

Matching Type IIB Moduli and M-Theory Cycles

Arthur Hebecker (Heidelberg)

based on work with

A. Braun, S. Gerigk, H. Triendl – 0912.1596

A. Braun, R. Ebert, R. Valandro – 0907.2691

as well as two earlier papers with

A. Braun, H. Triendl, R. Valandro, C. Lüdeling

Outline

- Motivation for the study of cycles in F-theory
- Explicit matching of type-IIB moduli and F-theory cycles in 3-folds
- Side remark on K3 (visualization of K3)
- Towards applications to 4-folds with fluxes

New paradigm: F-theory GUTs

Beasley, Heckman, Vafa '08

Donagi, Wijnholt '08

...

- Many phenomenological advantages
+ advantage of working in 'best' part of landscape
- Global models are an important challenge
Marsano, Saulina, Schafer-Nameki;
Blumenhagen, Grimm, Jurke, Weigand, Krause '09... '10
...
- Calculation of D3 tadpole is crucial
Blumenhagen, Grimm, Jurke, Weigand, Krause '09... '10
Chen, Knapp, Kreuzer, Mayrhofer '10
- Important to understand explicitly the structure of 4-cycles
(which govern the brane motion and support fluxes)

Thus: Strong motivation for study of M-theory 4-cycles in global models

- D7-brane stabilization on $K3 \times K3$

Denef, Douglas, Florea, Grassi, Kachru '05

Lüst, Mayr, Reffert, Stieberger '05

Braun, A.H., Lüdeling, Valandro '08

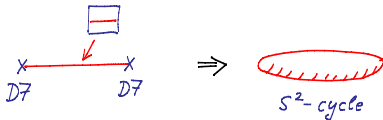
- We want to extend this to generic type IIB / F-theory models
(relate D7-brane periods and compl. structure of 4-folds)
- Eventually: Calculate and interpret D7-brane superpotentials

Alim, Hecht, Jockers, Mayr, Soroush, Walcher '08... '09

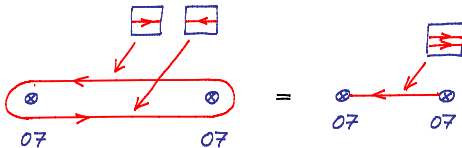
Grimm, Ha, Klemm, Klevers '09; Aganagic, Beem '09

Basic building block of our analysis

- 2-cycle between 2 D7-branes (in 10d):



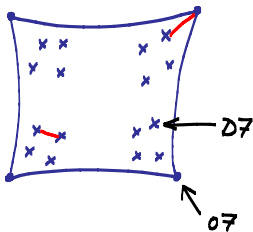
- Analogously, for two O7-planes:



- We explicitly see the 2-cycle governing the O7-O7-plane distance (there is an analogous 2-cycle with vertical lines in the fibre-torus)

Side remark:

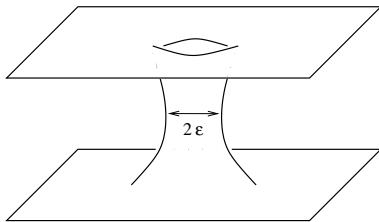
- In the case of $K3 \times K3$ [or $K3 \times (T^2/\mathbb{Z}_2)$] these building blocks can be used to explicitly stabilize a desired pattern of gauge enhancement.



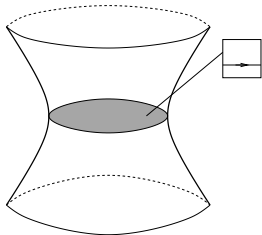
(the T^2/\mathbb{Z}_2 base of the elliptically fibred $K3$)

Recombination cycles

- Two recombining D7-branes in \mathbb{C}^2 are described by $xy = \epsilon$
- The “recombined” topology is



- The explicit (F-theory) recombination cycle (an S^3) is
- An analogous construction leads to **two** O7-O7 recombination cycles



- We will use this local understanding of D7/O7 cycles to build **all** middle-homology cycles of the global elliptic fibration.
- So far, we have only achieved this for **3-folds**

Global situation at the orientifold point for 3-folds:

