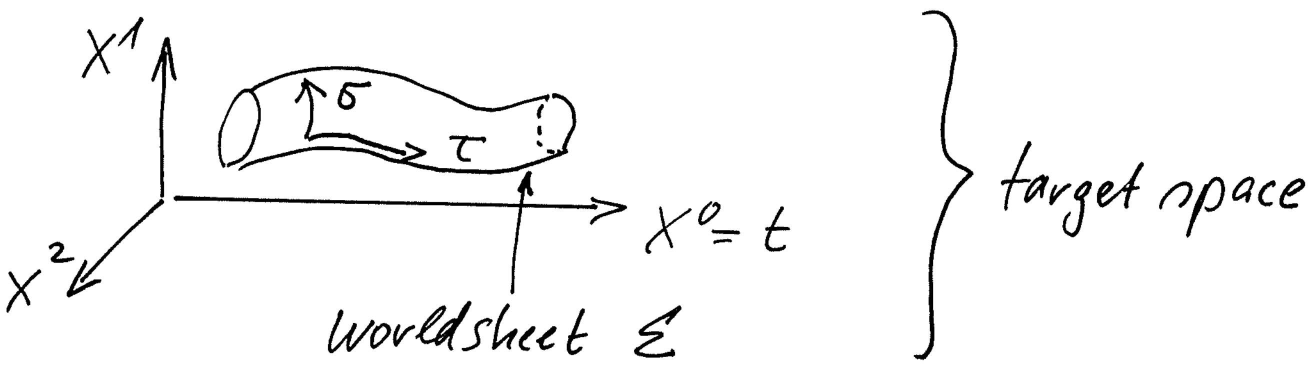
- Detailed study reveals:

This really only works for $D=10$ and with SUSY (in the 10d theory).

$\Rightarrow$ Need to work hard to get back to our real world in $D=4$ & ~~SUSY~~.

2 String Theory

2.1 Classical String



$$(\tau, \sigma) \equiv (\xi^0, \xi^1) \equiv \xi^a$$

- The most natural action is $S_{NG} = -T \int_\Sigma dA$.

$\uparrow$

"Nambu-Goto"

- To make it explicit, we must quantify the embedding of the WS in the target space:

$$X^\mu = X^\mu(\xi) \quad , \quad \mu = 0, \ldots, D-1$$

$$\Rightarrow S_{NG} = -T \int_\Sigma d^2\xi \sqrt{-G} \quad , \quad G \equiv \det(G_{ab})$$

$\uparrow$

induced metric on Σ

- Explicitly: $G_{ab} = \eta_{\mu\nu} \left(\frac{\partial X^\mu}{\partial \xi^a} \right) \left(\frac{\partial X^\nu}{\partial \xi^b} \right)$

- Because of $\sqrt{\cdots}$, this is very hard to quantize.

- Much better: $$\underset{\substack{\nearrow \\ \text{"Polyakov"}}}{S_P} = -\frac{T}{2}\int d^2\xi \sqrt{-h}\, \underbrace{h^{ab} G_{ab}}_{h^{ab}\eta_{\mu\nu}(\partial_a X^\mu)(\partial_b X^\nu)}$$

- Here h_{ab} is an independent "WS metric".

- For $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \cdots, 1)$, S_P is just the action of a QFT with D free scalars X^μ.

- Crucially, it is classically equivalent to S_{NG}:
$$T_{ab} \sim \frac{\delta S_P}{\delta h^{ab}} \sim G_{ab} - \frac{1}{2} h_{ab} (G_{cd} h^{cd})$$

- EOM for h: $T_{ab} = 0 \Rightarrow h_{ab} = G_{ab} \cdot \underset{\substack{\uparrow \\ \text{any function}}}{f(\xi)}$

- Plugging $h_{ab} = f \cdot G_{ab}$ into S_P immediately recovers S_{NG}.

- Thus, from now on, our fundamental action is $$S_P = -\frac{1}{4\pi\underset{\substack{\nearrow \\ \text{"Regge slope"}}}{\alpha'}} \int d^2\xi \sqrt{-h}\, \underbrace{(\partial X)^2}_{h^{ab}(\partial_a X^\mu)(\partial_b X^\nu)\eta_{\mu\nu}}$$

$\alpha' \sim l_s^2$ ("string length")

- S_P has three key symmetries:

  1) Diffeomorphisms: $\xi^a \to \xi'^a = \xi'^a(\xi^0, \xi^1)$

  2) Poincare: $X^\mu \to X'^\mu = \Lambda^\mu{}_\nu X^\nu + V^\mu$

  $(\Lambda \in SO(1, D-1))$

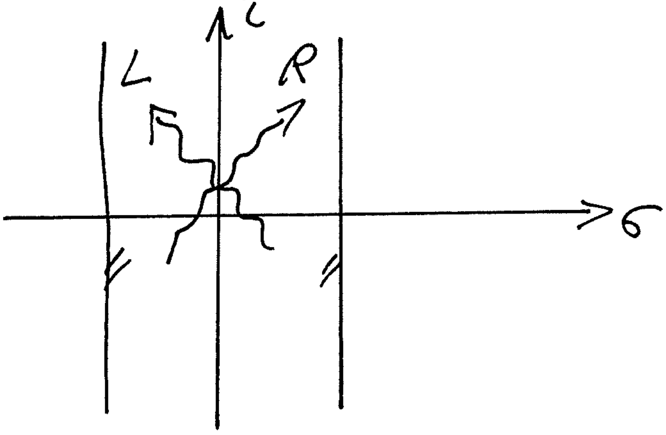
  [This is a global symmetry of our 2d QFT.]

  3) Weyl rescalings: $h_{ab}(\xi) \to h'_{ab}(\xi) = \varphi(\xi) h_{ab}(\xi)$

  [This key feature sets $d=2$ apart.]

- 1) & 3) are gauge symmetries.
  They may be used to choose a "flat gauge",
  i.e. $h_{ab} = \mathrm{diag}(-1, 1)$

$\Rightarrow$ All we need to do is to quantize free scalars (X^μ's) on a flat cylinder:



2.2 Quantization

- Use light-cone coordinates: $\sigma^\pm \equiv \tau \pm \sigma$
- Then: $\Box X^\mu = 0 \iff \partial_+ \partial_- X^\mu = 0$
- general solution: $X^\mu(\sigma^+,\sigma^-) = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-)$

with

$$X_L^\mu(\sigma^+) = \frac{1}{2} x^\mu + \alpha' p^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\tilde{\alpha}_n^\mu}{n} e^{-2\pi i n \sigma^+}$$

(and analogously for X_R^μ, with $\tilde{\alpha} \to \alpha$)

- Canonical quantization proceeds as usual:

$$[p^\mu, x^\nu] = -i\eta^{\mu\nu} \; ; \; [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n] = m\,\delta_{m+n}\,\eta^{\mu\nu} \; ; \text{ same for } \alpha\text{'s.}$$

$\uparrow \nearrow$ "left-movers"

$\uparrow$ $\equiv \delta_{m+n,0}$

$\uparrow$ "right-movers"

$$(\alpha_m)^\dagger = \alpha_{-m} \, , \quad m > 0 \Rightarrow \text{"annihilators"}$$

$\uparrow$ This part is special to string theory, because our space is compact: S^1.

x^μ – centre of mass ; p^μ – momentum (of motion of $\langle X^\mu \rangle$)

- <u>Fock space:</u> All linear combinations of

$$\underbrace{\alpha_m^{\mu}\,\alpha_n^{\nu}\cdots\tilde{\alpha}_k^{\rho}\,\tilde{\alpha}_l^{\sigma}\cdots}_{\substack{\text{any number of}\\ \text{creation operators}}}\,|0,\underset{\substack{\uparrow\\ \text{any } p}}{p}\rangle\;;\qquad \underset{\substack{\uparrow\\ \text{creators!}}}{m,n,k,l\ldots} < 0$$

of left- and right-moving excitations

- The proper Hilbert space is much smaller, analogously to the familiar case of gauge theory: $(\partial_\mu A^\mu)_{\text{annih.-part}}\,|\psi\rangle = 0$

- Here:

$$\underset{\substack{\uparrow\\ \text{Fourier modes of this}}}{(T_{ab})}_{\text{annih.-part}}\,|\psi\rangle = 0$$

≡ "Virasoro generators" $L_m,\ \tilde{L}_m$.

- In particular:

$$(L_0 - 1)|\psi\rangle = 0 \;;\quad L_0 = \frac{p^2}{4}\alpha' + \underbrace{\sum_{n=1}^{\infty}\alpha_{-n}\alpha_n}_{\text{"Level" } N}$$

[and same for $\tilde{L}_0$]

$\Rightarrow$ Must always excite left- & right-movers together ("level matching")

$\Rightarrow$ Can calculate the mass-squared ($= -p^2$) for any state.

Note: In developing this framework, one must carefully think about ordering-ambiguity in L_0 and other operators.

The "parameter" D plays a crucial role.

One discovers that $SO(1, D-1)$ is anomalous unless $D = 26$.

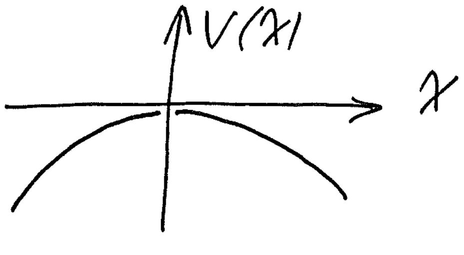
Alternatively, one may quantize in an $SO(1,D-1)$-covariant way. But then one discovers that 2d-Weyl-invariance is broken.

We focus here on "critical string", $D=26$

2.3 String states = 26d particles or fields

- **Vacuum:** $|0,p\rangle$; $m^2 = -4/\alpha'$

This is a "tachyon", i.e. this particle is an excitation of 26d-QFT with potential:



$\Rightarrow$ System unstable, but some short-time questions can be answered.

$\Rightarrow$ Bosonic string is just a toy model.

- <u>1st excited state:</u> $\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0,p\rangle \underbrace{\varepsilon(p)_{\mu\nu}}_{\text{polarization tensor}}$; $m^2=0$

1) traceless-symm. part: graviton, i.e. $\delta g_{\mu\nu}$ in 26d.

2) antisymm. part: "Kalb-Ramond potential" $B_{\mu\nu}$

[Analogue of "A_μ", i.e.

$H_{\mu\nu\rho} \sim \partial_{[\mu}B_{\nu\rho]}$, or better: $H = dB$,

just as $F_{\mu\nu} \sim \partial_{[\mu}A_{\nu]}$ or $F = dA$.

Such a B-field was expected since:

$$\underset{\text{Electron-WL}}{\int A_\mu dx^\mu} \longleftrightarrow \underset{\text{String-WS}}{\int B_{\mu\nu}\, dx^\mu dx^\nu}.$$]

3) trace of $\epsilon_{\mu\nu}$: "dilaton" ϕ – massless scalar

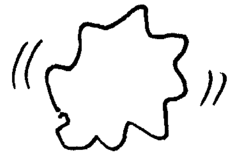
- <u>2nd excited state:</u> $(\alpha_{-1}^{\mu}\alpha_{-1}^{\nu}\tilde{\alpha}_{-1}^{\rho}\tilde{\alpha}_{-1}^{\sigma} + \alpha_{-2}^{\mu}\tilde{\alpha}_{-2}^{\nu})|0,p\rangle$

etc.

⋮

all heavy

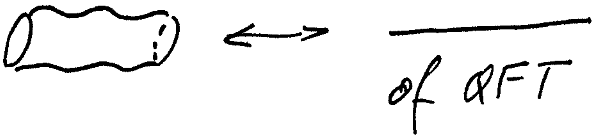
Intuitively, these are (highly) excited string states:



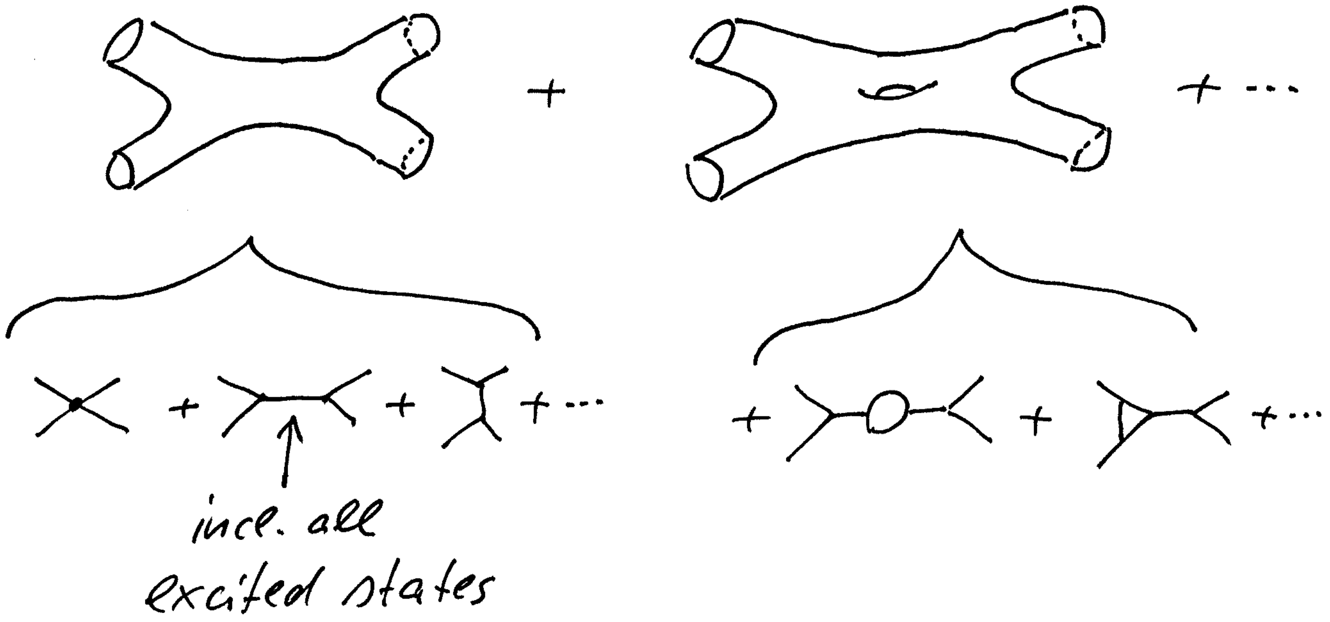
They are crucial for the consistency of the theory, but not important for us.

