Throats & Randall-Sundrum-Cosmology

or

The Klebanov-Strassler Throat as a Randall-Sundrum Model with Goldberger-Wise Stabilization

(based on work with

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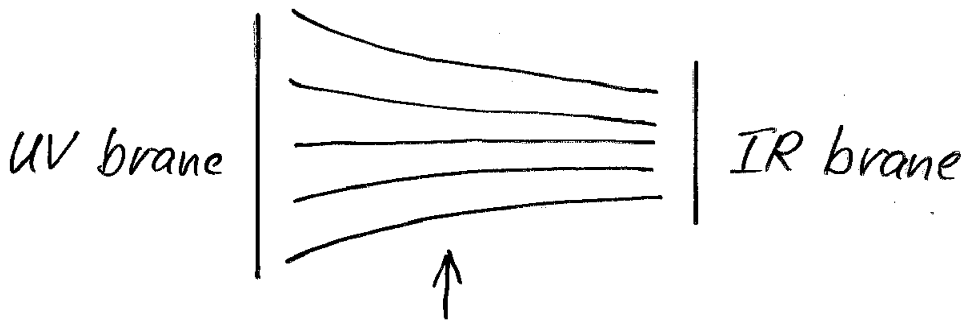
Outline

- Motivation for (5d) warped models
- Their realization in flux compactifications
- The hierarchy stabilization
- Equivalence to Goldberger-Wise mechanism
- The universal Kähler modulus as a UV-brane field
- Open issues (in particular 10d vs. 5d vs. 4d SUSY)

The Randall-Sundrum model



$$ds^2 = \underbrace{e^{2ky}}_{\text{warp factor}} dx^2 + dy^2$$

<u>5d lagrangian:</u> $\mathcal{L} = \frac{1}{2} M_5^3 \mathcal{R} - \Lambda_5$

$(\Lambda_5 < 0)$

<u>brane lagrangians:</u>

$$\mathcal{L}_{UV/IR} = \mathcal{L}(\psi_{UV/IR},\ \underset{\substack{\uparrow \\ \text{induced from } g_5}}{g_{4,UV/IR}})$$

$\Downarrow$

<u>exponential hierarchy of scales</u>

Motivation

a) Phenomenological

Hierarchy: $\underline{M_{EW}}$ vs. $\underline{M_{P,4}}$

IR-brane lagrangian

g_4 - zero mode is dominated by geometry near the UV brane

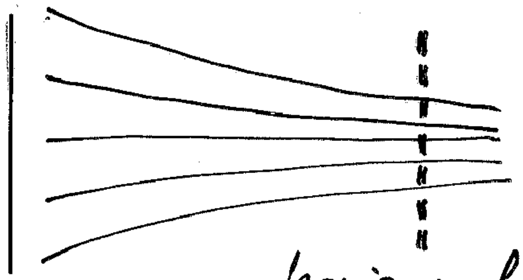
b) Cosmological

Need to understand the (unusual) high-scale cosmology of this model

- Expansion is effectively 5d at high T
  (→ various interesting implications for part. production & inflation)
- 1st order phase transition when IR brane disappears behind horizon
  (→ Creminelli, Nicolis, Rattazzi, '01)

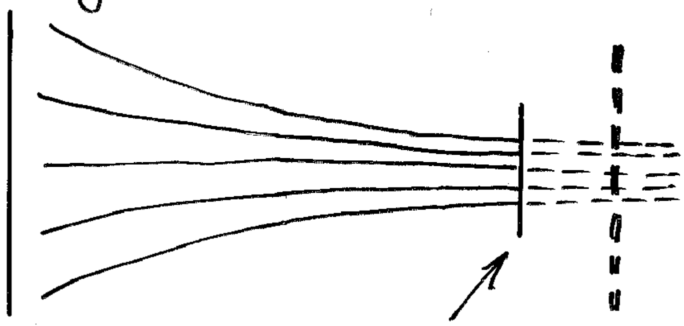
to understand this, recall the related 1-brane model:

RS II geometry

UV brane  non-compact

horizon forms at $T \neq 0$
(cf. dark radiation prediction
of A.H., March-Russell, '01)

After cooling in RS I case:

UV brane 

IR brane is "created"!

