

Grand Unification in the "Landscape"

(Based on work with Gero v. Gersdorff and Christian Groß)

Outline

- The "old" story: Low-energy observations are predicted by a simpler, more unique high-energy theory.
- The "new" story: Low-energy observations are the result of an accidental (anthropic?) selection from a very large set of consistent theories (the "landscape").
- How does the idea of "Grand Unified Theories" fit into these developments?
- A landscape-inspired mechanism "explaining" the smallness of the unified gauge coupling

The "old" story

- start from the "effective theory" of the known world
 - try to unify interactions and degrees of freedom ;
increase the symmetry
- ⇒ find a more unique theory predicting (or at least explaining) some of the low-energy observations

[Obviously, this has worked many times in the past.

Discovery of SUSY at LHC would be another example.

Grand Unification is then the next logical step...]

Supersymmetric Grand Unification

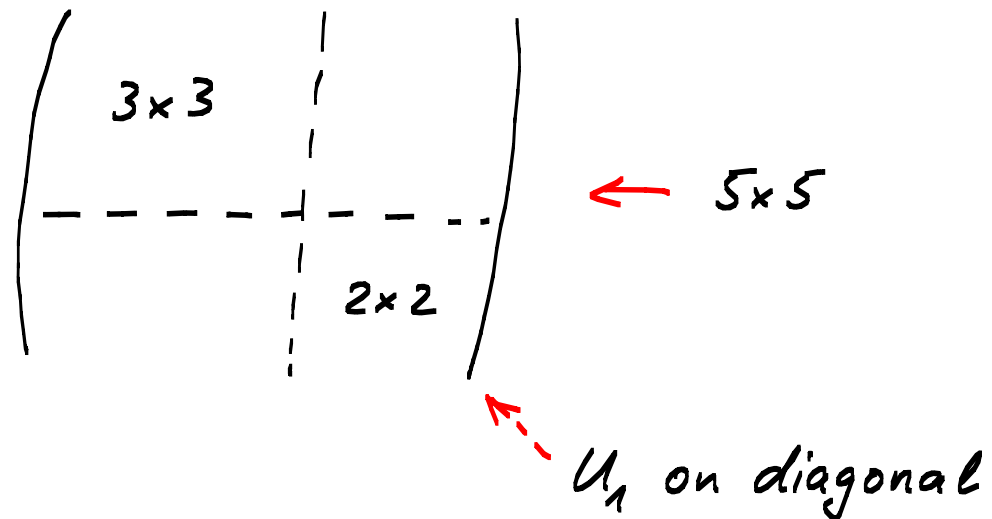
- Standard Model:

