Some Issues in F-Theory Geometry

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based on work with

A. Braun, S. Gerigk, H. Triendl – 0912.1596

A. Braun, R. Ebert, R. Valandro – 0907.2691

as well as two earlier papers with

A. Braun, H. Triendl, R. Valandro, C. Lüdeling

Outline

- Motivation for the study of cycles in F-theory
- Explicit matching of type-IIB moduli and F-theory cycles in 3-folds
- Side remark on K3 (nice visualization of K3)
- Conclusions / Outlook

General motivation

- Low-scale SUSY used to be the solution to fine-tuning
- Non-discovery of SUSY at LEP has weakened this point
- Gauge coupling unification is now arguably the main reason to hope for SUSY at the LHC
- We should take Grand Unification seriously, also in string model building

Conventional Approach: Heterotic $E_8 \times E_8$

- Grand Unification automatic; 'realistic' models exist
- However: Flux landscape poorly understood
- 'Landscape solution' to cosmological constant problem unclear, even in principle

New paradigm: F-theory GUTs

Beasley, Heckman, Vafa '08
Donagi, Wijnholt '08
...
Dudas, Palti '09

- GUTs in intersecting brane models possible but non-generic
- In particular, SU(5) D7-brane GUTs can be constructed

Blumenhagen, Braun, Grimm, Weigand '08
...

- However, the large top-Yukawa is a problem
- Also, exceptional groups and SO(10) with **16**-matter can not be built
- F-theory GUTs can resolve these issues
- Thus, using F-Theory (non-pert. type IIB), one can build realistic GUTs in the best-understood part of the landscape

GKP/KKLT, ..., Denef, Douglas '04

Motivation for the study of F-theory cycles

- Goal: Model building with fluxes in type IIB / F-theory
- Natural way to think about fluxes: 4-form fluxes in F/M-theory
- Thus: Need to understand brane motion in terms of F-theory 4-cycles

  Denef, Douglas, Florea, Grassi, Kachru '05
  Lüst, Mayr, Reffert, Stieberger '05
  Braun, A.H., Lüdeling, Valandro '08

  (D7-brane stabilization on K3$\times$K3)
- We want to extend this to generic type IIB / F-theory models (relate D7-brane periods and compl. structure of 4-folds)
- Eventually: Calculate and interpret D7-brane superpotentials

  Alim, Hecht, Jockers, Mayr, Soroush, Walcher '08...'09
  Grimm, Ha, Klemm, Klevers '09; Aganagic, Beem '09

Technical introduction

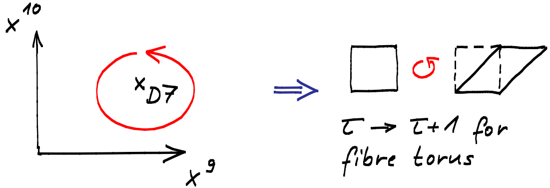
- M-theory on S^1 with small radius $\Rightarrow$
- Type IIA in 10d; compactify on another small S^1 $\Rightarrow$
- Type IIB in 9d $\times$ (large S^1) $=$ type IIB in 10d
- Thus: non-perturbative type IIB compactifications
  $=$ compactifications of a 12d theory (F-theory) on elliptic fibrations
- Complex dilaton $\tau(x^5, \ldots, x^{10})$
  $=$ complex structure of the fiber $S^1 \times S^1 = T^2$

Vafa; Witten; Sen '96

...

Technical introduction (continued)

- $\tau = C_0 + ie^{-\phi}$ has monodromy $\tau \to \tau + 1$ at D7-brane positions

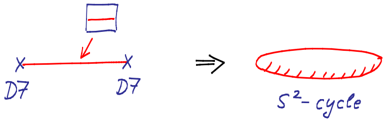


- Thus, the horizontal 1-cycle of T^2 shrinks at D7-brane locus

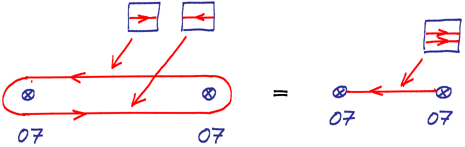
- Analogously, O7-planes represent monodromy points with an involution of the torus, $T^2 \to$ "$-T^2$"

Basic building block of our analysis

- Consider two D7-branes in 10d (as loci where the T^2 fibration of F-theory degenerates):



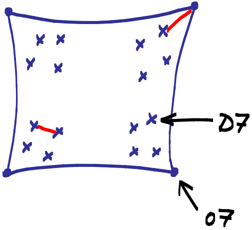
- Analogously, for two O7-planes:



- We explicitly see the 2-cycle governing the O7-O7-plane distance (there is an analogous 2-cycle with vertical lines in the fibre-torus)

Side remark:

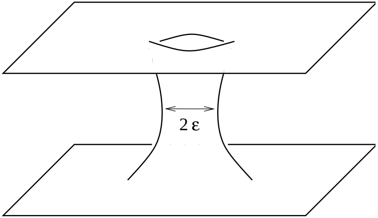
- In the case of $K3 \times K3$ [or $K3 \times (T^2/\mathbb{Z}_2)$] these building blocks can be used to explicitly stabilize a desired pattern of gauge enhancement.



