Precision Gauge Unification from Extra Yukawas

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Outline

- The 2-loop problem of SUSY GUTs
- Analytical understanding of the problem
- The effect of extra multiplets and extra Yukawas
- An analytically calculable model at strong GUT coupling
- Weakly coupled models

The 2-loop problem of SUSY GUTs

- With $\alpha_3 = 0.118$, the numbers look perfect at 1-loop:

$$\left(\frac{2\pi}{\alpha_3(m_Z)}\right)_{\rm exp} = 53.2 \qquad \text{vs.} \qquad \left(\frac{2\pi}{\alpha_3(m_Z)}\right)_{1-\rm loop} = 53.7$$

- At 2-loop, one finds

$$\Delta\left(\frac{2\pi}{\alpha_3(m_Z)}\right)_{2-\rm loop} = -5.4\,,$$

which is not easy to compensate by thresholds. For example,

$$\Delta\left(\frac{2\pi}{\alpha_3(m_Z)}\right)_{\rm Higgs-triplets} = \frac{9}{7}\ln\frac{m_{X,Y}}{m_3}$$

requires $m_{X,Y}/m_3 \sim 50$.

2-loop problem of SUSY GUTs (continued)

- Another possibility are SUSY thresholds:

$$\Delta\left(\frac{2\pi}{\alpha_3}\right)_{\mathrm{SUSY}} = \frac{19}{14}\ln\frac{M_{\mathrm{SUSY}}}{m_Z} \qquad \text{with} \qquad M_{\mathrm{SUSY}} \sim \left(\frac{m_{\tilde{W}}}{m_{\tilde{g}}}\right)^{\frac{28}{19}} m_{\tilde{H}}$$

Langacker, Polonsky '92. . . '95
Carena, Pokorski, Wagner '93

- In general, it is difficult to get a sufficiently large correction since **gluinos tend to be heavy**
- However, if one is willing to use **several competing contributions to gaugino masses**, precision unification can be realized

Raby, Ratz, Schmidt-Hoberg '09

2-loop effect in the holomorphic approach

- Let us understand the origin of the problem in detail.
- It is covenient to use a holomorphic Wilsonian action, where **gauge couplings run only at 1-loop**.

NSVZ '83...'86; Shifman '96
Arkani-Hamed, Murayama '97

- One gets a low-energy action with (1-loop) gauge couplings and (all-loop) Z-factors for matter fields
- To compare with data, one converts to the canonical scheme (accounting for **vector and Konishi anomaly**)
- The resulting 2-loop correction to the α_3 prediction is

$$\Delta\left(\frac{2\pi}{\alpha_3}\right) = \frac{24}{7}\ln g_2^2 - 3\ln g_3^2 + \sum_f c_f \ln Z_f$$

2-loop effects in the holomorphic approach (continued)

- The required 1-loop Z factors are known analytically, e.g.

$$Z_D = \left(\frac{\alpha_{\mathrm{GUT}}}{\alpha_1}\right)^{-2/99} \left(\frac{\alpha_{\mathrm{GUT}}}{\alpha_3}\right)^{8/9} .$$

- Putting everything together, one finds

$$\begin{aligned}\Delta\left(\frac{2\pi}{\alpha_3}\right) &= -\underbrace{4.08}_{\mathit{vectors}} - \underbrace{0.65}_{L} - \underbrace{5.51}_{Q} + \underbrace{0.21}_{E} + \underbrace{3.21}_{U} + \underbrace{1.82}_{D} - \underbrace{0.44}_{H} \\ &= -5.4\end{aligned}$$

- Thus, we just need to change one of those Z factors contributing negatively **from** $Z < 1$ **to** $Z \gg 1$**.**

Extra Yukawa couplings

- The Z factors (assumed to be $\mathcal{O}(1)$ at the GUT scale) are driven to smaller values by gauge interactions
- Yukawas have the opposite effect, but even the top contribution is far too small
- However, we can introduce **extra multiplets** ($5 + \overline{5}$ or $10 + \overline{10}$) with **extra Yukawa couplings**
- An independent motivation of extra multiplets is the **messenger sector** of gauge mediation
- Another motivation is the **fine tuning of the MSSM**, which can be improved using **precisely** the extra Yukawa couplings we need

Moroi, Okada '91
Babu, Gogoladze, Rehman, Shafi '08
Martin '09
Graham, Ismail, Rajendran, Saraswat '09 . . .