$$SU_3 \times SU_2 \times U_1 + \text{Matter} + \text{Higgs}$$

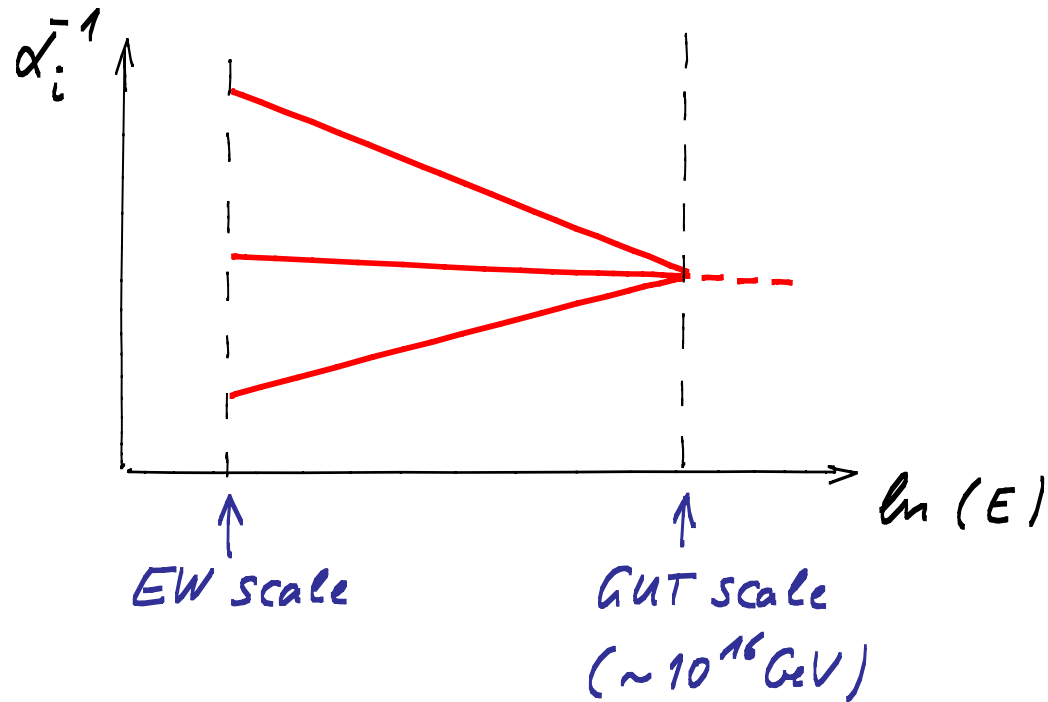
fits perfectly into SU_5 ;
with matter in $3 \times (\bar{5} + 10)$

is special anyway...

- Quick reminder:



Crucial quantitative evidence:



\Rightarrow prediction of α_3
from α_1 & α_2 :

$$\alpha_{3, \text{exp}} \approx 0.12$$

$$\alpha_{3, \text{GUT}} \approx 0.13$$

(This corresponds to a $+O(1)$
error in α_3^{-1} , as expected
from unknown thresholds)

How seriously should we take this?

- expect $0 < \alpha_3 < 1$
- hit 0.12 with error 0.01

\Rightarrow $\sim 3\text{-}\sigma$ -effect!

The "new" story

- Compactify the (essentially unique) 10d-Superstring-Theory to 4d using a 6d compact space

including fluxes, it is easy to see that $\sim 10^{500}$ or more "geometries" are available



arguably, any reasonable 4d theory arises as a metastable solution of string theory

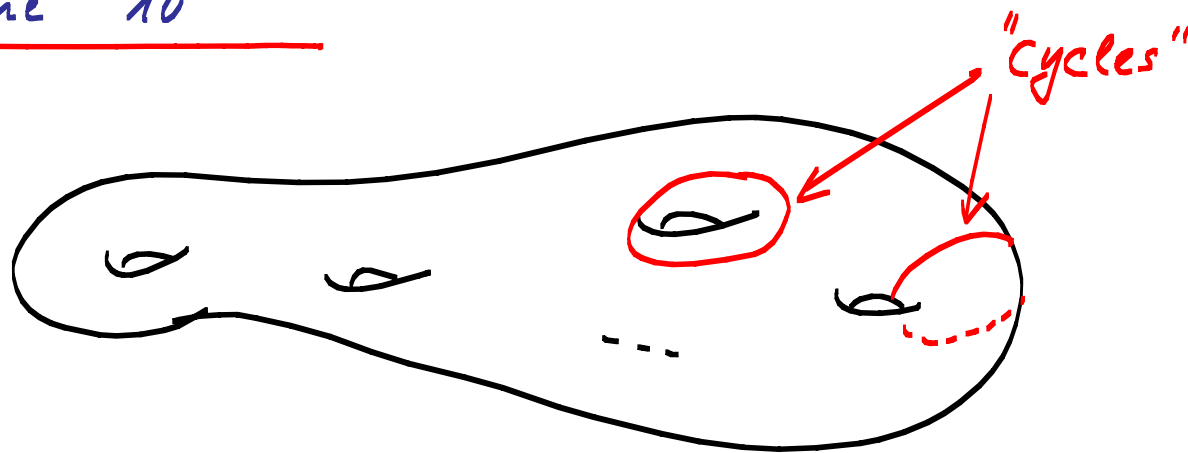
("The Landscape")



- Thus, $SU_3 \times SU_2 \times U_1$ with "our" matter & couplings could be just "accidental".