2.4 Amplitudes & 26d-QFT

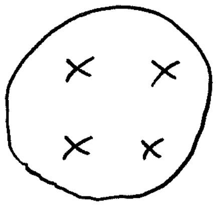
- We now understand:



- We need also:



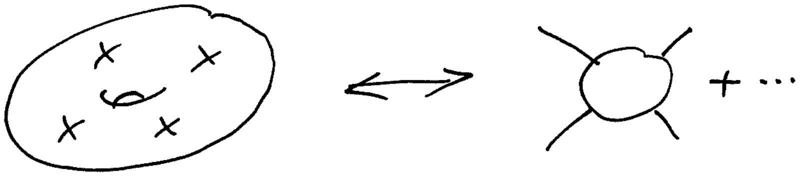
- Method:

$\rightarrow$  $\sim \int DX^\mu Dh_{ab}\, e^{-S_P[X,h]}\, \mathcal{O}_1(p_1)\cdots\mathcal{O}_4(p_4)$

- The operators $O_i(p_i)$ on S^2 characterize incoming / outgoing string states: "Vertex operators".
- Result of a long analysis:

  Such scattering amplitudes can be calculated and display the expected "soft" UV behaviour: $\mathcal{A}(s,t) \longrightarrow 0$

  as $s \to \infty$ at fixed $\frac{s}{t}$.
- Loops are finite:

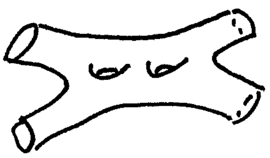


- At low s,t (i.e. $\ll 1/\alpha' \hat{=} M_s^2$), these amplitudes coincide with the amplitudes of a 26d QFT:

$$S = \frac{1}{\kappa^2}\int d^{26}x\sqrt{-g}\, e^{-2\phi}\left[R[g] - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + 4(\partial\phi)^2\right]$$

$$\frac{1}{\kappa^2} \searrow (\alpha')^{-12} \searrow (M_{P,26})^{24}, \qquad e^{-2\phi} \downarrow (M_{P,26})^{24}$$

$\Rightarrow$ $\langle\phi\rangle$ governs the ratio $M_s/M_{P,26}$.

If $\phi \to -\infty$, string is "weakly coupled" in the sense that:  $\ll$ 

One writes: $g_s \equiv e^{\phi}$

Important final comment:

One can study this theory in backgrounds where $g_{\mu\nu} \neq \eta_{\mu\nu}$; $B_{\mu\nu} \neq 0$; $\phi \neq 0$ (as just noted).

The background may change:

$$g_{\mu\nu} \to g_{\mu\nu} + \underbrace{\delta g_{\mu\nu}}$$

corresponds to "condensate" of elementary quanta of $\delta g_{\mu\nu}$, i.e. of massless string modes (just like $\langle E \rangle$ is coherent state of photons).

2.5 Superstring

$$S \sim \int_{\Sigma} (\partial_a X^{\mu})(\partial^a X^{\nu}) \eta_{\mu\nu} + i \bar{\psi}^{\mu} \not{\partial} \psi_{\mu} \equiv S_B + S_F$$

ψ^{μ} is fermionic partner of WS scalar X^{μ}

- <u>More precisely</u>: We have a 2d supergravity theory on the WS, with $h_{ab} \leftrightarrow$ gravitino
  $X^\mu \leftrightarrow \psi^\mu$

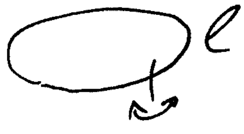
- Can choose gauge $h_{ab} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; gravitino $= 0$.

- $\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix} \begin{matrix} \leftarrow \\ \leftarrow \end{matrix}$ both real (Maj.-Weyl-spinors)

- Use light-cone coordinates on WS: $\sigma^\pm \equiv \tau \pm \sigma$

$$\Rightarrow S_F \sim \int d^2\sigma \, (\underbrace{\psi_-^\mu \partial_+ \psi_{-\mu}}_{\substack{\downarrow \\ \partial_+ \psi_- = 0 \\ \text{(right-movers)}}} + \underbrace{\psi_+^\mu \partial_- \psi_{+\mu}}_{\substack{\downarrow \\ \partial_- \psi_+ = 0 \\ \text{(left-movers)}}})$$

- <u>Novelty</u>: Can choose different boundary conditions for ψ's on S^1:



$\psi_+(\sigma+\ell) = +\psi_+(\sigma)$; $\psi_-(\sigma+\ell) = +\psi_-(\sigma)$ R-R

$\psi_+(\sigma+\ell) = +\psi_+(\sigma)$; $\psi_-(\sigma+\ell) = -\psi_-(\sigma)$ R-NS

⋮ NS-R

NS-NS

(Ramond,
Neveu-Schwarz)

- Quantization: $[\alpha_m^\mu, \alpha_n^\nu] = m\,\delta_{m+n}\,\eta^{\mu\nu}$

$$\{\psi_r^\mu, \psi_s^\nu\} = \delta_{r+s}\,\eta^{\mu\nu} \begin{cases} r,s \in \mathbb{Z} & (R) \\ r,s \in \mathbb{Z}+\frac{1}{2} & (NS) \end{cases}$$

- $D=26 \xrightarrow{SUSY} \underline{\underline{D=10}}$ (for consistency)
- As before: create states by applying α_m^μ, ψ_r^μ $(m,r<0)$ to $|0,p\rangle$
- Derive masses from $(T_{ab})_{\text{annih.-part}}|\Psi\rangle = 0$

NS-NS (r./l.)

$|0,p\rangle$ tachyon

$\epsilon_{\mu\nu}\,\psi^\mu_{-1/2}\tilde{\psi}^\nu_{-1/2}|0,p\rangle$ massless $\longrightarrow$ graviton, $B_{\mu\nu}$, ϕ

all higher states massive

R-NS (r./l.)

$|0,p\rangle \xrightarrow[R]{} |0,p,\alpha\rangle$

↩ spinor index

(This is necessary since vacuum must carry repres. of ψ_0^μ's:

$\{\psi_r^\mu, \psi_s^\nu\} = \delta_{r+s}\,\eta^{\mu\nu}$, Cliff.-alg. for $r=s=0$.)

By level matching, this must be combined with the 1st excited level of l.-moving sector:

$$\epsilon_\mu \psi^\mu_{-1/2} |0,p\rangle \quad ; \quad \text{massless} \rightarrow \text{gravitini (etc.)}$$

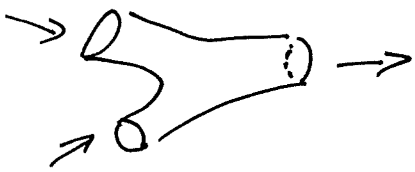
Analogously: $\underline{NS-R}$, $\underline{R-R}$
(10d fermions) (10d bosons)

- For each (r & l) subsector we can also define a projector: $P = \frac{1}{2}(1 + (-1)^F)$
  fermion number
  $\Downarrow$
  splitting in even/odd subspace of P

- In total, this gives 10 consistent pairings

$$\left.\begin{array}{l}(NS-, NS-)\\(NS+, NS-)\\(NS+, R-)\\(R-, NS+)\\ \vdots\end{array}\right\} 2^{10} \text{ possibilities to choose a subset of 10}$$

- Not all choices consistent with interactions:



- Need selection of sectors or "<u>GSO projection</u>"

$\Rightarrow$ Two consistent choices w/o tachyons exist:

$\underline{\underline{\text{type IIA}}}$; $\underline{\underline{\text{type IIB}}}$

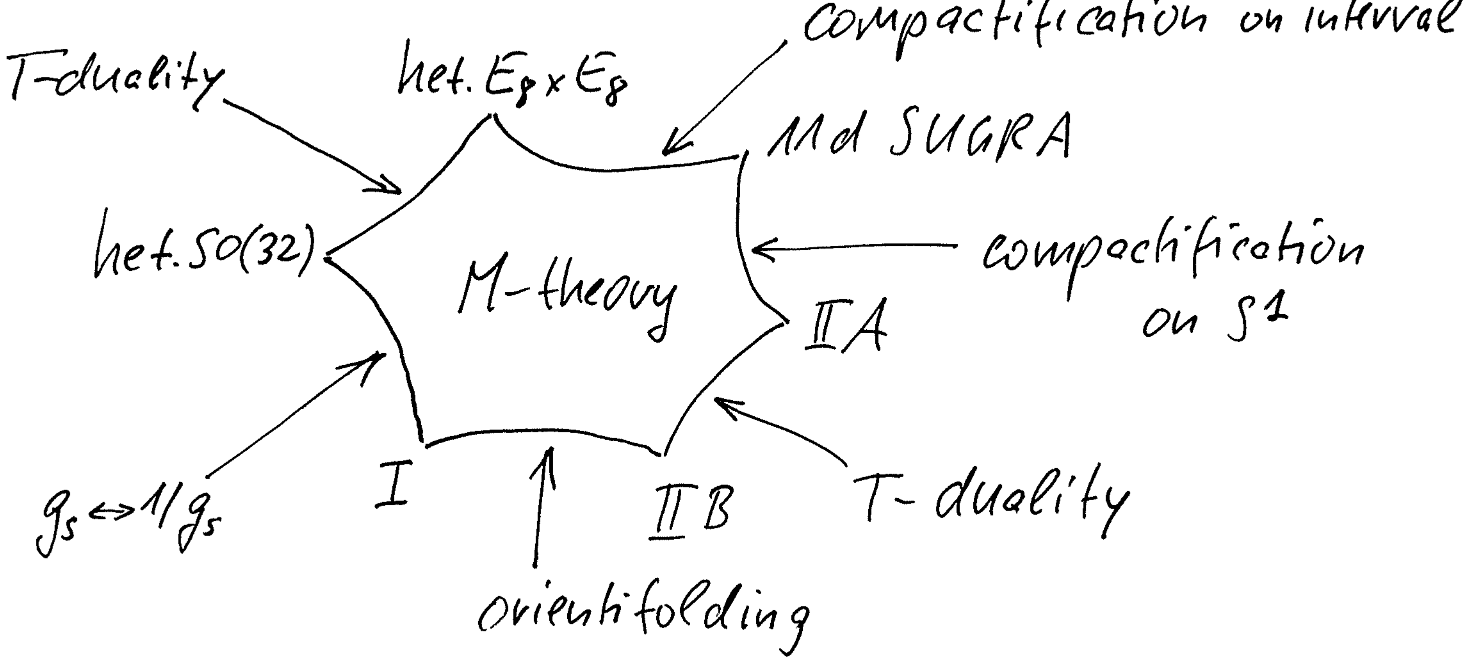
(Here "II" means: $\mathcal{N}=2$ SUSY in 10d, i.e.
$$\begin{tikzcd}[row sep=tiny] & \lambda^\mu \\ g_{\mu\nu} \arrow[ur, leftrightarrow] \arrow[dr, leftrightarrow] & \\ & \lambda'^\mu. \end{tikzcd}$$
)

- A small set of further consistent constructions exist:
  - unoriented string ("type I")
  - supersymmetrize only l. or only r.-movers (heterotic $E_8 \times E_8$ & heterotic $SO(32)$)
  - 11d SUGRA with 3d-objects (M2-branes) as fundamental degrees of freedom.
- They are all related by dualities

$\uparrow$ (different descriptions of the same physical theory)

[Example: $F = dA$ electric
$*F = \tilde{F} = d\tilde{A}$ magnetic > formulation of electrodynamics]

$$\Downarrow$$



3 Type IIB supergravity

3.1 10d action

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\,\Big\{e^{-2\phi}\Big[\mathcal{R} + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}H_3^2\Big]$$

$$- \frac{1}{2}F_1^2 - \frac{1}{2\cdot 3!}\tilde{F}_3^2 - \frac{1}{4\cdot 5!}\tilde{F}_5^2\Big\} + S_{CS} + S_{loc.} + S_F$$

$$2\kappa_{10}^2 = (2\pi)^7\alpha'^4$$

- The key novelty w.r.t. the bosonic string are

<u>p-form-fields:</u> $\quad F_p = dC_{p-1}$

$$\Big(\text{just like } \underset{\substack{\uparrow \\ H_{\mu\nu\rho}}}{H_3} = d\underset{\substack{\uparrow \\ B_{\mu\nu}}}{B_2}\Big)$$

- Explicitly:

$$\tilde{F}_3 = F_3 - C_0\wedge H_3 \quad ; \quad \tilde{F}_5 = F_5 - \frac{1}{2}\underbrace{C_2\wedge H_3}_{\sim (C_2)_{[\mu_1\mu_2}(H_3)_{\mu_3\mu_4\mu_5]}} + \frac{1}{2}B_2\wedge F_3$$

- This implies special gauge trfs., e.g. $C_2 \to C_2 + d\lambda_1$ & $C_4 \to C_4 + \frac{1}{2}\lambda_1\wedge H_3$

- <u>Note:</u> The above action and its IIA-cousin are the <u>only</u> existing 10d SUSY actions.

$$S_{CS} = \frac{1}{4\kappa_{10}^2} \int \underbrace{C_4 \wedge H_3 \wedge F_3}_{\text{10-form; can be integrated over 10d spacetime without } \sqrt{-g}.}$$

- Think of the above as "$A_\mu j^\mu$"

$$\overset{\nearrow}{\text{gauge field } C_4} \qquad \overset{\nwarrow}{\text{source } H_3 \wedge F_3}$$

(This structure will be crucial below.)