- The fibration base is $B = K3/\mathbb{Z}_2$
up to a coordinate change $z \leftrightarrow z^2$ making
 B smooth at the O7-plane locus
- All such non-symplectic involutions of $K3$ are known

Nikulin '79...'83

- They are classified in terms of “characteristic triplets”

$$(r, a, \delta)$$

describing the **even sublattice** of the 2-cycle-lattice of $K3$

Global situation at the orientifold point for 3-folds (continued)

- The triplet determines the topology of the O7-plane:

$$O7 = C_g + \sum_{i=1}^k E_i,$$

where C_g is a Riemann surface of genus g , each E_i is a separate S^2 , and

$$g = \frac{1}{2}(22 - r - a) \quad , \quad k = \frac{1}{2}(r - a).$$

Global situation at the orientifold point for 3-folds (continued)

- Thus, we have the following generic O-plane:



- As described before, each of the $2g$ 1-cycles of the O-plane corresponds to a relative-homology cycle of the O-plane in B .
- Each of these relative-homology cycles induces two 3-cycles of the 3-fold (using the T^2 -fibration structure).
- In addition, B may have absolute 2-cycles (independently of the O-plane), some of which lift to 3-cycles of the 3-fold:

Global situation at the orientifold point for 3-folds (continued)

- Each of the (rigid) CP^1 s of the O-plane corresponds to two such cycles. These **do not** lift to 3-fold-cycles.
- The other 2-cycles of B **do** lift to 3-fold-cycles:

$$b_3(\text{3-fold}) = 4g + 2(b_2(B) - 2k).$$

- This result is consistent with the

M-theory-orbifold perspective:

- The orbifold- \mathbb{Z}_2 acts on T^2 as $z \rightarrow -z$. We have

$$\text{3-fold} = (K3 \times T^2)/\mathbb{Z}_2$$

with

$$H_3(\text{3-fold}) = H_2^-(K3) \otimes H_1(T^2).$$

Before discussing generic D7-brane positions, we make a

Side remark on M-theory orbifolds

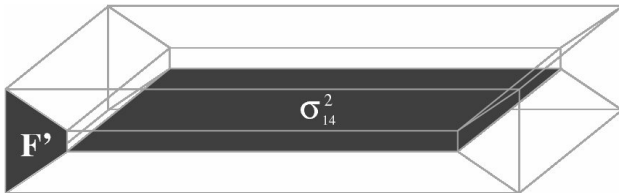
- Viewing the 3-fold as $(K3 \times T^2)/\mathbb{Z}_2$, the O-plane is a curve of four A_1 -singularities (each is locally $\mathbb{C}^2/\mathbb{Z}_2$ and carries gauge group SU_2).
- By contrast, the **Weierstraß model** description sees the O-plane as a curve with a D_4 singularity (and gauge group SO_8).
- **How is this apparent contradiction resolved?**
- To understand this, we take a simpler example: an elliptic 2-fold, i.e. once again $K3$.
- We view it as $T^4/\mathbb{Z}_2 = (T^2 \times T^2)/\mathbb{Z}_2$.

Side remark on M-theory orbifolds (continued)

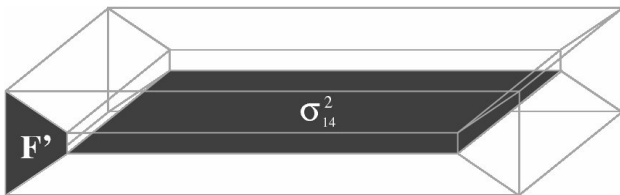
- The cycles of $K3 = T^4/\mathbb{Z}_2$, including the blowup-cycles of the singularities, can be explicitly identified with the known $K3$ -lattice.

Braun, Ebert, A.H., Valandro '09

- Thus, $K3$ can be visualized as a 4-dimensional hypercube:



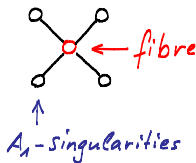
Side remark on M-theory orbifolds (continued)



- Each vertex: A_1 -singularity.
- Each 2-dimensional face: One of the sections of the many possible elliptic fibrations.
- σ_{14}^2 : The base of the F-theory fibration.
- F' : One of the four O-planes with its degenerate T^2 -fibre.