(cf. e.g. statistical analysis of Blumenhagen, Gmeiner, Honecker, Lüst, Weigand, 2005)

Idea behind the "10⁵⁰⁰"

"CY"



- Some of the fields living on this space take values in "non-trivial bundles" (associated with cycles)
- e.g. for 1-dim. cycles we can have  or .
- for a 2-dim. cycle, such as S^2 on which a gauge-field lives, the "twisting" is characterized by the "monopole number"
 - $\dots, -1, 0, 1, 2, \dots$
- typical CY's have hundreds of cycles...

So why bother with Grand Unification?

- Because the "3-5-effect" mentioned earlier remains
- Because "Grand Unification" is generic in certain branches of the landscape:

a) Heterotic string: one gauge coupling in 10d (group $E_8 \times E_8$ or SO_{32})

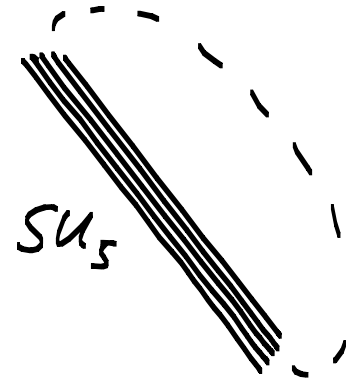
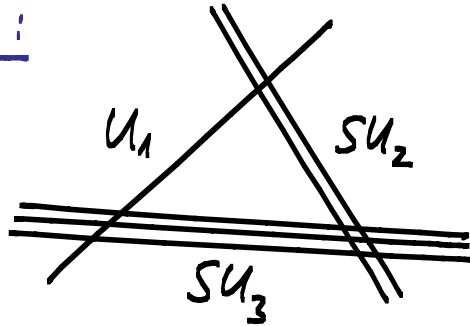
(This is however not the best-understood part of the landscape as far as moduli stabilization / vacuum energy is concerned)

b) Type IIA / IIB with D-branes: each (stack-of) D-branes has its own, independent gauge group

(Moduli stabilization / vacuum energy well-understood, but coupling unification not necessary)

→ GKP/KKLT, ..., specifically in F-theory: Lüst, Mayr, Reffert, Stieberger

compare:



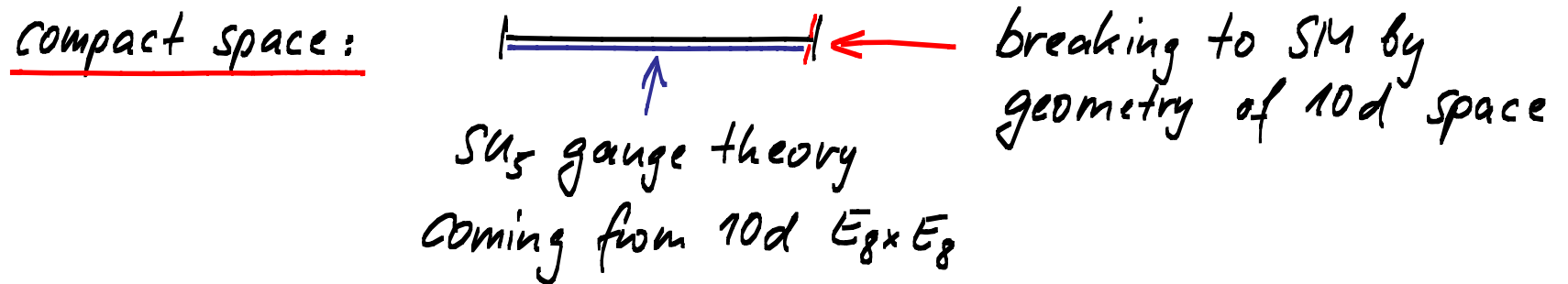
Broken to $SU_3 \times SU_2 \times U_1$
e.g. by compact
geometry of this stack

- Thus, in the "intersecting brane" setting, Grand Unification is not generic, but an interesting possibility.
 - Recently, promising constructions have emerged in the type IIB / D7-brane case. (Blumenhagen, Braun, Grimm, Weigand)
 - Moreover, in the F-theory-setting (a non-pert. generalization of the IIB / D7-brane case), certain problems of brane/GUT models have been overcome (Heckman, Vafa, '08; ...)
- [Top-Yukawa coupling, E_6 -GUTs, ...]