- S_F - fermionic terms ("gravitini/dilatini")
- Note: NS-NS ⟶ $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ

  R-R ⟶ C_p

  R-NS ⟶ fermions

3.2 D-branes and their action

Recall basic electrodynamics:

A_μ or $A_1 \equiv (A_1)_\mu dx^\mu$ ⟹ demands existence of e^- with:

$$S_{e^-} = \int_{WL} A - m \int_{WL} d\tau$$

worldline ↗ (WL) ↑ $\hat{=} A_\mu dx^\mu$ ↖ eigentime

- Similar logic applies to the <u>bosonic string</u>:

B_2 demands existence of a 2d object with

$$S = \int_{WS} B_2 - T \int_{WS} d\Sigma$$

$\uparrow$ surface element

(of course, we already know this object. It is the "<u>fundamental string</u>".)

- For the <u>10d superstring</u> we found:

gauge fields C_p — p even for IIB
$\quad\quad\quad\quad\quad\quad\;\;$ \ p odd for IIA

$\Rightarrow$ demands existence of "$D(p-1)$-branes"

counts only spatial dimensions, s.t.

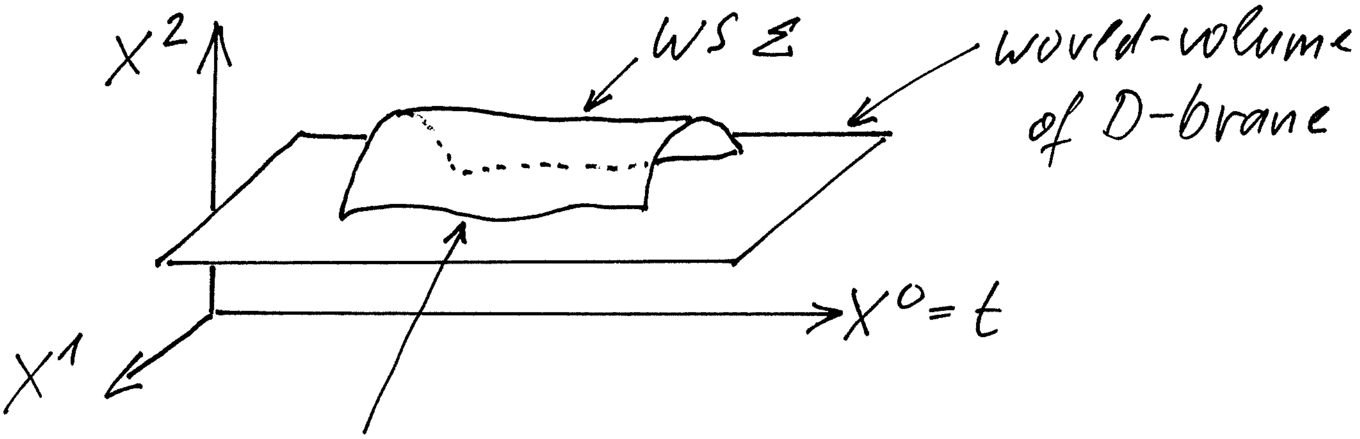
$D0$ = particle ; $D1$ = "string" ; $D2$ = membrane etc.

- The actions are "natural", as above. E.g.:

$$S_{loc} \supset S_{D3} = \frac{1}{2\pi^3 \alpha'^2} \int_{D3} C_4 - \int_{D3} d^4\xi \sqrt{-g}\, T_3 \quad ; \quad T_3 = \frac{e^{-\phi}}{(2\pi)^3 \alpha'^2}$$

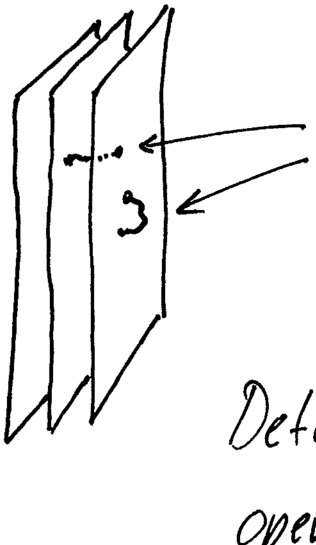
- Thus, Dp-branes couple to C_{p+1}-forms.

- "D" stands for "Dirichlet" since they can also be understood as defining boundary conditions for open strings:



$$X^2(\partial\Sigma) = X^2_{D\text{-brane}} = \text{fixed.}$$

- One can "<u>stack</u>" D-branes:



strings can end on the same or on different branes

$\Downarrow$

Detailed analysis of this type of open superstrings reveals:

A stack of N Dp branes realizes an SU(N) (p+1)-dimensional super-Yang-Mills-theory

4 Compactifications

To describe the real world, consider target space $\mathcal{M}_{10} = \mathbb{R}^{1,3} \times X_6$; X_6 - compact.

4.1 5d → 4d toy model

$$\mathcal{M} = \mathbb{R}^{1,3} \times S^1$$

$$S = \int_{\mathcal{M}} d^5x \; \frac{1}{2} (\partial_M \phi)(\partial^M \phi) \quad ; \quad M \in \{0, 1, 2, 3, 5\}$$

$$x^\mu = \{x^0, \dots, x^3\} \in \mathbb{R}^{1,3} \quad ; \quad x^5 \equiv y \in (0, 2\pi R)$$

- Ansatz: $\phi(x,y) = \sum_{n=0}^{\infty} \phi_n^c(x) \cos\left(\frac{ny}{R}\right) + \sum_{n=1}^{\infty} \phi_n^s(x) \sin\left(\frac{ny}{R}\right)$

- Performing $\int dy$, one finds a 4d action with one massless and many massive scalars ("Kaluza-Klein modes")

$$S = 2\pi R \int d^4x \left[\frac{1}{2} (\partial \phi_0^c)^2 + \frac{1}{4} \sum_{n=1}^{\infty} \left\{ (\partial \phi_n^c)^2 + m_n^2 (\phi_n^c)^2 + (\partial \phi_n^s)^2 + m_n^2 (\partial \phi_n^s)^2 \right\} \right]$$

- ϕ_0^c has no potential at all - can take any value ("it is a modulus")

$m_n = \frac{n}{R}$ $\Rightarrow$ KK-tower is irrelevant for low-energy ($E \ll 1/R$) observer

4.2 The "historical" Kaluza-Klein theory

$\mathcal{M} = \mathbb{R}^{1,3} \times S^1$, $\quad S = \frac{M_{P,5}^3}{2} \int d^4x \, dy \sqrt{-g} \, \mathcal{R}_5$

Ansatz:

$$(g_5)_{MN} = \left(\begin{array}{c:c} g_{\mu\nu} + (2/M_P^2)\,\phi^2 A_\mu A_\nu & (\sqrt{2}/M_P)\,\phi^2 A_\mu \\ \hdashline (\sqrt{2}/M_P)\,\phi^2 A_\nu & \phi^2 \end{array} \right)$$

$M, N, \dots \in \{0, \dots, 3, 5\}$; $\mu, \nu, \dots \in \{0, \dots, 3\}$

Let $\langle (g_5)_{MN} \rangle = \left(\begin{array}{c:c} \eta_{\mu\nu} & 0 \\ \hdashline 0 & \phi^2 \end{array} \right)$, $\quad M_P^2 = 2\pi R \phi M_{P,5}^3$,

$\Downarrow$ 4d

$$S = \int d^4x \sqrt{-g} \, \phi \left(\frac{M_P^2}{2} \mathcal{R} - \frac{1}{4} \phi^2 F_{\mu\nu} F^{\mu\nu} + \frac{M_P^2}{3} \frac{(\partial\phi)^2}{\phi^2} \right)$$

[Can be brought to Einstein-form by $g_{\mu\nu} \to g_{\mu\nu}/\phi$.]

Lessons:
- $U(1)$-symm. of $\mathcal{M}$ $\Rightarrow$ $U(1)$ gauge-symm. in 4d
- The "modulus" ϕ determines $\mathrm{Vol}(S^1) = 2\pi R \phi$.

4.3 T-duality

A very deep and important feature of string theory emerges upon compactification on S^1:

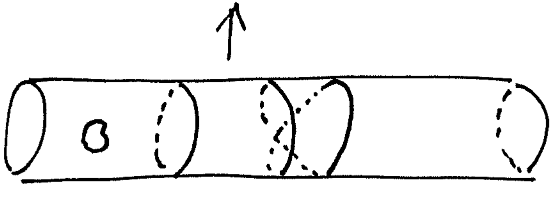
IIA on $\mathbb{R}^{1,8} \times S^1$ ↘
IIB on $\mathbb{R}^{1,8} \times S^1$ ↗ Same theory in 9d, including all massive states & interactions!

But: This identification is non-trivial.

It requires IIA on S^1 with R
IIB on S^1 with $R' = \frac{\alpha'}{R}$

The matching works as follows:

$$\begin{tikzcd}[column sep=large] \text{(A)} & \text{KK-tower} \arrow[dr, leftrightarrow] & \text{KK-tower} & \text{(B)} \\ & \text{winding-string tower} \arrow[ur, leftrightarrow] & \text{winding-string tower} & \end{tikzcd}$$



Implication: String theory does not allow distances $\ll l_s$. Making an S^1 small implies that a large S^1 "opens up" in the dual description.

4.4 Calabi-Yaus — basic facts

- We want $\mathbb{R}^{1,3} \times X_6$ to solve Einstein equs., s.t. our 4d world is not immediately unstable.

- This needs $R_{MN}(g_{6d}) = 0$, i.e. "Ricci-flatness". (Note that $R_{MNP}{}^{Q} \neq 0$ is OK.)

- Many examples of such manifolds are "Calabi-Yau". (In some sense most.)

- More formally, CYs are defined by a number of features:

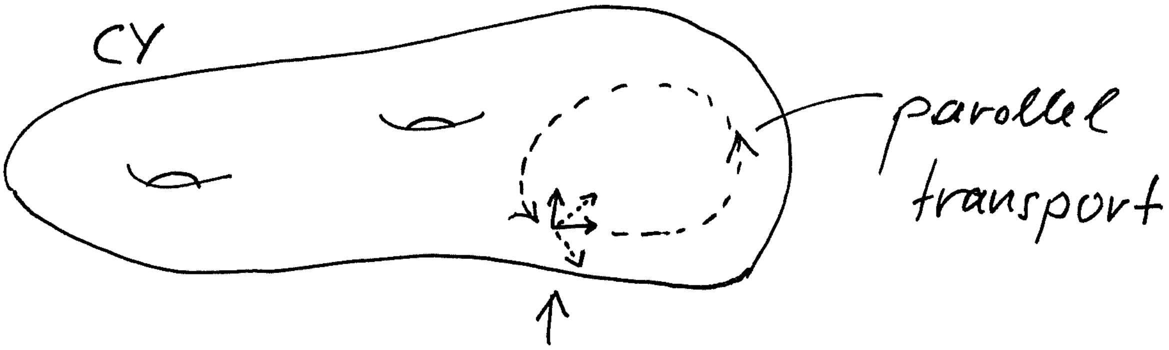
1) They are complex (3d) manifolds, i.e. the transition maps are $z'^{i} = z'^{i}(\underbrace{z^1, z^2, z^3}_{\text{no } \bar{z}\text{'s here!}})$

2) They are "Kahler", i.e.

$$g_{\underset{\substack{\uparrow \\ 1,2,3}}{i}\underset{\substack{\uparrow \\ \bar{1},\bar{2},\bar{3}}}{\bar{j}}} = \frac{\partial^2 K}{\partial z^i \partial \bar{z}^{\bar{j}}} \longleftarrow \text{Kahler potential}$$

$$(\& \; g_{ij} = g_{\bar{i}\bar{j}} = 0)$$

3) CY's have $\underline{\underline{SU(3) \text{ holonomy}}}$, i.e.



Transformation in tangent space is in

$$\underset{\underline{\underline{\text{CY !}}}}{\underset{\uparrow}{SU(3)}} \subset \underset{\substack{\text{generic case}\\ \text{for 6d manif.}}}{\underset{\uparrow}{SO(6)}}$$

<u>One can show:</u>

This is equivalent to the existence of a covariantly constant spinor and to unbroken ($\mathcal{N}=2$) SUSY after KK reduction.

- CYs can be deformed in many ways without losing their crucial feature $R_{mn}=0$.

  $\Rightarrow$ "<u>Moduli space</u>" of a CY.

  (Morally this corresponds to the field space of $\phi \sim Vol(S^1)$ of our S^1 toy model)

- "<u>Moduli</u>" are 4d scalars and the <u>moduli space</u> is the space in which they take their values. (cf. "sigma model", where this space is $SU(2)$.)

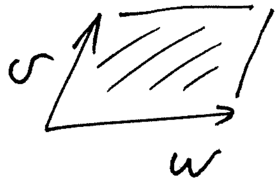
4.5 Differential forms / homology / cohomology

- Let $A = A_\mu dx^\mu$ be a 1-form.
- It "naturally" acts on a vector $v = v^\mu \partial_\mu$ by: $$A(v) = A_\mu v^\nu \underbrace{dx^\mu(\partial_\nu)}_{\delta^\mu{}_\nu} = A_\mu v^\mu$$
- This allows us to integrate A along a "1-cycle" C_1:

$$\int \;\to\; \underset{\overset{\uparrow}{\text{all infinitesimal vectors } v}}{\sum} A(v)$$

- Similarly, a 2-form $B_2 = \frac{1}{2} B_{\mu\nu} dx \wedge dx^\nu$ acts on two vectors, "measuring" the area of the corresponding parallelepiped:

$B(v,w) = B_{\mu\nu} v^\mu w^\nu$



$$\Rightarrow \int_{C_2} B_2 \equiv \sum_{\substack{\text{infinites.}\\ \text{parallelepipeds}}} \text{"areas"}$$

(no metric needed!)