Side remark on M-theory orbifolds (continued)

- This fibre corresponds to the central root in the (extended) SO_8 Dynkin diagram:
- It shrinks only in the F-theory limit, where the singularity indeed becomes D_4 .

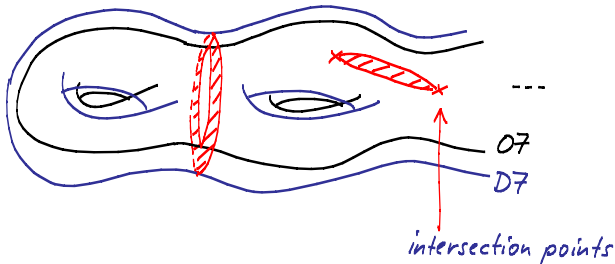


(In the paper we have used these methods to understand how a $K3$ with **Weierstraß description** becomes consistent with an **Enriques involution** in the F-theory limit.)

Returning to our main line of development:

Moving 1...3 D7-branes off the O7-plane

- Two types of 'relative cycles' between D-brane and O-plane emerge:



- Their counting agrees with the counting of sections of the normal bundle of the corresponding D7-brane(s)

A possible formal definition:

- Consider relative homology cycles of base with (generic) branes
- These cycles 'take values' in the T^2 lattice
- These 'values' have to be consistent with the boundary-branes (i.e., they have to be in the sublattice of cycles shrinking at the boundary)
- This is the the 'F-Theory part' of the 4-cycle homology of the global model

We now turn to the

Generic D7-brane configuration

- Generically, we have a single (fully recombined) D-brane and a 'naked' O-plane
- We first ignore the O-plane
- The D-brane is just a Riemann surface of very high (but easily calculable) genus:



- Each of the $2g$ 1-cycles gives rise to a relative 2-cycle which (naively) lifts to a 3-cycle of the 3-fold.

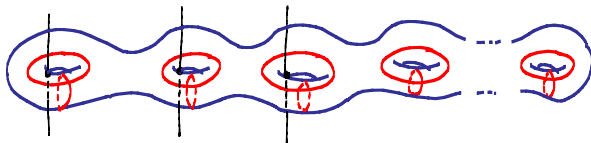
(closely related to Jockers, Louis '04)

- This **overcounts** the number of d.o.f. (known from the Weierstraß model) **by two**.

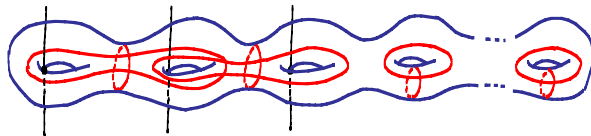
Global picture

(due to the constraints from the O7-plane – cf. talk of H. Triendl)

naive:



correct:

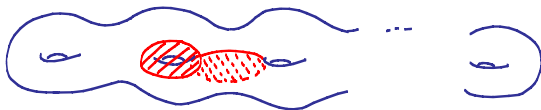


- **Crucial:** Due to the torus involution at the O-plane, the number of D-brane 1-cycles which can be lifted to 3-cycles is reduced by two, (as required the Weierstraß model).

Towards flux tadpole in 4-folds

(work in progress in Heidelberg)

- Consider I_1 -brane in global F-theory model



- Riemann surface \rightarrow complex surface
- Curve \rightarrow complex curve
- Relative 2-cycle \rightarrow relative 3-cycle = M-theory 4-cycle
- $G_4 \wedge G_4$ corresponds to intersection number of boundary curves
- Crucial issues: Monodromies; different T^2 cycles

Summary

- We have explicitly constructed all 3-cycles of a generic F-theory 3-fold using the topology of D-branes and O-planes as divisors of the base.
- The counting can be performed in terms of only the genus of the O-plane (known e.g. from Nikulin's classification of $K3$ -involutions).
- We hope to extend (some of) these results to F-theory 4-folds
- In particular, it appears to be possible to use explicit understanding of 4-fold 4-cycles for analysis of gauge fluxes in F-theory