A realistic GUT-coupling from a "micro-landscape"

- In the following, I focus on the "classical" heterotic case
- Moreover, I will only discuss the "last step" of compactification

$$5d, SU_5 \longrightarrow 4d, SU_3 \times SU_2 \times U_1$$



- We now consider the 5d gauge theory in more detail:

usually, one writes $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad ; \quad D_\mu = \partial_\mu + ig_5 A_\mu$$

It is more natural to redefine $g_5 A_\mu \rightarrow A_\mu$,
giving:

$$\begin{aligned}
 S &= -\int d^5x \frac{1}{4g_5^2} F_{\mu\nu} F^{\mu\nu} \\
 &= -\int dy \int d^4x \frac{1}{4g_5^2} F_{\mu\nu} F^{\mu\nu} \\
 &= -\int d^4x \frac{2\pi R}{4g_5^2} F_{\mu\nu} F^{\mu\nu} \\
 &= -\int d^4x \frac{1}{4g_4^2} F_{\mu\nu} F^{\mu\nu}
 \end{aligned}$$

Hence:

$$g_4^2 = \frac{g_5^2}{2\pi R}$$

$\Rightarrow g_5^2$ has dimension [length]

Moreover: $\alpha_{\text{GUT}} \approx \frac{g_4^2}{4\pi} \approx \frac{1}{25}$ ("exper." value)

requires a mild hierarchy:

$$R \gg g_5^2$$

Given that $R_6 \dots R_{10} \sim \ell_{\text{string}}$; $g_5^2 \sim \ell_{\text{string}}$,

what makes $R = R_5 \gg \ell_{\text{string}}$?

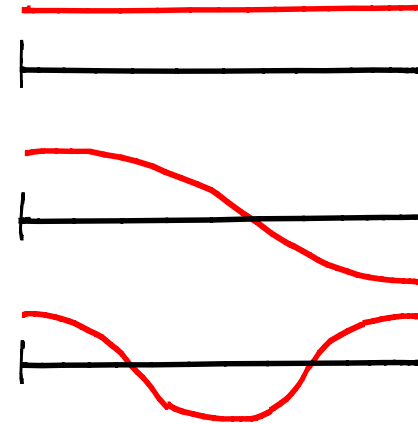
(In other words, what makes the last extra dims. somewhat larger, thereby explaining the weak coupling we observe?)

- Clearly, this requires understanding what stabilizes the size of this 5th extra dimension.

2-loop Casimir stabilization

Kaluza-Klein modes on interval:

- each of these modes corresponds to 4d field with R -dependent mass



etc.

- each such field is a set of oscillators
- each oscillator has a zero-point energy