Also:

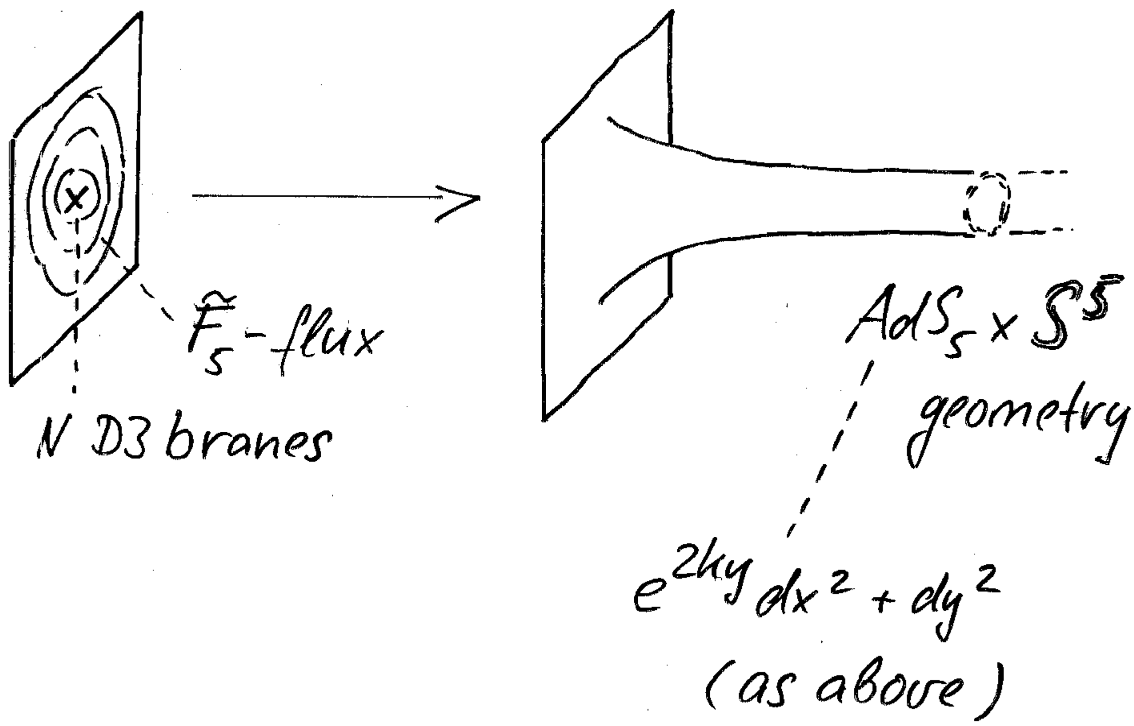
- KK dark matter (→ e.g. Agashe, Servant, '04)
- IR-brane dark matter
- various issues in "brane cosmology" (e.g. in DGP-like scenarios)

c) Fundamental

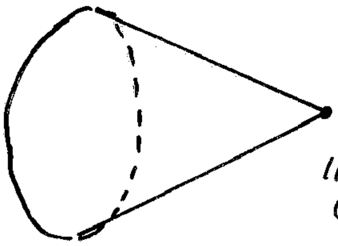
> Randall-Sundrum-like models are natural (and "common") in flux compactifications

Verlinde, '99
Klebanov, Tseytlin, '00
Klebanov, Strassler, '00
"GKP", '01

Basic idea:



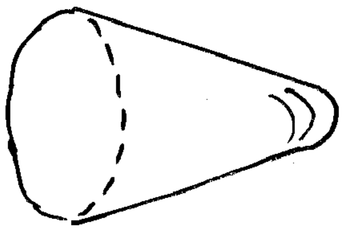
To create <u>finite</u> hierarchy:



cone: $\mathbb{R}_+ \times \underbrace{T^{1,1}}_{\text{topologically} \sim S^3 \times S^2}$

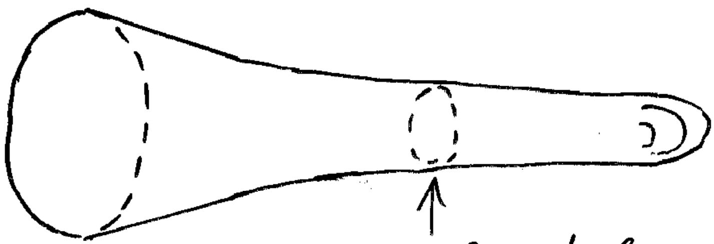
"conifold"