- We mainly care about "closed" forms, i.e. with

$$dB_2 = \frac{1}{2}\partial_\mu(B_{\nu\rho})dx^\mu \wedge dx^\nu \wedge dx^\rho = 0$$

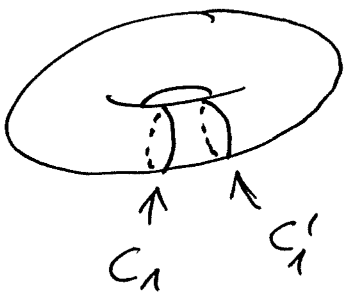
- We identify forms differing by an "exact" form (i.e. by a "gauge transformation" dX):

$$B_2 \sim B_2' \quad \text{if} \quad B_2 - B_2' = dA_1$$

$$A_1 \sim A_1' \quad \text{if} \quad A_1 - A_1' = dX_0 \qquad \text{etc.}$$

- This reduced space of forms is called "cohomology" and has finite dimension for a compact manifold.

- It is dual to the space of cycles, identified if they can be deformed into each other:



$$C_1 \sim C_1'$$

- The vector-space-duality works by integration:

$$C_1(A_1) = \int_{C_1} A_1 \quad \longleftrightarrow \quad \underset{\substack{\uparrow\\ V}}{X^\mu} \cdot \underset{\substack{\uparrow\\ V^*}}{Y_\mu}$$

4.6 CY moduli space

For a CY, there are two key forms:

1) The Kahler form

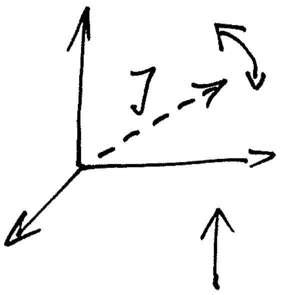
$$J = i\, g_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}} \quad \rightarrow \quad \text{"metric information"}$$

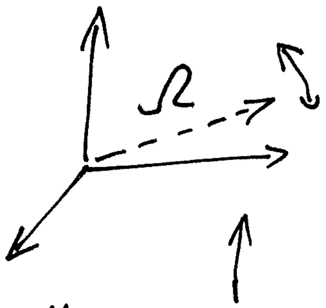
2) The holomorphic 3-form

$$\Omega = \frac{1}{3!}\, \Omega_{ijk}(z^1, z^2, z^3)\, dz^i \wedge dz^j \wedge dz^k \sim \underset{\uparrow}{\bar{\psi}}\, \gamma^i \gamma^j \gamma^k \underset{\uparrow}{\psi} \cdots$$

→ "complex-structure information"

covariantly constant spinor





The dimensions of the spaces are fixed by the numbers of 2-cycles and 3-cycles of our CY.

- Coordinates for explicit parametrization:

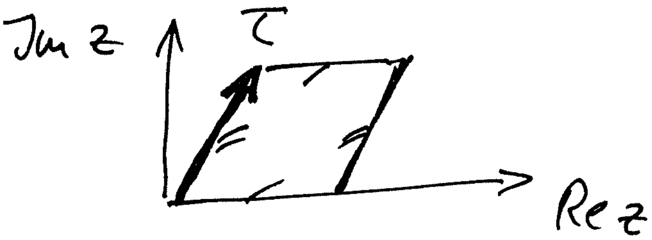
$$\int_{C_2^\alpha} J \sim t^\alpha \qquad \text{(2-cycle volumes)}$$

$$\int_{C_3^a} \Omega \sim \Pi^a(z) \qquad \text{(3-cycle volumes} \sim \text{"periods")}$$

4.7 Torus toy model

T^2 is a "CY 1-fold": $\quad z = x + \tau y$

$x, y \in [0, 1]$



$$\Omega = \alpha\, dz \longrightarrow \Pi_1 = \int_{y=\text{const.}} \Omega = \alpha$$

$$\longrightarrow \Pi_2 = \int_{x=\text{const.}} \Omega = \alpha\tau$$

$$J = t\, dx \wedge dy \longrightarrow Vol(T^2) = \int_{T^2} J = t$$

$$g_{ab} = \frac{t}{\mathrm{Im}(\Pi_2/\Pi_1)} \begin{pmatrix} 1 & \mathrm{Re}(\Pi_2/\Pi_1) \\ \mathrm{Re}(\Pi_2/\Pi_1) & |\Pi_2/\Pi_1|^2 \end{pmatrix}$$

Remark: The moduli space is the space of non-equivalent metrics. It can be described in terms of (the components of) J & Ω.

4.8 Explicit CY moduli spaces

Use $\mathcal{N}=1$ SUGRA language, s.t.

$$\mathcal{L} = \underset{\uparrow}{K_{I\bar{J}}} (\partial\varphi^I)(\partial\bar{\varphi}^{\bar{J}}) + \text{gauge} + \text{fermions} + \cdots$$

Derivatives of 4d Kahler potential
(not to be confused with Kahler pot. of CY metric)

① $J = t^\alpha \underset{\uparrow}{\omega_\alpha}$, $\alpha = 1, \dots, h^{1,1}$

basis 2-forms

CY volume: $\mathcal{V} = \frac{1}{6}\int_X J\wedge J\wedge J = \frac{1}{6} \underset{\uparrow}{\kappa_{\alpha\beta\gamma}} t^\alpha t^\beta t^\gamma$

topolog. data

A useful (for us) change of variables:

$$t^\alpha \longrightarrow \tau_\alpha = \frac{1}{2}\int_{C^\alpha_{(4)}} J\wedge J = \frac{\partial \mathcal{V}}{\partial t^\alpha}$$

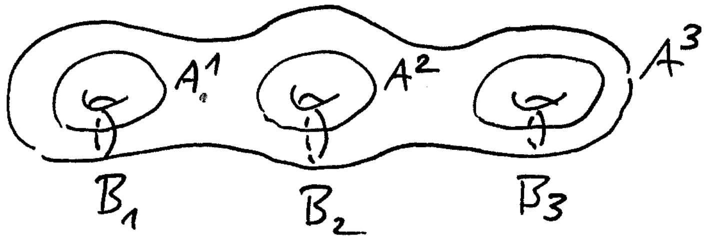
$$\Rightarrow \underset{\uparrow}{K_K} = -2\ln \mathcal{V}(T_\alpha, \bar{T}_\alpha)$$

"Kahler" $T_\alpha = \tau_\alpha + i c_\alpha$; $c_\alpha = \int_{C^\alpha_{(4)}} C_4$

---

② $\Omega \xrightarrow{\text{periods:}} \cdots$ $z^a = \int_{A^a_{(3)}} \Omega$; $\mathcal{G}_b(z) = \int_{B_{(3)b}} \Omega$

A's & B's : symplectic basis of 3-cycles, like...



$$\underset{\substack{\uparrow\\ \text{"compl. structure"}}}{K_{cs}} = -\ln\left(i\int_X \Omega\wedge\bar{\Omega}\right) = -\ln\left(-i\bar{z}^a \underset{\uparrow}{\mathcal{G}_a}(z) + c.c.\right)$$

calculated by solving PDEs for each CY "by hand".

Finally:

$$K = K_K(T^\alpha, \bar{T}^{\bar{\alpha}}) + K_{cs}(z^a, \bar{z}^{\bar{a}}) - \ln\left(-i(S-\bar{S})\right)$$

$$S \equiv C_0 + \frac{i}{g_s} \leftarrow \text{"axio-dilaton"}.$$

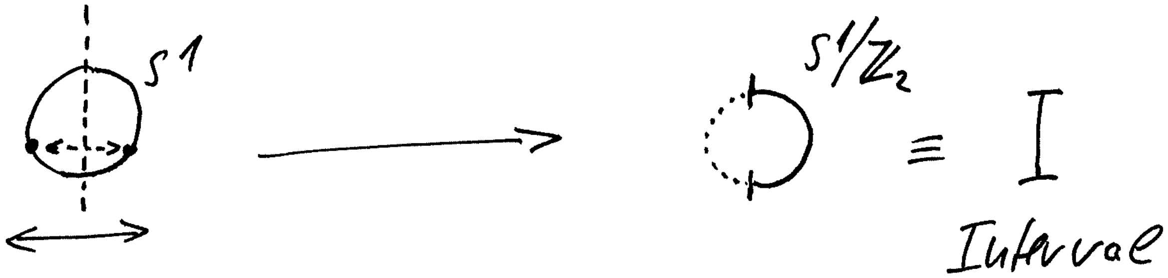
$M_{P,4} = 1$; All CY-related volumes are measured in units of $l_s = 2\pi\sqrt{\alpha'}$.

5 The flux landscape

(Bousso/Polchinski ; Giddings/Kachru/Polchinski)
"GKP"

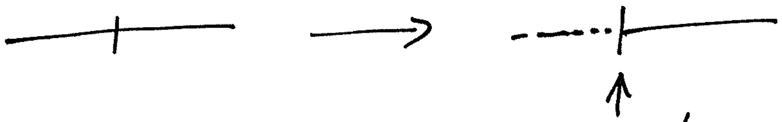
5.1 Orbifolds

A useful way of constructing new compact spaces is by "modding out" a discrete symmetry:



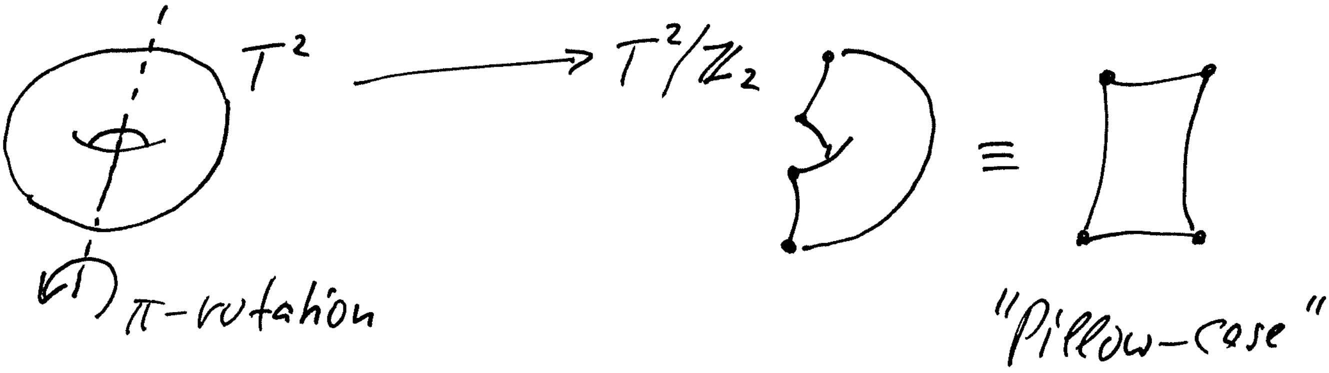
locally, this is just

$$\mathbb{R} \longrightarrow \mathbb{R}_+ = \mathbb{R}/\mathbb{Z}_2$$



a singularity, in this case a boundary, is created

A more interesting, 2d example:



[Singularities of this type are OK for the stringy UV completion as long as some SUSY is preserved.]

Even better: $CY \longrightarrow CY/\mathbb{Z}_2$

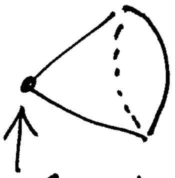
with $\mathbb{Z}_2$ locally e.g. $(z^1, z^2, z^3) \longrightarrow (-z^1, -z^2, -z^3)$

5.2 Orientifolds

- Replace $\mathbb{Z}_2$-action by the same geometric action, e.g. $y \to -y$, combined with an orientation change of the string: $(\tau, \sigma) \to (\tau, -\sigma)$.

- The 10d geometry is then modified in the same way as for orbifolds, e.g.

or



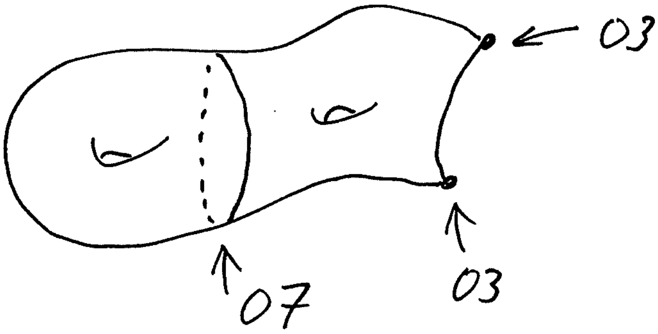
etc.

- But, crucially, the singular locus acquires non-trivial charges / couplings:

The induced Op-planes carry opposite charge w.r.t. the corresponding Dp-branes, e.g.

An O7 plane needs 4 D7 branes to be placed "on top" to be neutral w.r.t. the gauge potential C_8.

- Most relevant for us: O7, O3 (with D7, D3 on top).
- Thus, we consider type IIB on $CY/\mathbb{Z}_2$ ("CY-orientifolds") with O3/O7-planes:



- Need to include D7/D3s to make the whole compact geometry charge-neutral and hence consistent (technical term: "Tadpole cancellation").
- Crucially: One may choose not to cancel the "D3 tadpole" by D3-branes but instead by fluxes:

$$\boxed{\text{Background-values (quantized) of } F_3, H_3: \quad \langle F_3 \rangle, \langle H_3 \rangle}$$

- To see this, recall:

$$S_{IIB} \supset \int C_4 \wedge \underbrace{H_3 \wedge F_3}_{\text{"}j_{D3}\text{"}} + \int_{D3} C_4$$

$\Rightarrow$ $\langle H_3 \wedge F_3 \rangle \;\hat{=}\;$ D3's in CY

$\Rightarrow$ IIB orientifolds allow for the presence of H_3, F_3 - flux with a specific, limited "tadpole", i.e. $\int H_3 \wedge F_3 \leq |Q_3| \in \mathbb{Z}$

5.3 Flux compactifications "at GKP level"

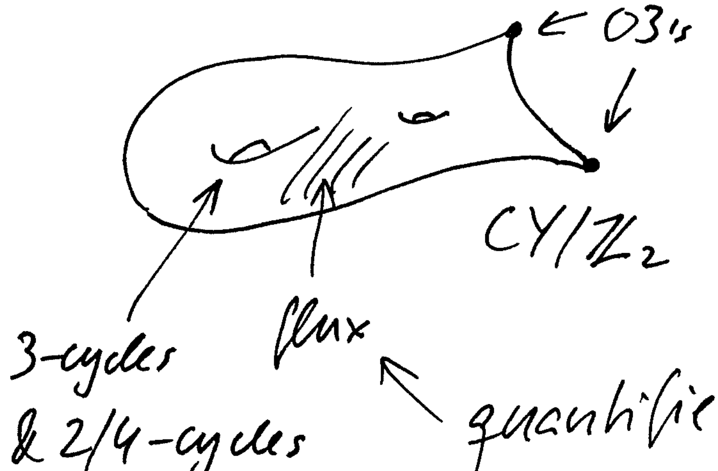
- Key fact: H_3, F_3 are quantized in the sense that $$\int_{C_{(3)}} H_3 \,, \int_{C_{(3)}} F_3 \sim \in \mathbb{Z}$$
- This is completely analogous to the magnetic charge quantization familiar from electrodynamics:

[Reason: $\Psi_{\text{electron}}(\theta, \varphi)$ can otherwise not be a well-defined fct. on the whole S^2.]