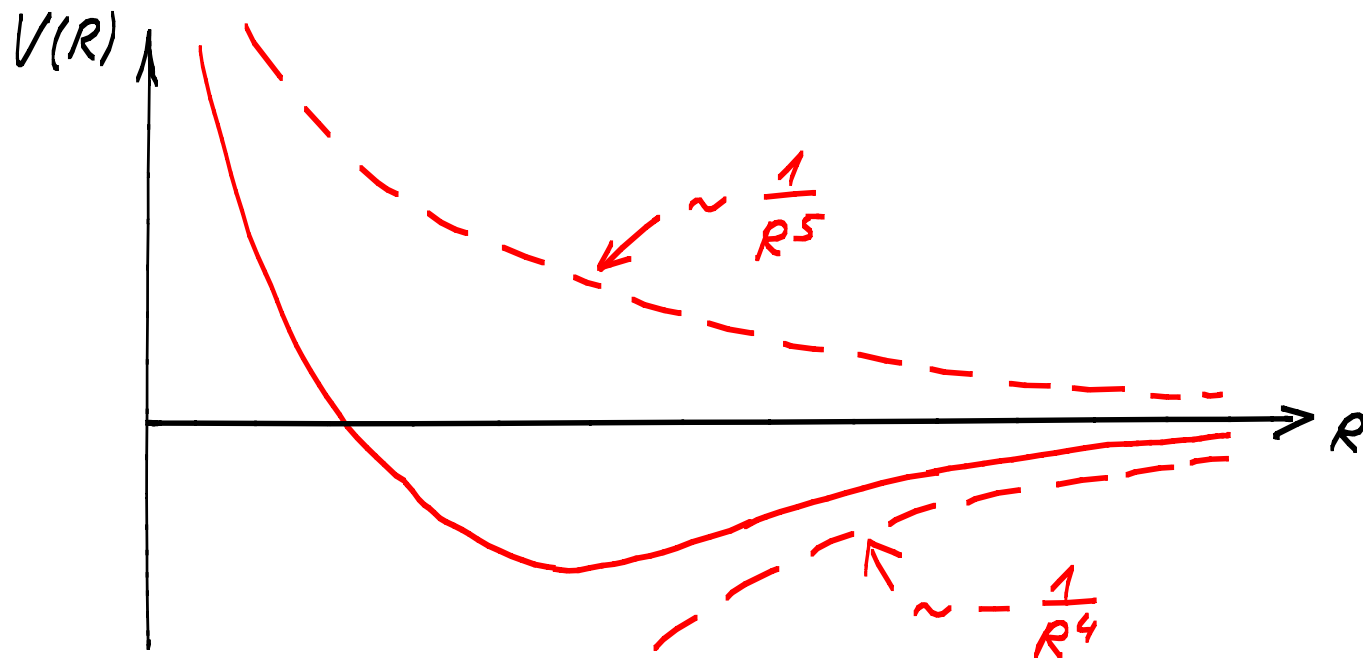
While the full energy is divergent, the R -dependence is calculable.

In fact, by simple dimensional analysis

$$V(R) = V_{1\text{-loop}}(R) + V_{2\text{-loop}}(R) + \dots$$

$$= \frac{1}{R^4} \left(c^{(1)} + c^{(2)} \frac{g^2}{R} + \dots \right)$$

If $c^{(1)} < 0$ and $c^{(2)} > 0$, it is easy to get a 2-loop minimum:



- Actually, one finds:

$$R_{\min} = -\frac{5}{4} \frac{c^{(2)}}{c^{(1)}} g_5^2$$

- Problem: If all $c^{(i)} \sim O(1)$, $g_5^2/R_{\min} \sim O(1)$ and the 2-loop minimum is not reliable
- However: If by an accidental cancellation $c^{(1)} \ll 1$, $g_5^2/R_{\min} \ll 1$, and higher loops do not affect $V(R)$ near R_{\min}

(Note: This is also precisely what we need to get $\alpha_{\text{GUT}} \ll 1$)

Side remark: A similar situation occurs for $\beta(g^2)$ in QCD-like theories, leading to a non-trivial IR-fixed-point (Banks-Zaks fixed point)

Actually: The above holds only for compactifications on a circle, S^1 . On an interval, S^1/\mathbb{Z}_2 , one has

$$V(R) = \frac{1}{R^4} \left(c^{(1)} + c^{(2)} \frac{g_5^2}{R} \left[\ln \frac{R}{g_5^2} + O(1) \right] + \dots \right)$$

The log is associated with UV-divergences arising on the boundaries (branes). One naively gets $\ln(\Lambda_{\text{cutoff}} \cdot R)$.

It is then natural to set $\Lambda_{\text{cutoff}} \sim 1/g_5^2$.

(But a slightly different choice does not affect the result.)

The previous parametric analysis remains correct.