<u>Deformation</u>



smooth (CY-) geometry where S^3 does <u>not</u> shrink to zero size

<u>Put M units of F_3-flux on S^3-cycle</u>



approximately $AdS_5 \times T^{1,1}$

$$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$$

N_{eff} flux units

2-form potential on S^2

M units on S^3

<u>recall:</u> $$\boxed{\begin{array}{c} F_3 = dC_2 \quad , \quad H_3 = dB_2 \\ F_5 = dC_4 \end{array}}$$

$$\tilde{F}_5 = F_5 - \tfrac{1}{2} C_2 \wedge H_3 + \tfrac{1}{2} B_2 \wedge F_3$$

sourced by D3 branes

let $\underline{\underline{r}}$ be the radial coordinate of cone

$$N_{eff}(r) \sim \int_{T^{1,1}} \tilde{F}_5 \sim \underbrace{\left(\int_{S^3} F_3\right)}_{M} \underbrace{\left(\int_{S^2} B_2\right)}_{\text{can vary with } r:}$$

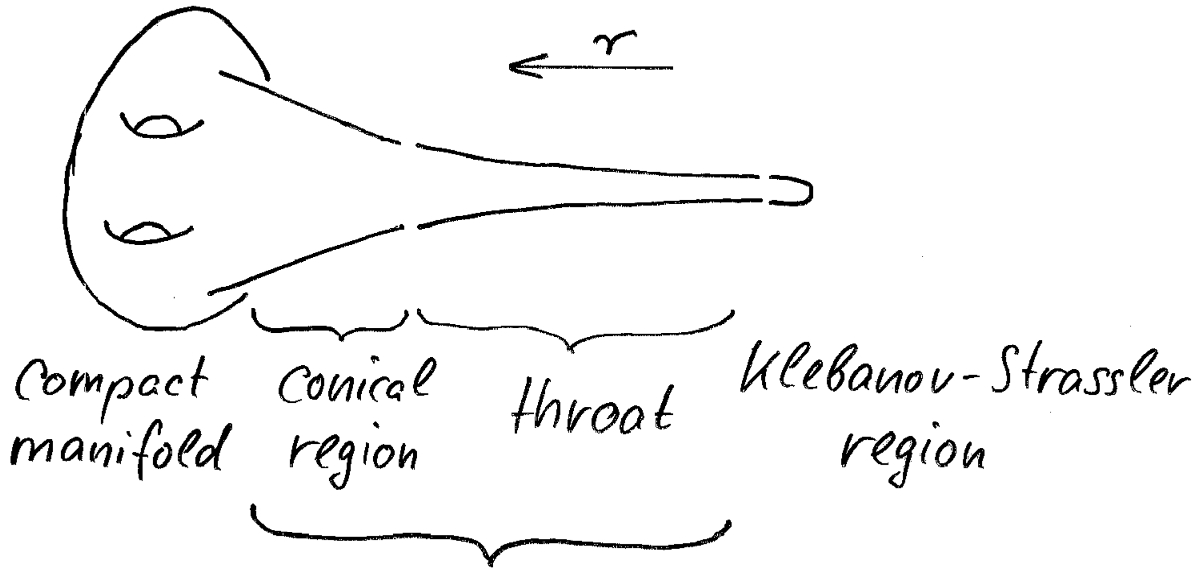
$$\frac{d}{dr}\left(\int_{S^2} B_2\right) \sim H_3 \underset{\text{by EOM}}{\sim} *F_3 \sim \frac{M}{r}$$

$$\Rightarrow \boxed{N_{eff}(r) = \frac{3}{2\pi} g_s M^2 \ln(r/r_s)}$$

This also determines $R_{T^{1,1}}$ as fct. of r

constant of integration

Overall picture



$$ds^2 = h(r)^{-1/2}\,dx^2 + h(r)^{1/2}\left(dr^2 + r^2 ds^2_{T^{1,1}}\right)$$

with

$$h(r) = 1 + \frac{\alpha'^2 g_s^2 M^2 \ln(r/r_s)}{r^4}$$

The throat is governed by one scalar degree of freedom:

$$\left(\int_{S^2} B_2\right) \quad \text{or} \quad N_{eff}(r) \quad \text{or} \quad R_{eff}(r)$$