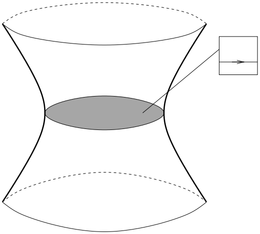
(the $T^2/\mathbb{Z}_2$ base of the elliptically fibred $K3$)

Recombination cycles

- Two recombining D7-branes in $\mathbb{C}^2$ are described by $xy = \epsilon$
- The "recombined" topology is



- The explicit (F-theory) recombination cycle (an S^3) is



- An analogous construction leads to two O7-O7 recombination cycles

- We will use this local understanding of D7/O7 cycles to build all middle-homology cycles of the global elliptic fibration.
- So far, we have only achieved this for 3-folds

Global situation at the orientifold point for 3-folds

- The fibration base is $B = K3/\mathbb{Z}_2$
  up to a coordinate change $z \leftrightarrow z^2$ making
  B smooth at the O7-plane locus
- All such non-symplectic involutions of $K3$ are known

Nikulin '79. . .'83

- They are classified in terms of "characteristic triplets"

$$(r, a, \delta)$$

  describing the even sublattice of the 2-cycle-lattice of $K3$

Global situation at the orientifold point for 3-folds (continued)

- The triplet determines the topology of the O7-plane:

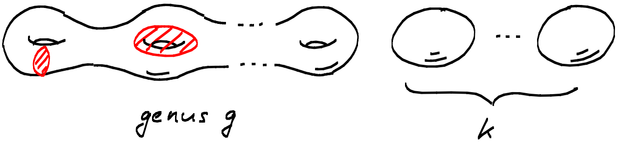
$$O7 = C_g + \sum_{i=1}^{k} E_i \,,$$

where C_g is a Riemann surface of genus g, each E_i is a separate S^2, and

$$g = \frac{1}{2}(22 - r - a) \qquad , \qquad k = \frac{1}{2}(r - a)\,.$$

Global situation at the orientifold point for 3-folds (continued)

- Thus, we have the following generic O-plane:



- As described before, each of the 2g 1-cycles of the O-plane corresponds to a relative-homology cycle of the O-plane in B.
- Each of these relative-homology cycles induces two 3-cycles of the 3-fold (using the T^2-fibration structure).
- In addition, B may have absolute 2-cycles (independently of the O-plane), some of which lift to 3-cycles of the 3-fold:

Global situation at the orientifold point for 3-folds (continued)

- Each of the (rigid) CP^1s of the O-plane corresponds to two such cycles. These do not lift to 3-fold-cycles.
- The other 2-cycles of B do lift to 3-fold-cycles:

$$b_3(\text{3-fold}) = 4g + 2(b_2(B) - 2k)\,.$$

- This result is consistent with the

M-theory-orbifold perspective:

- The orbifold-$\mathbb{Z}_2$ acts on T^2 as $z \to -z$. We have

$$\text{3-fold} = (K3 \times T^2)/\mathbb{Z}_2$$

with $$H_3(\text{3-fold}) = H_2^-(K3) \otimes H_1(T^2)\,.$$

Before discussing generic D7-brane positions, we make a

Side remark on M-theory orbifolds

- Viewing the 3-fold as $(K3 \times T^2)/\mathbb{Z}_2$, the O-plane is a curve of four A_1-singularities
(each is locally $\mathbb{C}^2/\mathbb{Z}_2$ and carries gauge group SU_2).
- By contrast, the Weierstraß model description sees the O-plane as a curve with a D_4 singularity
(and gauge group SO_8).

- How is this apparent contradiction resolved?

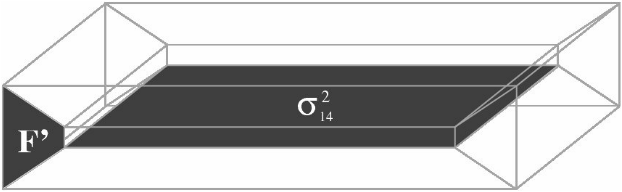
- To understand this, we take a simpler example:
an elliptic 2-fold, i.e. once again $K3$.
- We view it as $T^4/\mathbb{Z}_2 = (T^2 \times T^2)/\mathbb{Z}_2$.

Side remark on M-theory orbifolds (continued)

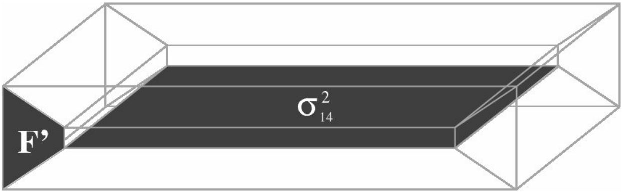
- The cycles of $K3 = T^4/\mathbb{Z}_2$, including the blowup-cycles of the singularities, can be explicitly identified with the known $K3$-lattice.