- In our case: $\Psi_{D1/F1}$ must be well-defined

  ↑ fund. string

- Thus, our model is specified by a CY-orientifold with a certain (negative) tadpole:



$\Longrightarrow$ $Q_3 < 0$

quantified by integer-valued vectors $f, h \in (\mathbb{Z})^{h^3}$

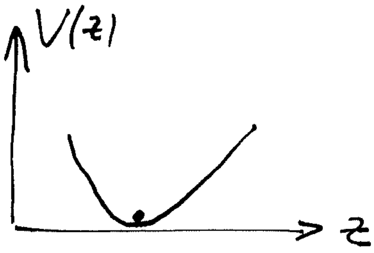
- Must obey:

$$\int H_3 \wedge F_3 \sim \underset{\substack{\uparrow \\ \text{symplectic} \\ \text{metric}}}{f \cdot h} = -Q_3 - \underset{\substack{\uparrow \\ \text{(pos.) number} \\ \text{of D3 branes}}}{N_{D3}}$$

- Since $S \supset \underbrace{\int |F_3|^2 + |H_3|^2 e^{-2\phi}}_{\text{involves metric}}$,

the presence of fluxes "stabilizes" 3-cycle moduli and axio-dilaton

$$\underset{\substack{\uparrow \\ \text{"}z^a\text{"}}}{\text{moduli}} \quad \text{and} \quad \underset{\substack{\uparrow \\ \text{"}S = C_0 + \frac{i}{g_s}\text{"}}}{\text{axio-dilaton}}$$



- To quantify this, we employ 4d $N=1$ supergravity language:

$$\mathcal{L} = \frac{1}{2}R + K_{I\bar{J}}(\partial\varphi^{I})(\partial\bar{\varphi}^{\bar{J}}) - V(\varphi,\bar{\varphi}) + \cdots$$

with

$$K = K_K(T^{\alpha},\bar{T}^{\bar{\alpha}}) + K_{cs}(z^{a},\bar{z}^{\bar{a}}) - \ln(-i(S-\bar{S}))$$

&

$$\underset{\substack{\uparrow \\ \text{"Gukov-Vafa-Witten superpotential"}}}{W} = \int \underset{\substack{\uparrow \\ \equiv F_3 - SH_3}}{G_3} \wedge \Omega_3 = (2\pi)^2 (f - Sh)\cdot \underset{\substack{\uparrow \\ \begin{pmatrix} z^{a} \\ \mathcal{G}_b(z) \end{pmatrix}}}{\Pi(z)}$$

&

$$V = e^{K}\left(K^{I\bar{J}}(D_I W)(\overline{D_J W}) - 3|W|^2\right)$$

$$\text{with } I \to (\underset{\substack{\uparrow \\ \text{Kähler}}}{\alpha}, \underset{\substack{\uparrow \\ \text{compl.-structure}}}{a}, \underset{\substack{\uparrow \\ \text{axio-dilaton}}}{S})$$

& $D_I W = \partial_I W + K_I W$

- The following crucial cancellation occurs:

$$V \supset e^{K}\left(K_K^{\alpha\bar{\beta}}(D_\alpha W)(\overline{D_\beta W}) - 3|W|^2\right) \equiv 0$$

if: • W is independent of T^α

• $K_K = -2 \ln \underbrace{\mathcal{V}(T^\alpha, \bar{T}^{\bar{\alpha}})}_{\text{homogeneous fct. of degree } 3/2}$.

[This is the famous "no-scale structure" of this type of supergravity model.]

$$\Rightarrow \quad V = e^K K^{i\bar{j}} (D_i W)(\overline{D_j W})$$

with $i \to (\alpha, S)$, i.e. just compl.-structure moduli & axio-dilaton

- This V is manifestly ≥ 0.
- Minima are at $D_i W = 0$, which generically stabilizes ("fixes") z^α & S.

$\Rightarrow$ Fluxes stabilize 3-cycle-ratios, as intuitively expected:

fluxes on two different 1-cycles of a torus.

$\Rightarrow V \equiv 0$ (independently of T^α)

$\Rightarrow$ T^α's are not stabilized – their potential remains flat.

$\Rightarrow$ Volume (and all 4-cycle ratios) unfixed.

~~SUSY~~-scale can change freely

$\Rightarrow$ name "no-scale model".

Note: $D^i W \equiv F^i = 0$ $\Rightarrow$ no ~~SUSY~~ in compl.-structure sector

$D^\alpha W \equiv F^\alpha \neq 0$ $\Rightarrow$ ~~SUSY~~ in Kahler sector

$(|D^\alpha W|^2 = 3|W|^2)$

$\Rightarrow$ Loop-corrections will induce a potential also for the T^α's, but this is hard to calculate.

Summary: ("GKP")

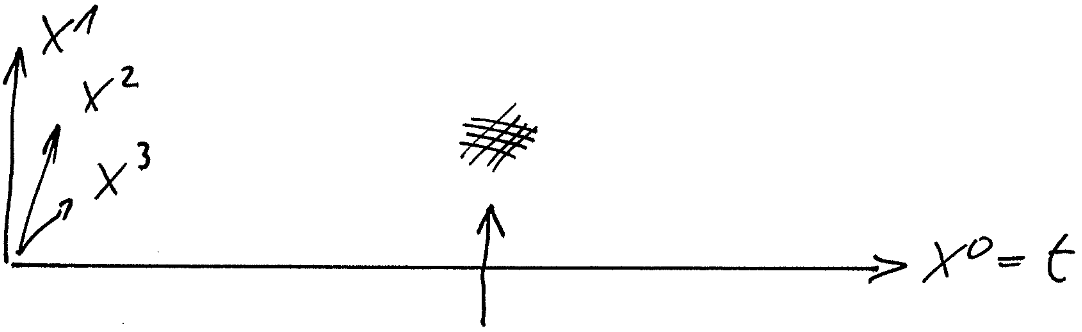
z^a, S fixed ; T^α free ; ~~SUSY~~ ;

$V \equiv 0$ (at LO) $\Rightarrow$ not bad!

6 Volume stabilization and the cosmological constant

6.1 Non-perturbative effects

A) Gauge instantons (e.g. in QCD)

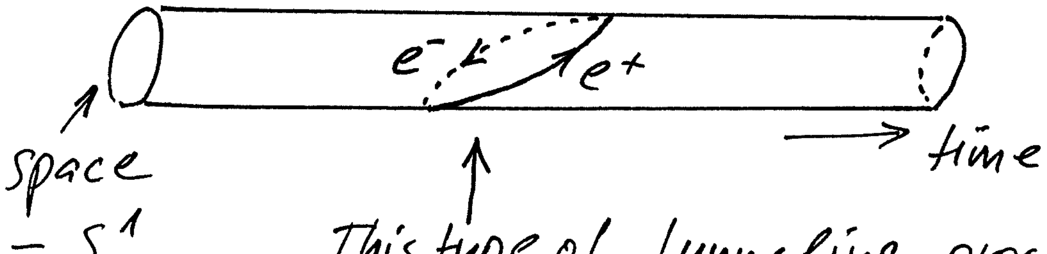


region with $F_{\mu\nu} \neq 0$

(specifically $\frac{1}{8\pi^2}\int tr(F \wedge F) = 1$.)

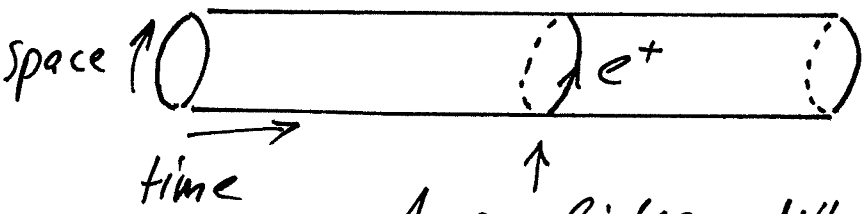
This is a localized tunneling process or "instanton". [Summing over all of them famously induces a potential for the axion field.]

B) QED in $d=2$ compactified on S^1



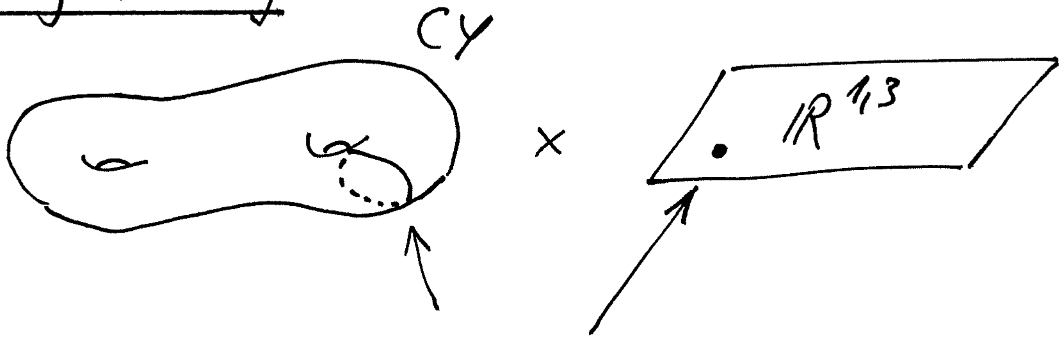
This type of tunneling process is the simplest "brane-instanton".

- A better way to think of this:



A euclidean WL of an electron is wrapped on a 1-cycle of the compact space at a certain locus in (non-compact) time.

c) String theory



Euclidean D3-brane ("E3-brane") wrapped on 4-cycle of CY at a certain locus in $\mathbb{R}^{1,3}$.

- In all cases the potential receives corrections $\sim e^{-S_{inst.}}$

- In the last case: $S_{inst.} \sim Vol(4\text{-cycle})$.

- Concretely we know: $Vol(C^{\alpha}_{(4)}) = Re\, T^{\alpha}$

- In our case δV is induced by a superpotential-correction δW:

Fact: $$\delta W = \sum_{\text{4-cycles}} A(z)\, e^{-2\pi T^{\alpha}}$$

[Note: A similar δW-effect is induced by gaugino condensation in SU(N) gauge theories realized by D7-brane stacks on 4-cycles.]

6.2 SUSY-KKLT

(Kachru/Kallosh/Linde/Trivedi)

- Start with a CY-orientifold with a single Kähler modulus:

$$K_k = -2\ln(T+\bar{T})^{3/2} = \underline{\underline{-3\ln(T+\bar{T})}}.$$

- $W = W(S,z)$ due to flux
- $K = K_k(T,\bar{T}) + K_{CS+AD}(S,\bar{S},z,\bar{z})$

$$z = \{z^1, \dots, z^n\}$$

- As explained above, the scalar potential (or equivalently the SUSY conditions $D_z W = D_S W = 0$) fix the moduli S, z.