(size of $T^{1,1}$)

$\Rightarrow$ Natural to look for effective 5d description with one scalar field $\underline{H}$:

$$\mathcal{L}_5 = \frac{1}{2} M_5^3 \mathcal{R}_5 - \frac{1}{2} (\partial H)^2 - V(H)$$

Step 1: identify coordinate y of 5d Einstein frame:

$$M_5 \, dy = M_{5,\mathrm{eff}}(r) \sqrt{g_{rr}} \, dr$$

$$\sim \left[M_{10}^8 R_{\mathrm{eff}}(r)^5 \right]^{1/3} \left[\frac{R_{\mathrm{eff}}(r)}{r} \right] dr$$

$$\Rightarrow \boxed{(y/R_s) = \left(\ln (r/r_s) \right)^{5/3}}$$

5d Einstein frame metric:

$$ds_5^2 = e^{2A(y)} dx^2 + dy^2$$

$$A(y) = (y/R_s)^{3/5} + O(\ln (y/R_s))$$

(cf. $A(y) = k \cdot y$ in RS model)

Step 2:

Identify potential $V(H)$ leading to this geometry (via back reaction of H on metric)

$$D_y^2 H - \frac{\partial V}{\partial H} = 0$$

$$\left[\partial_y^2 + 4A'(y)\partial_y \right] H - \frac{\partial V}{\partial H} = 0$$

$$\boxed{k(y)\, \partial_y H \sim \frac{\partial V}{\partial H}}$$

with $A(y) \sim k(y) \cdot y$; $-\underbrace{k^2(y)}_{\text{AdS}_5 \text{ curvature scale}} M_5^3 \sim V(H)$

$\Longrightarrow$ Differential equation for $V(H)$;

Solution: $\boxed{V(H) \sim -M_5^7 R_s^{-2} H^{-8/3}}$

where $R_s \sim M_5^{-1} (g_s^2 M^2)^{4/3}$

Step 3:

Identify boundary conditions for H to complete Goldberger-Wise-like stabilization model:

$$H \sim M_5^{3/2} (R_{eff}/R_s)^2 \sim M_5^{3/2} (N_{eff}/N_s)^{1/2}$$

$$(N_s \sim g_s M^2)$$

$$N_{IR} \sim N_s + \underbrace{N_{D3}}_{\text{explicit D3 branes in KS region}} \sim g_s M^2 + N_{D3}$$

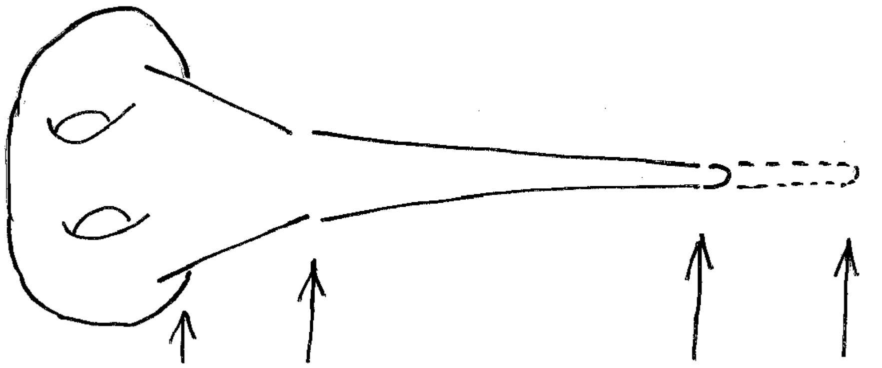
$$N_{UV} \sim \int_{T^{1,1} \text{ at UV-end of the throat}} \tilde{F}_5 \quad \leftarrow \quad \text{determined by D3 branes \& flux in "compact manifold"}$$

(N_{UV} is similar to "$N = M \cdot K$" in literature:

$$M \leftarrow \int_{S^3} F_3 \qquad K \leftarrow \int_{\tilde{S}^3} H_3 \ \text{(dual cycle)}$$

)

Overall picture



N $\qquad$ $\boxed{N_{UV}}$ $\qquad\qquad\qquad$ $\boxed{N_{IR}}$ $\qquad$ N_s

R_c $\qquad$ R_{UV} $\qquad\qquad\qquad$ R_{IR} $\qquad$ R_s

$\qquad\quad$ r_{UV} $\qquad\qquad\qquad$ r_{IR} $\qquad$ r_s

compactif. scale (UV-brane thickness)