SUSY - Breaking

- In fact, $V(R) \equiv 0$ if SUSY is unbroken
- We assume "Scherk-Schwarz" SUSY breaking, where the boundary conditions for bosons / fermions differ by a small number $\omega \ll 1$.
- This can also be understood using a "radion superfield"

$$T = R + iA_5 + \dots + \theta^2 F_T$$

- ω is then related to a brane-localized superpotential and induces an "F-term" for T :

$$\omega \sim F_T \sim \frac{|W_0|}{M_{P,5}^3}$$

(\rightarrow Marti/Pomaral ; Kaplan/Weiner ; Chacko/Luty)

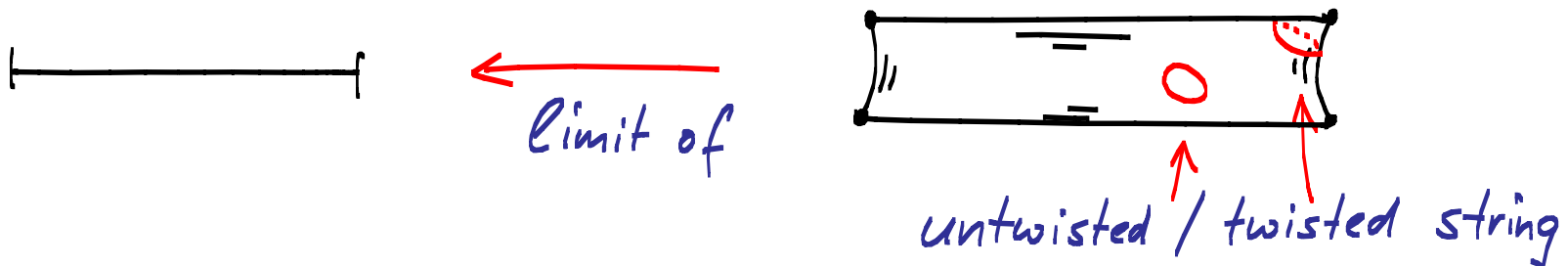
Since both terms in $V(R) \sim \omega^2$, the SUSY-breaking scale is irrelevant for the position of the minimum

Thus, all that was said before remains true for (very weakly broken) SUSY.

- We now turn to specific 5d SUSY GUT models
("Orbifold GUTs")

→ Kawamura; Hall/Nomura; ... ~ 2001

- Matter can "live" in bulk or on branes:



5d SUSY-GUT models

GUT-breaking: $SU_5 \rightarrow SU_3 \times SU_2 \times U_1$ at one boundary

- Higgs on SM-brane
- Matter in bulk or on "Standard-Model brane"

In detail: $\bar{5}$ -matter

$$\bar{5} = \underbrace{(\bar{3}, 1)}_u + \underbrace{(1, 2)}_s$$

← counts # of bulk fields of this type

10-matter

$$10 = \underbrace{(3, 2)}_t + \underbrace{(\bar{3}, 1) + (1, 1)}_r$$

$$\Rightarrow r, s, t, u \in (0, 1, 2, 3)$$

parameterize our "micro landscape" of $4^4 = 256$ models

• We find:

$$\frac{c^{(1)}}{c^{(2)}} = \pi^3 \frac{-160 - 16r - 8s + 96t + 48u}{\frac{2424}{5} - 108r - 36s + 36t + 12u}$$

• From this ratio,

the value of R_{\min} and hence α_{GUT} follow.

(We need $c^{(1)} < 0$, $c^{(2)} > 0$, and small ratio to get small g_5^2 / R_{\min} .)

We find a pert. controlled minimum for $\sim 1/3$ of the models

We find $\frac{1}{20} \gtrsim \alpha_{\text{GUT}} \gtrsim \frac{1}{30}$ for 12 models

Uplifting

- One problem still remains: Our minimum is always at $V < 0$ (neg. cosmol. constant) and needs to be "uplifted".
- Because of SUSY, we can not simply add a pos. contribution to V .
- However, we can add $\Lambda_5 < 0$ in bulk and compensate by $\Lambda_4 > 0$ at branes. ("warped space" or "supersymmetric Randall-Sundrum model")
- We can then slightly "detune" Λ_4 & Λ_5 , maintaining SUSY and introducing a small positive contribution to V
(\rightarrow Bagger / Belyaev '02)
- This completes our construction.