$\Rightarrow$ Our low-energy EFT has:

$$K = -3\ln(T+\bar{T}) \quad ; \quad W = \underset{\substack{\uparrow \\ \equiv W(S_0, z_0) \text{ as just explained}}}{W_0} = \text{const.}$$

$$V = e^K\left(K^{T\bar{T}}|D_T W|^2 - 3|W|^2\right) \equiv 0.$$

- Including instanton corrections, we have:

$$W = W_0 + \delta W = W_0 + A(S_0, z_0)\, e^{-2\pi T}$$

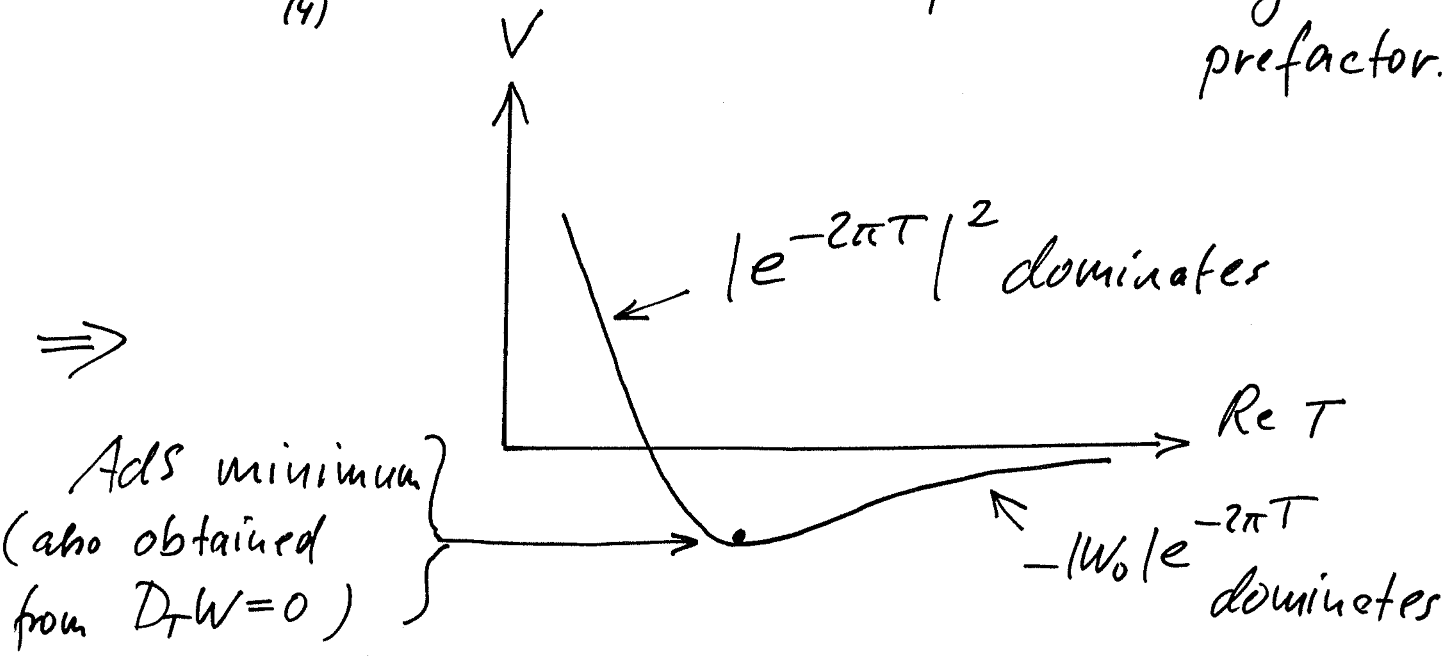
$$\Rightarrow V \sim e^K K^{T\bar{T}}\left(|Ae^{-2\pi T}|^2 + \underbrace{A\bar{W}_0 e^{-2\pi T} + \text{h.c.}}\right)$$

$T = \tau + ic$

$c \sim \int_{C_{(4)}} C_4$

The phase $\exp(-2\pi i c)$ adjusts such that this term acquires a negative prefactor.

$\Rightarrow$



- In this vacuum: $\mathcal{V} \sim T^{3/2} \sim \ln^{3/2}(1/|W_0|)$

  $\Rightarrow$ Need $|W_0| \ll 1$ for "control" in 10d SUGRA.

- This is achievable by flux choice, which determines S_0, z_0 and hence W_0

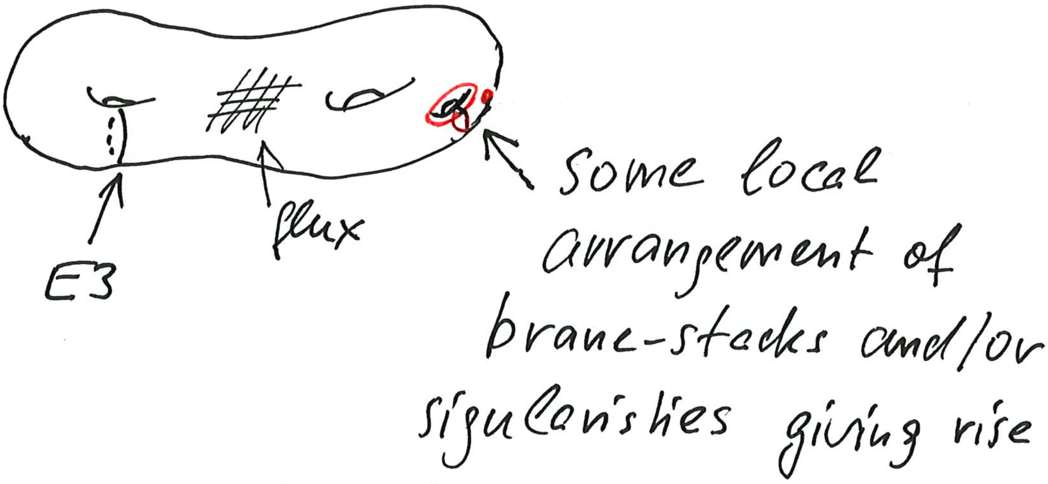
  $\Rightarrow$ Huge "landscape" of SUSY-AdS vacua with all moduli stabilized.

- Key open problems:
  - cosm. const. $\Lambda < 0$
  - SUSY unbroken

6.3 Uplift - general idea & toy model

- Need positive contribution to potential

- Idea:



E.g.:

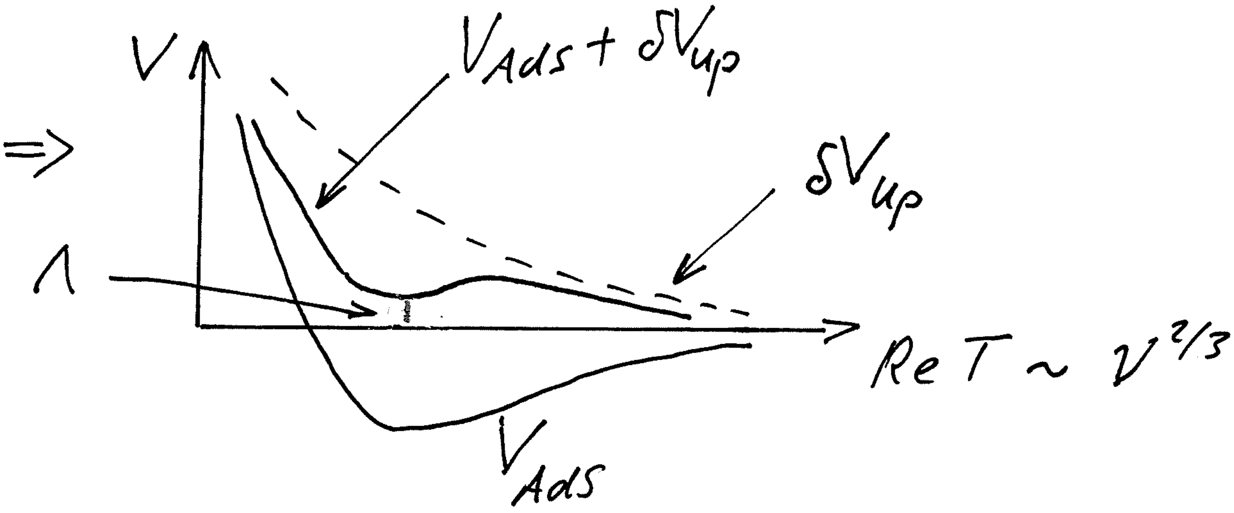
$$\delta K = X\bar{X} - (X\bar{X})^2 \quad ; \quad \delta W = \alpha X$$

$$\Rightarrow \quad |D_X W| \neq 0 \text{ in vacuum.}$$

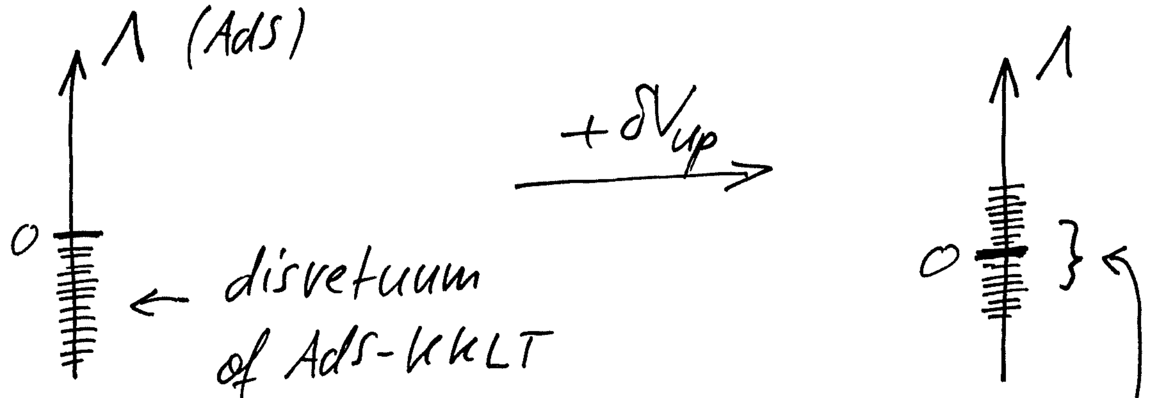
$$\Rightarrow \delta V = \underset{\overset{\uparrow}{\sim 1/\mathcal{V}^2}}{e^K} K^{X\bar{X}} |D_X W|^2 > 0$$

(Potential in units of M_p^4 goes to zero because as $\mathcal{V} \to \infty$ because $M_p^2 \sim \mathcal{V}$.)

$\Rightarrow$



$$\Lambda = \delta V_{up} + V_{AdS} = \delta V_{up} - c\,|W_0|^2$$



disretuum of AdS-KKLT vacua

many vacua with very small $|\Lambda|$ are expected (depending on size of landscape)

(The homogeneity of this distribution has been argued for by Denef / Douglas.)

6.4 Tuning the cosmological constant

- The key object is the flux vector $\vec{n} = \begin{pmatrix} f \\ h \end{pmatrix}$
- It lives in a lattice of high dimension:

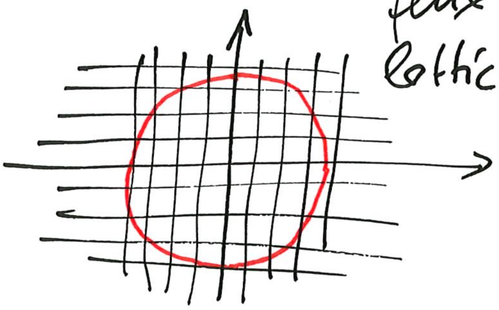
$$\vec{n} \in (\mathbb{Z})^{2h_3}$$

- Its length is constrained by the available negative tadpole: $f \cdot h \leq |Q_3|$

$$\Rightarrow \mathcal{N}_{vac.} \sim |Q_3|^{h_3} / h_3!$$

(This corresponds to the scaling of the volume of a sphere.)

in very high dimension! →



- The historical estimate was roughly

$$\underbrace{|Q_3| \sim 10^4 \, ; \, h_3 \sim 300}_{} \Rightarrow \mathcal{N}_{vac.} \sim 10^{600}$$

By now much larger values known in non-pert. branch of type-IIB ("F-theory").

- To appreciate size of landscape, note that there are $\sim 10^{80}$ atoms in visible universe.

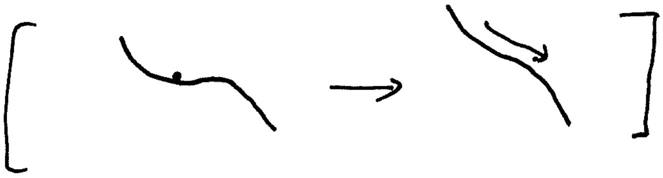
- If our logic works, we can find vacua with $|\Lambda|$ as small as $10^{-600} M_p^4$ (i.e. much smaller than the observed value $10^{-120} M_p^4$).

Note:
- This does not say anything about why we live in such a "very special" vacuum. (Cf. however Weinberg's prediction of Λ and "anthropic" arguments.)
- Our arguments were only about the existence of such vacua in spite of them being strongly fine-tuned.
- Even if $1/10^{120}$ of the 10^{600} vacua have the right Λ, these are still $\underline{\underline{10^{480}}}$.

$\Rightarrow$ Many further fine-tunings, e.g. for $m_H \ll m_{SUSY}$, are possible.

6.5 Uplift – anti-D3s and Beyond

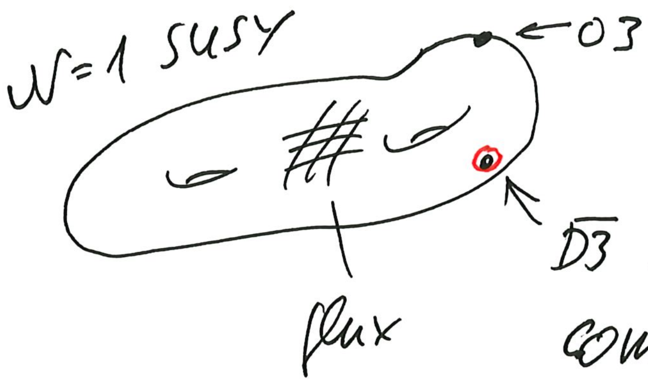
- Since $-V_{AdS} \sim |W_0|^2 \ll 1$, the uplift must be very small. Otherwise, no meta-stable minimum arises:



- So far, no explicit stringy version of our "QFT toy model uplift" has been built.
- The most explicit uplift is the anti-D3 (or "$\overline{D3}$") of KKLT / KPV:
- Starting point:

$F_3 \wedge H_3$ & D3:	$\overline{D3}$:
same D3-charge & preserve same $\mathcal{N}=1$ SUSY in our CY.	opposite D3-charge and preserve opposite $\mathcal{N}=1$ SUSY

of $\mathcal{N}=2$ of CY



$\overline{D3}$ breaks 4d SUSY completely ($\mathcal{N}=0$)

↑

It also "uplifts" by $T_{D3} \sim M_s^4$, which is much too high

- Idea: look more carefully at CY geometries in presence of fluxes:

(cf. e.g. "GKP")

↑

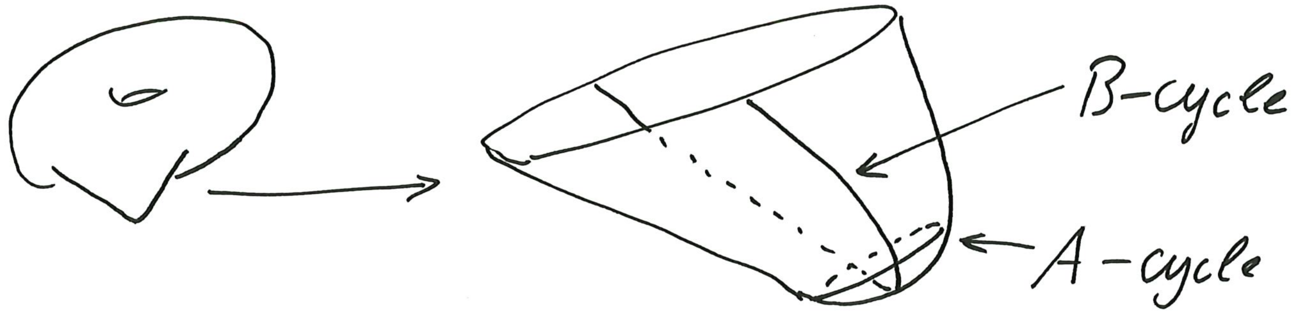
Giddings / Kachru / Polchinski

$$ds^2 = h^{-1/2}(y)\, \underset{\underset{\mathbb{R}^{1,3}}{\uparrow}}{dx^2} + h^{1/2}(y)\, \underset{\underset{\text{CY metric}}{\uparrow}}{\tilde{g}_{mn}(y)}\, dy^m dy^n$$

This is not $\mathbb{R}^{1,3} \times CY$ due to warp-factor $h(y)$.