$\underbrace{r_{UV} \quad \cdots \quad r_{IR}}_{\text{5d model}}$

$\underbrace{r_s}_{\text{throat end if no explicit D3 branes present}}$

N & N_{UV} differ by $\sim \underbrace{g_s M^2 \ln(R_c/R_{uv})}_{\text{negligible if conical region not too large}}$

(R_c not too large)

corresponds to univ. Kähler modulus

The universal Kähler modulus

without fluxes: scaling of compact space in throat & conical region:

$$ds^2 = h(r)^{-1/2} dx^2 + h(r)^{1/2}\left(dr^2 + r^2 ds^2_{T^{1,1}}\right)$$

$$h(r) = 1 + \frac{\alpha'^4 g_s^2 M^2 \ln(r/r_s)}{r^4}$$

modulus has to correspond to variation of ratio $r_s/\sqrt{\alpha'}$

- consider $r_s \longrightarrow r_s\, c^{1/4}$
- after appropriate reparameterization of r and x this means

$$h(r) \to c + \frac{\alpha'^4 g_s^2 M^2 \ln(r/r_s)}{r^4}$$

(cf. explicit calculation of Giddings, Maharana, '05)

We keep parameterization where

$$h(r) = 1 + \cdots$$

In this case, the univ. Kähler modulus governs size of $\underline{R_c}$ since

$$N = g_s M^2 \ln (R_c / r_s)$$

can be read as

$$R_c = R_c(r_s) \quad \text{or} \quad r_s = r_s(R_c)$$

$\Rightarrow$ warp factor as fct. of $\underline{R_c}$:

$$h(r) = 1 + \frac{\alpha'^2 g_s \left(N - g_s M^2 \ln (R_c / r)\right)}{r^4}$$

This defines the UV end of the throat as

$$r_{UV}^4 \simeq \alpha'^2 g_s \Big[N - \frac{g_s M^2}{4} \underbrace{\ln \Big(\frac{R_c^4}{\alpha'^2 g_s N} \Big)}_{\text{weak } R_c\text{-dependence}} \Big]$$

(weak as long as

$$R_c / R_{c,min} \ll \exp \Big(\frac{N}{g_s M^2} \Big) \Big)$$

To see that, in spite of this variation, R_c should be thought of as a <u>brane-field</u>, consider

$$R_c \longrightarrow (1+\epsilon)\cdot R_c$$

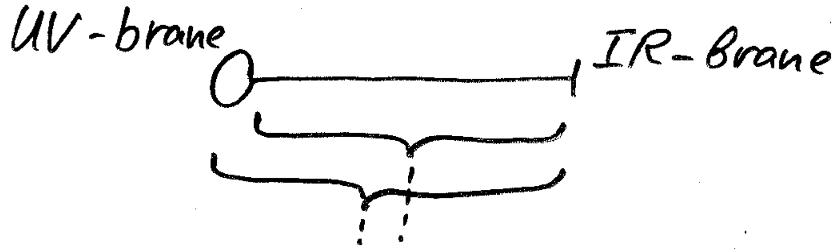
<u>Effect 1)</u>

UV brane thickness R_c changes by Δ_1

<u>Effect 2)</u>

Throat length $(y_{UV} - y_{IR})$ changes by Δ_2

$$\text{Ratio:} \quad \frac{\Delta_1}{\Delta_2} \sim \frac{R_c}{R_{c,min}} > 1$$



5d Interval length defined in these two ways <u>grows</u> / <u>shrinks</u> with growing R_c

$\Rightarrow$ logically impossible to associate R_c with 5d interval length (= RS radion); Kähler modulus is UV brane field!