Braun, Ebert, A.H., Valandro '09

- Thus, $K3$ can be visualized as a 4-dimensional hypercube:



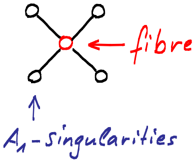
Side remark on M-theory orbifolds (continued)



- Each vertex: A_1-singularity.
- Each 2-dimensional face: One of the sections of the many possible elliptic fibrations.
- σ^2_{14}: The base of the F-theory fibration.
- F': One of the four O-planes with its degenerate T^2-fibre.

Side remark on M-theory orbifolds (continued)

- This fibre corresponds to the central root in the (extended) SO_8 Dynkin diagram:



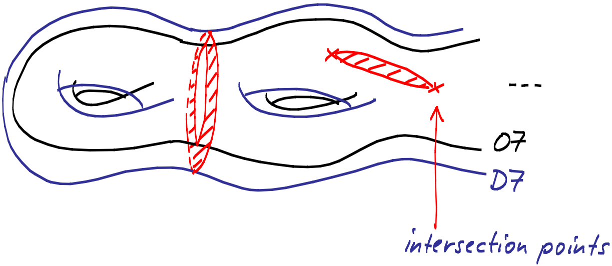
- It shrinks only in the F-theory limit, where the singularity indeed becomes D_4.

(In the paper we have used these methods to understand how a $K3$ with Weierstraß description becomes consistent with an Enriques involution in the F-theory limit.)

Returning to our main line of development:

Moving 1. . .3 D7-branes off the O7-plane

- Two types of ‘relative cycles’ between D-brane and O-plane emerge:

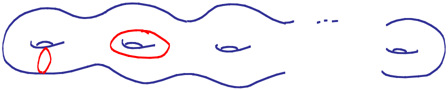


- Their counting agrees with the counting of sections of the normal bundle of the corresponding D7-brane(s)

We now turn to the

Generic D7-brane configuration

- Generically, we have a single (fully recombined) D-brane and a 'naked' O-plane
- We first ignore the O-plane
- The D-brane is just a Riemann surface of very high (but easily calculable) genus:



- Each of the $2g$ 1-cycles gives rise to a relative 2-cycle which (naively) lifts to a 3-cycle of the 3-fold.

(closely related to Jockers, Louis '04)

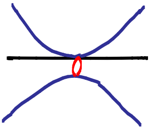
- This overcounts the number of d.o.f. (known from the Weierstraß model) by two.

Generic D7-brane configuration (continued)

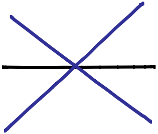
- The presence of the O-plane must be taken into account.
- We know from the Weierstraß model that a naked O-plane can only have double intersection points with a D-brane.

Braun/A.H./Triendl; Collinucci/Denef/Esole; Brunner/Herbst '08

generic:



special point:

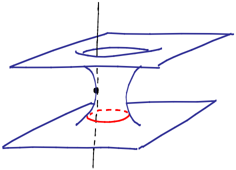


O-plane: $z = 0$;   D-brane: $x^2 = z(z - \delta)$

(The D-brane deformations live in a $\mathbb{Z}_2$-twisted version of the canonical bundle discussed e.g. in Beasley, Heckman, Vafa '08 and Cordova '09)

Generic D7-brane configuration (continued)

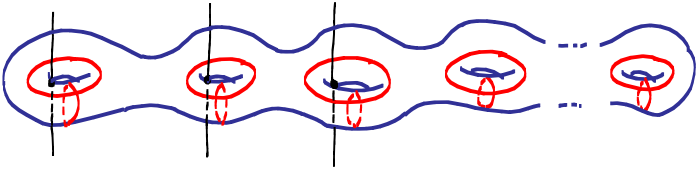
- From the explicit equations near the recombination point, the following topology can be derived:



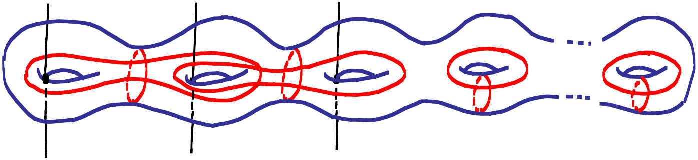
- In particular, the D-brane 1-cycle governing the recombination has winding number one w.r.t. the O-plane
- This prohibits the construction of the corresponding 3-cycle (since the fibre part is ill-defined in the vicinity of the O-plane due to the torus-involution).

Finally, the following global picture emerges:

naive:



correct:



- Crucial: Due to the torus involution at the O-plane, the number of D-brane 1-cycles which can be lifted to 3-cycles is reduced by two,

  (as required by the d.o.f. of the Weierstraß model).

Summary

- We have explicitly constructed all 3-cycles of a generic F-theory 3-fold using the topology of D-branes and O-planes as divisors of the base.
- The counting can be performed in terms of only the genus of the O-plane (known e.g. from Nikulin's classification of $K3$-involutions).
- We hope to extend (some of) these results to F-theory 4-folds
- In particular, we expect such an explicit understanding of 4-fold 4-cyles (3-fold 3-cycles) to be useful for analysing the interplay between fluxes and gauge-enhancements.