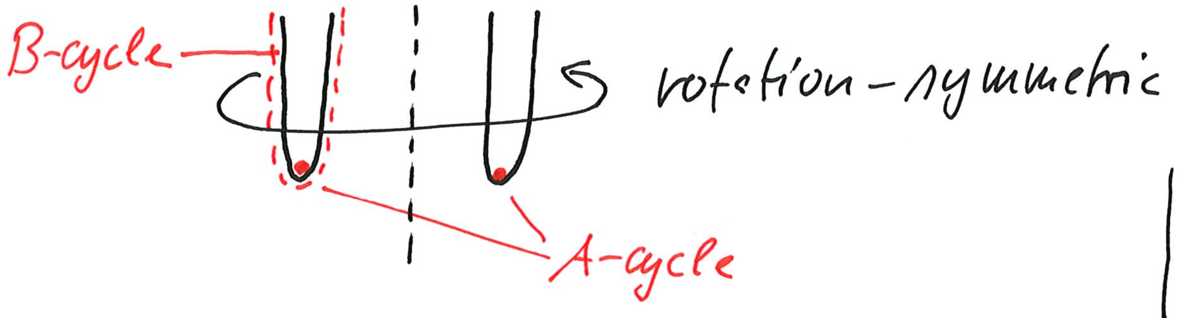
Its EOM: $$-\tilde{\nabla}^2 h(y) = g_s\, \underset{\underset{\sim F_3 \wedge H_3 + \sum_i \delta^6(y - y^i_{D3}) + \dots}{\uparrow}}{\rho_{D3}(y)}$$

- **Fact:** CYs generically possess "conifold regions":



two 3-cycles (A, B) with one of them (A) potentially shrinking to zero size:

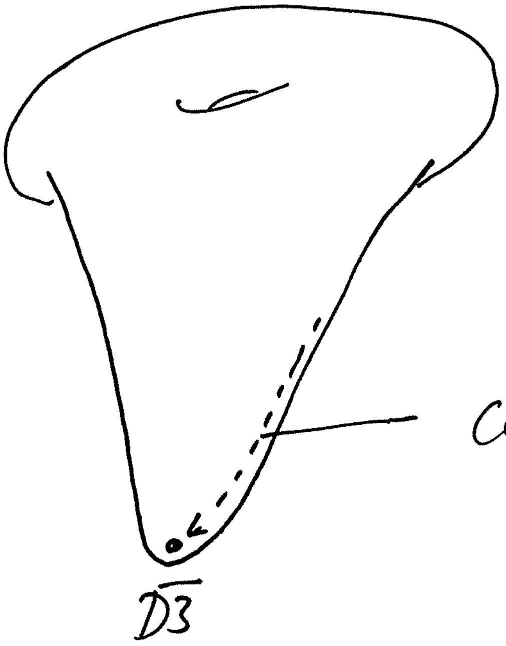
Illustration by a toy model:



- Now place M units of F_3-flux and K units of H_3-flux on A & B cycle respectively. Let $K \gg M$.

$\Longrightarrow$

- Key conceptual point: $h^{-1/2}(y)\, dx^2$ makes 4d effective mass scales small ("red shifted") in regions of CY where $h \gg 1$.



The $\overline{D3}$ "sinks" to the bottom and stays there, contributing a potential

$$\delta V_{up} \sim T_{D3}\, h^{-1}$$

$$\sim T_{D3} \exp\left(-\frac{8\pi K}{3 g_s M}\right)$$

- This (very explicit) uplifting mechanism, together with "AdS-KKLT" is a celebrated result and historically a cornerstone of the "landscape".

- Various concerns about the (meta-) stability of the $\overline{D3}$ were raised but never substantiated (→ Bena, Graña, Van Riet, Shiu, ...)

- More recently, in the "Swampland" context, these de Sitter constructions have been scrutinized again.
- It was in particular found that:
(Carta/Moritz/Westphal; Gao/AH/Junghans)

a) The constraints on the various parameters enforce $R_{\text{throat}} \gtrsim R_{CY}$

("strange geometry")

b) This, in turn, induces such a strong warping (i.e. strongly varying h), that singular regions ($h < 1$) arise.

$\Downarrow$

"Singular bulk problem"

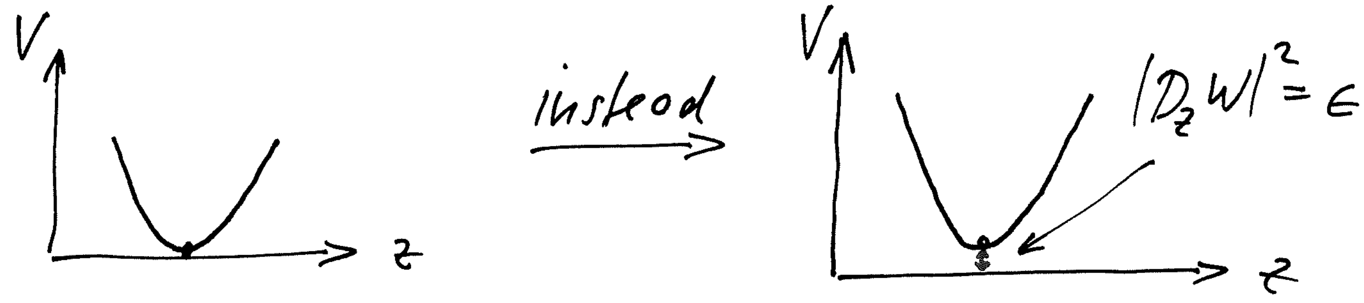
--- c.f. also subsequent work by McAllister, Moritz, ...

$\Rightarrow$ Maybe worth exploring alternative idea:

Complex-Structure F-term uplift

(Saltman/Silverstein)

- Here, one does *not* start with $|D_zW|^2=0.$



- This relies on the existence of non-SUSY local minima of
$$V(z,\bar{z}) \sim |D_zW|^2 \equiv \epsilon \quad \text{with} \quad \epsilon \ll 1.$$

(numerical evidence exists).

Also: statistical analysis by Denef/Douglas

- Other uplift-ideas exist. More work needed!

6.6 Large Volume scenario

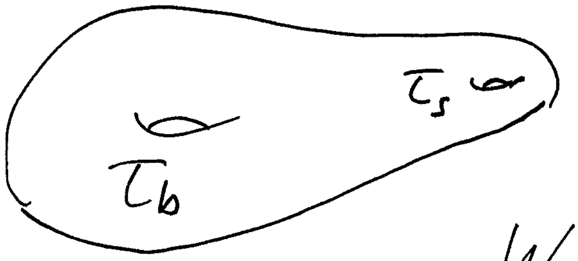
(Balasubramanian/Berglund/Conlon/Quevedo)

- Consider CYs with a minimally more complicated 4-cycle lattice:

$$K_k = -2\ln V = -2\ln\left(\tau_b^{3/2} - \tau_s^{3/2}\right)$$

$$\text{with}\quad \tau_b = (T_b + \overline{T}_b)/2$$

$$\tau_s = (T_s + \overline{T}_s)/2$$



$$W = W_0 + A e^{-2\pi T_s}$$

(corresponding T_b-term irrelevant)

$$K = K_k + \delta K = -2\ln\Big(V + \underbrace{\xi/g_s^{3/2}}_{\text{"}\alpha'\text{-correction"}}\Big)$$

[Follows from R^4-term in 10d lagrangian. ξ explicitly known in terms of topological features of CY.]

- One can now explicitly minimize the 4-field scalar potential

$$V = e^K\left(|D_{T_i}W|^2 - 3|W|^2\right).$$

- Result:

$$\tau_s \sim \xi^{2/3}/g_s \quad ; \quad \mathcal{V} \sim \tau_b^{3/2} \sim |W_0| e^{2\pi\tau_s}$$

tunable to $g_s \ll 1$ in landscape

This is <u>exponentially</u> large in τ_s and hence $\mathcal{V} \gg \ell_s^6$ is easy to achieve.

<u>But:</u> Finer effects (like R^4-term) were needed in the analysis.

$\Rightarrow$ Warping-related "control issues" (like in KKLT) still exist. Reliability unclear.

[$\hookrightarrow$ Junghans; AH/Venken/Schreyer]

<u>Summary:</u> My bets are on LVS + "complex-structure F-term uplift." KKLT too fragile to warping corrections.

7 Multiverse

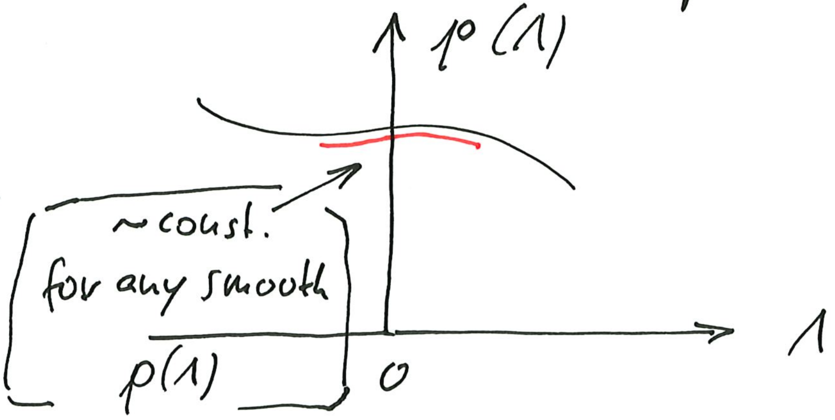
7.1 Predictions & measure problem

- So far, we only argued about the existence of certain solutions (e.g. $0 < \Lambda \ll 1$) in the landscape.
- No probabilities were assigned to values of parameters. Hence, no predictions were discussed.
- Famously, Weinberg made a prediction for Λ using "landscape/multiverse" logic.

Rough idea: (in my reading)

- $|\Lambda| < \Lambda_c$ experimentally
- Assume many universes with different Λ (where $\Lambda = 0$ is not special).

, i.e.



$\Rightarrow$ Exploring the region $\Lambda < \Lambda_c$, we expect to find $|\Lambda_{obs}| \sim \Lambda_c$

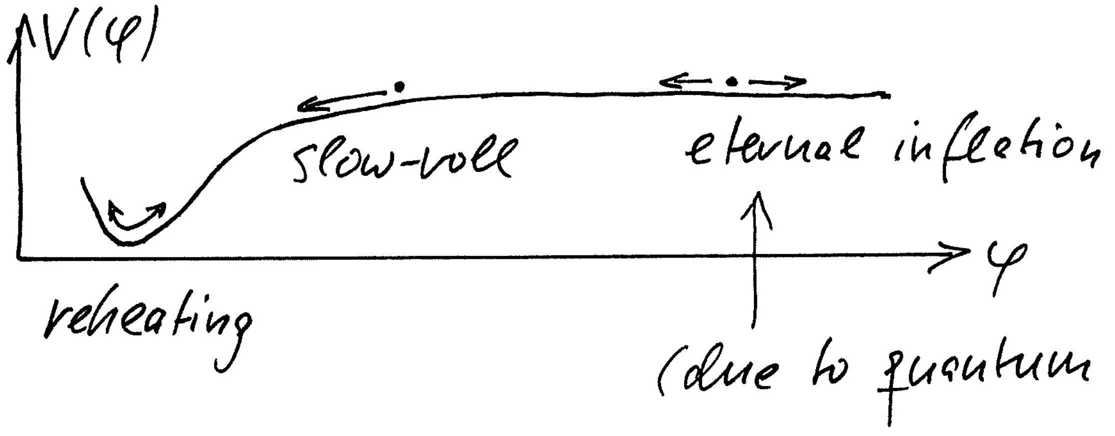
(<u>not</u> $|\Lambda_{obs}| \ll \Lambda_c$)

- <u>However:</u> This does not come for free. It uses a measure ($p(\Lambda)$ or the assumption that each vacuum is equally likely.)
- What is the basis for this (or any other) measure? ("Measure problem")

<u>7.2 Eternal Inflation</u>

- To quantify the problem, we must consider early cosmology and hence inflation.
- <u>Reminder:</u> $ds^2 = -dt^2 + a^2(t)\, \underset{\substack{\uparrow \\ \text{max. symmetric} \\ \text{3d space.}}}{g_{ij}}\, dx^i dx^j$

$$\mathcal{L} = \frac{1}{2} R - \frac{1}{2} (\partial\varphi)^2 - V(\varphi)$$



- **Slow-roll:** $\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$; $\eta \equiv \frac{V''}{V} \ll 1$

  (EOM: $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{V}{3}$; $3H\dot{\varphi} = -V'$)

- **Eternal:** $\epsilon < V$

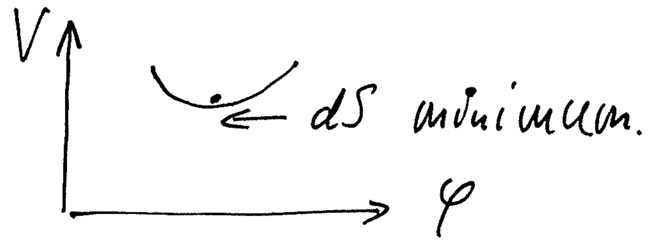
  (EOM: $H =$ const. ("de Sitter")

  or $a \sim \exp(Ht)$)

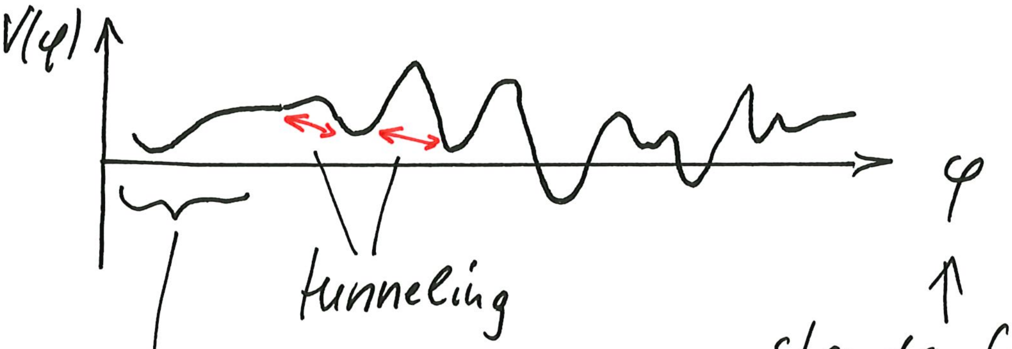
- But this is <u>not</u> the type of eternal inflation relevant for the string multiverse.