Complete 5d action:

(with $H \to \tilde{H} = g_s M H$)

$$S_{5d} = \int_{y_{IR}}^{y_{UV}} d^5x \sqrt{-g_5} \left(\frac{1}{2} M_5^3 \mathcal{R}_5 - \underline{\frac{1}{(g_s M)^2} (\partial \tilde{H})^2} + \underline{M_5^9 \tilde{H}^{-8/3}} + \cdots \right)$$

$$+ \underbrace{\int \sqrt{-g_{4,UV}}\, \mathcal{L}_{UV}}_{\text{UV brane}} + \underbrace{\int \sqrt{-g_{4,IR}}\, \mathcal{L}_{IR}}_{\text{IR brane}}$$

where

$$\mathcal{L}_{IR} = - V_{IR}(\tilde{H}) - \Lambda_{4,IR} + \cdots$$

potential fixing $\tilde{H}$ at $M_5^{3/2} \cdot (g_s N_{IR})^{1/2}$

(e.g. $V_{IR} = \mu^2 \left[\tilde{H} - M_5^{3/2} (g_s N_{IR})^{1/2} \right]^2$)

"Large" scale

$\mathcal{L}_{UV}$ is more interesting:

$$\mathcal{L}_{UV} = - V_{UV}(\tilde{H}) - \Lambda_{4,UV}$$ (in complete analogy to IR)

$$+ M_5^2 (g_s N_{UV})^{-10/3} \cdot \Big\{ \underline{30 (R_c M_5)^6 (\partial \ln R_c)^2}$$

$$+ \underline{\underline{\mathcal{R}_4 \big((R_c M_5)^6}} - \underline{\underline{\underline{(g_s N_{UV})^4}}} \big) \Big\}$$

"—" brane-localized kinetic term for R_c following directly from $\mathcal{R}_{10}$

"=" brane-localized Einstein term that grows with R_c^6 (as in GDP & related scenarios)

"≡" correction required to preserve "no scale" character of R_c after 5d bulk contribution to $\mathcal{R}_4$ is included

All together

- $\tilde{H}$ – Goldberger-Wise scalar
- $\tilde{H}_{UV/IR}$ are fixed "topologically" through
  N, N_{D3}, M
- kinetic term $\sim \frac{1}{M^2} (\partial \tilde{H})^2$ fixes "speed" of variation and hence $\Delta y = y_{UV} - y_{IR}$
- potentially large UV-brane term
  $\sim R_c^6 \cdot \mathcal{R}_4$

  (Note: going to 4d Einstein frame, the brane-field-character of R_c is hidden)

Relation to usual moduli notation:

$$K(\rho, z) = -3 \ln\left(-i(\rho - \bar{\rho})\right) - \ln\left(-i \int \Omega \wedge \bar{\Omega}\right)$$

$$W(z) = \int G_3 \wedge \Omega$$

$$z = \int_{S^3} \Omega$$

$$\operatorname{Im} \rho \sim R_c^{\,4}$$

$$z^{1/3} \sim \exp\left[-(M_s \Delta y)^{3/5} (g_s M)^{-4/5}\right]$$

Can a link to 5d SUSY and the 5d radion T ($\mathrm{Re}\,T \sim \Delta y$) be established?

Following Lalak, '01:

$$K_{5d} \simeq -3 \ln \Big[\underbrace{\int_{\mathrm{Re}\,T} dy\, e^{2A(y)}}_{\text{coeff. of } \mathcal{R}_4} \Big]$$

<u>for const. warping:</u> $\Rightarrow K_{5d} \sim |z|^{2/3}$

<u>for GKP with "weak" fluxes:</u>

$$\int_{\tilde{S}^3} \Omega = \frac{z}{2\pi i} \ln z + \cdots$$

$$\Rightarrow K \sim |z|^{2/3} \ln |z|$$

(reasonable at small z)

<u>Attempt to do better using $A(y) \sim y^{3/5}$:</u>

$$\Rightarrow K_{5d} \sim |z|^{2/3} (\ln |z|)^{2/3}$$

Problem!

- Need back reaction $\int \Omega \wedge \bar{\Omega}$ - formula?
- Need improvement of Lalak's K_{5d} - formula?

Summary / Outlook

- Simple understanding of throat as RS model with Goldberger-Wise stabilization
- univ. Kähler modulus is UV brane field governing 1) UV brane thickness
  2) brane-localized 4d-curvature term

- explicit connection with 5d SUSY (in 4d N=1 superfield description) desirable (e.g. for SUSY-mediation)
- Can start exploring brane cosmology of "fundamental" throat model in 5d RS - Goldberger-Wise language

cf. KKLMMT
Burgess et al.
Kofman, Yi
Chialva, Shiu, Underwood
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