see also: Barbieri, Hall, Papaioannou, Pappadopulo, Rychkov '07

An analytically calculable model

- Extra multiplets do not affect the 1-loop prediction for α_3
- The valua of $\alpha_{\rm GUT}$ grows with $n = n_{5+\overline{5}} + 3n_{10+\overline{10}}$
- If the mass of the extra multiplets is low, $M \sim m_Z$, one formally finds $\alpha_{\rm GUT} = 1$ for $n = 4.45$ (corresponding to $n = 5$ and some $M > m_Z$)
- This enhances the 2-loop correction from -5.4 to -7.9
- Let us assume strong GUT coupling and at least one pair $10 + \overline{10}$ with

$$W \supset \kappa Q_e U_e H_u + \bar{\kappa} \bar{Q}_e \bar{U}_e H_d$$

- The Higgs Z-factor is corrected by Z_H^Y with

$$2\pi \frac{d \ln Z_H^Y}{dt} = -3\alpha_\kappa$$

Analytically calculable model (continued)

- The extra Yukawa obeys

$$2\pi \frac{d \ln \alpha_\kappa}{dt} = 6\,\alpha_\kappa - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1$$

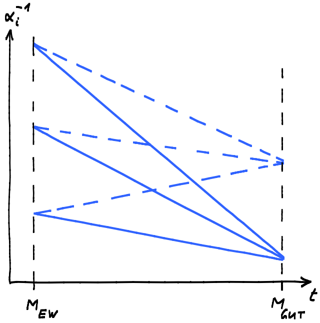
- For strong GUT coupling, at all lower scales we have approximately

$$\alpha_2 = (b_3'/b_2')\alpha_3 \qquad , \qquad \alpha_1 = (b_3'/b_1')\alpha_3\,,$$

- This gives an RGE for α_κ/α_3 with a fixed point:

$$2\pi \frac{d \ln(\alpha_\kappa/\alpha_3)}{dt} = 6\alpha_\kappa - \alpha_3 \left(\frac{16}{3} + \frac{3 b_3'}{b_2'} + \frac{13 b_3'}{15 b_1'} + b_3' \right)$$

Analytically calculable model (continued)



Analytically calculable model (continued)

- If α_κ is also large at $M_{\rm GUT}$, the fixed point regime $\alpha_\kappa = 1.4\alpha_3$ is quickly reached
- The Yukawa correction to the Higgs Z factor can then be obtained by simple integration:

$$\ln Z_H^Y(m_Z) = 2.8 \ln(\alpha_{GUT}/\alpha_3(m_Z)) = 6.0\,.$$

- This gives the 'Yukawa correction'

$$\Delta\left(\frac{2\pi}{\alpha_3}\right)_Y = \frac{9}{7}\ln Z_H^Y(m_Z) = 7.8\,,$$

precisely compensating the gauge-2-loop correction

On the strong-coupling assumption

- Going to strong(ish) GUT coupling has been discussed before

Kolda, March-Russell '96
Ghilencea, Lanzagorta, Ross '97
Amelino-Camelia, Ghilencea, Ross '98
...
J.L. Jones '08

- Calculability is clearly a critical issue
- We argue against 'precision loss' using the stringy relation

$$f_i(S,T) = k_i S + \Delta_i(T)$$

Nilles '86

- Based on this, one can shift S to strong coupling without enhancing threshold effects

On the strong coupling assumption (continued)

- However, non-perturbative effects can provide corrections

$$\sim C_i \exp(-a_i S)$$

- Thus, we probably need at least the exponential of $-4\pi/\alpha_{\rm GUT}$ to remain small
- This may be (at least marginally) consistent with our 'analytical model' discussed above

Weakly coupled scenarios

- Let us finally dump the strong-coupling assumption and analytical calculability
- We now do a 'proper job' solving (Yukawa) RGEs numerically, varying n and M and including the top effect
- For example, with $n = 4$ and $M = 500$ GeV we get $\alpha_{\rm GUT} = 0.23$
- Using $\alpha_\kappa(M_{\rm GUT}) = 0.9$, we find $(2\pi/\alpha_3) = 52.0$
- If we are willing to go up to $\alpha_\kappa(M_{\rm GUT}) = 6$, we find $(2\pi/\alpha_3) = 53.2$
- Using $n = 5$ and $M = 250$ TeV, very similar numbers are obtained

Weakly coupled scenarios (continued)

- Finally, for $n = 6$, we can have two pairs $10 + \overline{10}$ and double the Yukawa effect.
- However, except for the increased mass scale of the extra multiplets, $M = 17 \times 10^3$ TeV, the α_3 predictions remain roughly unchanged.
- Thus, quite generically, scenarios with large GUT coupling and large extra Yukawa couplings (but with both couplings still perturbative) bring the α_3 prediction in line with experiment
- Interestingly, they can not move it 'beyond' the experimental value

Summary

- Extra GUT multiplets can be used to make the GUT coupling stronger without sacrificing precision
- Large extra top-like Yukawa couplings to the MSSM Higgs fields bring the α_3 prediction in line with experiment
- The actual ‘strong coupling regime’ is distinguished by (easy) analytical calculability
- It may also be ‘natural’ (e.g. in the string theory ‘landscape’)
- However, the numerical effect does not rely on the strongly coupled regime