Uplifting in superfield language

$N=1$ supergravity perspective:

$$\Omega \sim - (T + \bar{T}) + \Delta\Omega_{\text{loop}} \quad ; \quad W = W_0$$

$$\Delta\Omega_{\text{loop}} \sim \frac{1}{(T + \bar{T})^2} + \frac{-g^2}{(T + \bar{T})^3} \ln(M(T + \bar{T}))$$

(cf. "Almost no-scale" proposal of Luty/Okada)

$$V(R_{\min}) \sim \frac{\omega^2}{R_{\min}^4} \text{ is calculable \& } < 0.$$

To "uplift", allow for a small warping ($\Lambda_5 \neq 0$):

$$\Lambda_5 = -6k^2 M_{P,5}^3 \quad ; \quad e^{-k(T + \bar{T})} = 1 - \text{"small"}$$

- Neglecting the loop-induced Kähler-correction for the moment, we have:

$$\Omega \sim e^{-k(T+\bar{T})} - 1 \quad ; \quad W = W_0 e^{-3kT}$$

(\rightarrow Luty/Sundrum)

(constant superpotential
at IR-brane)

- A Kähler-Weyl rescaling gives

$$\Omega \sim 1 - e^{k(T+\bar{T})} \quad ; \quad W = W_0$$

$$\sim -(T+\bar{T}) + \Delta\Omega_w$$



$$\Delta\Omega_w \sim (T+\bar{T})^2$$

- This has to be combined with our $\Delta\Omega_{loop}$

$$\Rightarrow \Delta\Omega_{\text{tot}} \sim -M_{P,5}^3 k (T+\bar{T})^2 + \frac{1}{(T+\bar{T})^2} + \frac{g^2}{(T+\bar{T})^3} \ln(M(T+\bar{T}))$$



$$\delta V \sim \frac{|W_0|^2}{M_{P,5}^6} (\Delta\Omega)_{T\bar{T}}$$

$$\delta V \sim \underline{\underline{\text{const.}}} + \frac{1}{R^4} + \frac{g^2}{R^5} \ln(MR)$$

positive
constant
contribution $\sim \omega^2 M_{P,5}^3 k$

- Uplifting to $\Lambda_4 \approx 0$ implies

$$k(T+\bar{T}) \sim \frac{1}{(RM_{P,5})^3}$$

i.e. Warping is indeed small and our unwarped calculation is Ok.

Phenomenology

- The main phenomenological implications (in addition to those of 5d SUSY GUTs) come from the fact that the radion superfield (i.e. F_T) dominates SUSY-breaking.
- It couples primarily to gauginos, making them heavy (gaugino mediation)
- Furthermore, if the Higgs comes from an enlarged gauge sector (e.g. SU_6), a very constraint scenario involving $m_{1/2}$, μ , $B\mu$ arises (Burdman/Nomura '02, Choi et al. '03, A.H., March-Russell, Ziegler '08)
- We have determined observational consequences of such a setting in a detailed study (Brümmer, Fichtel, A.H., Kraml '09)

Summary / Conclusions

- Two-loop Casimir stabilization occurs naturally in 5d orbifolds (given only F_T -dominance & suitable matter content).
- With a very mild discrete tuning, it can "explain" the smallness of $\alpha_{\text{GUT}} \sim 1/25$
- An elegant way of uplifting the stabilized AdS_4 -vacua is to appeal to a small warping ($\Lambda_5 \neq 0$)
- It would be interesting to find 2-loop-Casimir-stabil. in explicit heterotic models, in particular to allow for more moduli
(cf. e.g. 6d analysis of Buchmüller / Catena / Schmidt-Hoberg)