- In the string landscape, flat potentials at $V > 0$ are very rare (or non-existent).

- More probable:



- Or many such minima:

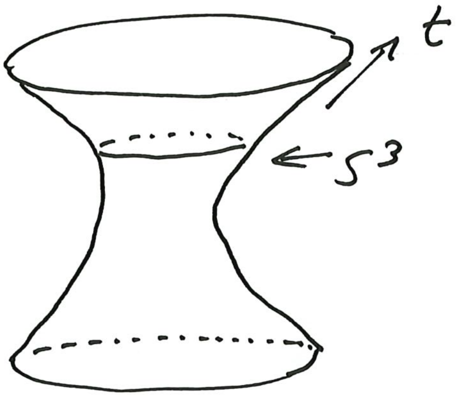


"Our" vacuum with its inflationary plateau

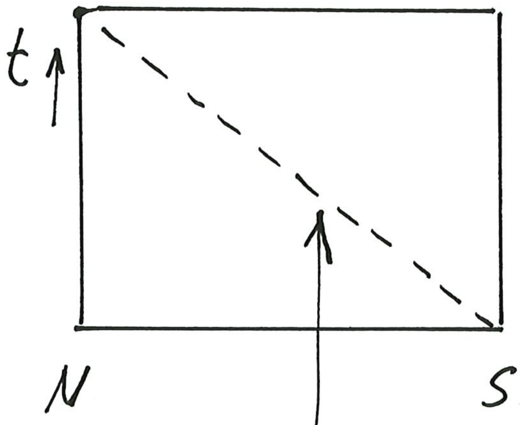
stands for many moduli:

$$\varphi = \{\varphi^1, \dots, \varphi^m\}$$

- If φ just sits in one of those minima with $\Lambda > 0$, the world is dS:

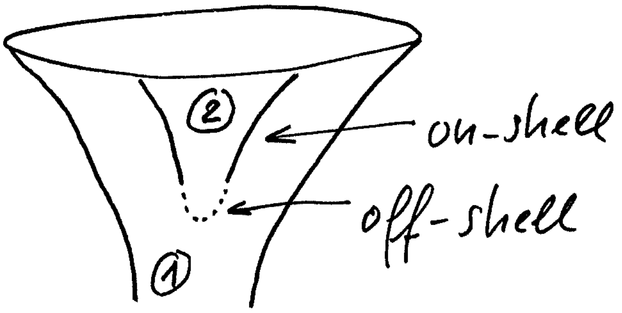


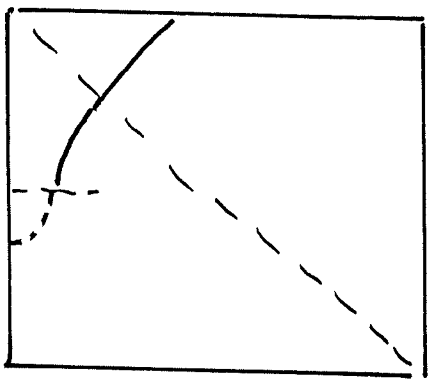
Penrose:



horizon of observer sitting at north pole

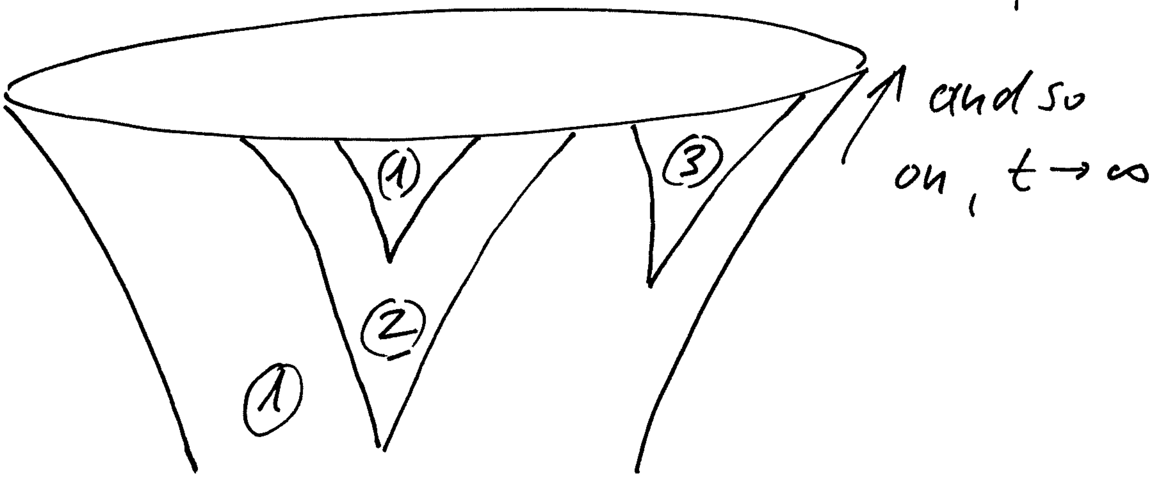
- As in 1st order phase transition: Spontaneous creation of a supercritical bubble of a new vacuum is possible.





→ Famous "Coleman-De Luccia" calculation of nucleation rate.

- This bubble nucleation can repeat itself:

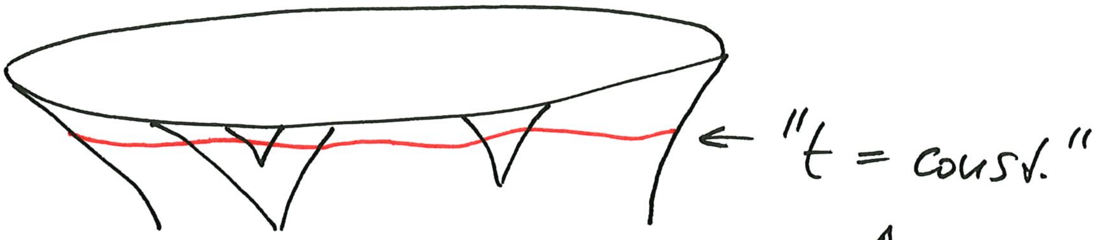


⇒ multiverse made of bubbles in bubbles in bubbles.....

⇒ "Everything that can happen will happen an infinite number of times" (Guth)

- How to ascribe probabilities in this situation? This is the real, hard measure problem!

- Technical issue: Choice of slicing



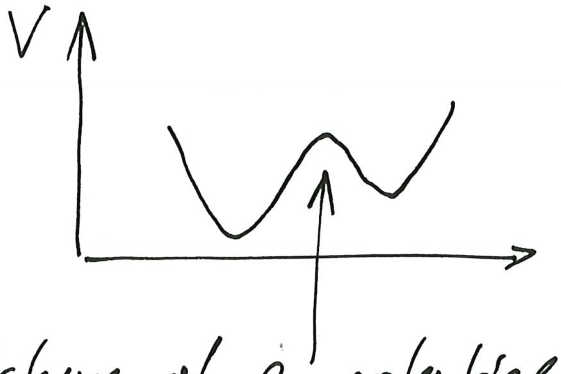
great freedom due to diffeomorphism-invariance.

- Two main choices:
  - Volume weighted measures
  - Local measures [do not reward volume growth]

(→ Linde, Vilenkin, Guth, Bousso, Freivogel, ...)

also: Hassfeld / AH / Westphal

- An extra issue:

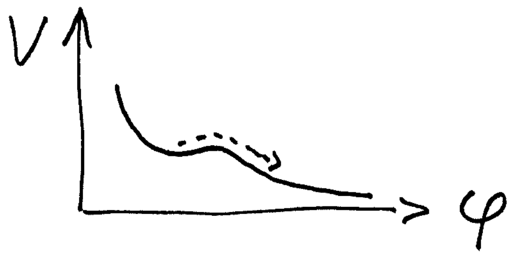


This picture of a potential barrier separating vacua is wrong!

Correct picture: Singular domain walls, specifically D5 branes wrapped on 3-cycles of CY, separate vacua.

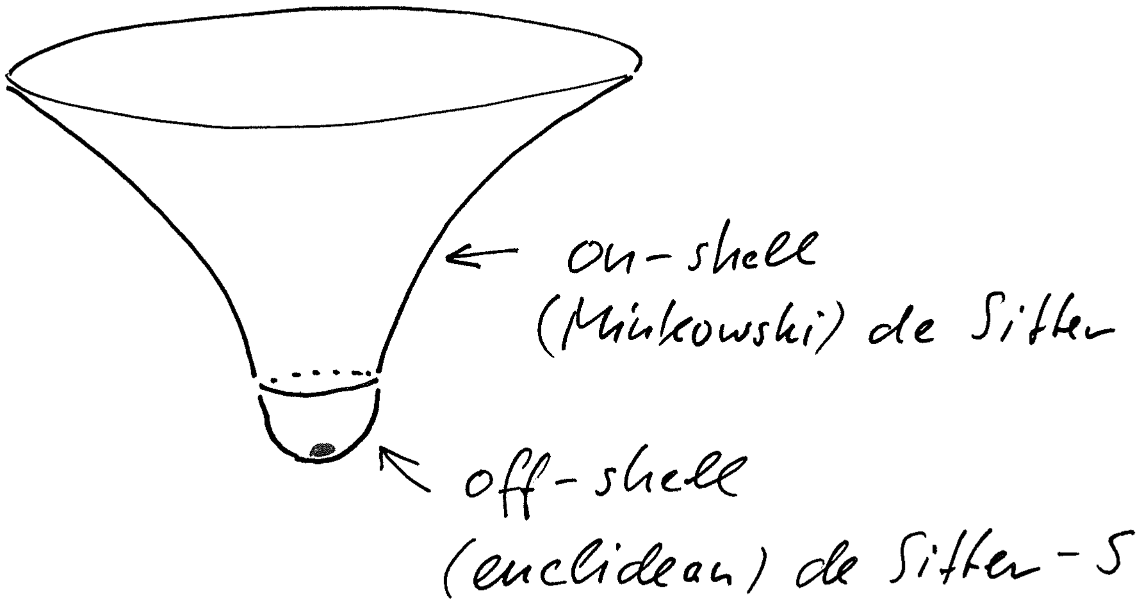
Problem: This wall may destabilize the (very fragile) LVS or KKLT vacua adjacent to it

(→ Larfors/Johnson)



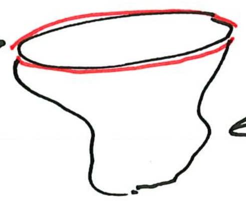
7.3 Creation from Nothing

An alternative (and at the moment very popular) picture is that of Hartle/Hawking or Linde/Vilenkin:



LV-logic: Small universe (•) "quantum fluctuates" out of nothing, tunnels to large size, goes on-shell and expands classically.

HH-Logic: Think of the above picture as defining initial conditions in a euclidean path integral approach to the "wave function of the universe".

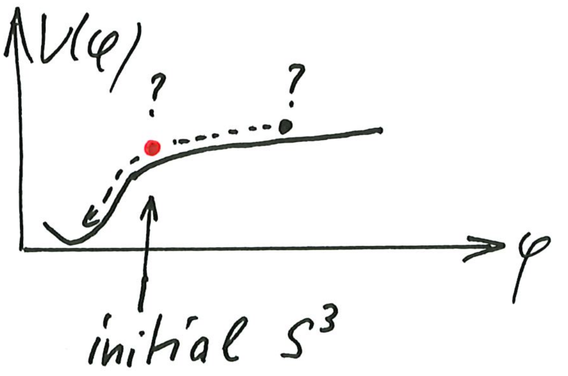


Difference LV/HH:

$$\text{prob.} \sim \exp(\mp \underset{\substack{\uparrow \\ \sim 1/V(\varphi)}}{S_{dS}})$$

Situation at the moment:

- Community prefers HH point of view.
- But: This gives a wrong "post-diction" even with just a single vacuum: The universe tends to start very low on the inflationary plateau:



$\Rightarrow$
- too small universe
- too much curvature

(cf. Maldacena '24)

- In principle, eternal inflation and creation from nothing can both be relevant together.
- One should work out all the details of this, combine with what we know about the string landscape, and make predictions.
- One key target: SUSY scale (and details of SUSY)

  → e.g. many papers by Baer

But: • There is much technical uncertainty, both on the stringy and on the dS / Cosmo side.

- Much more work is needed!