Gauge unification in anisotropic string compactifications

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Outline

- the string-scale / GUT scale problem
- a possible solution: non-local gauge-symm. breaking in anisotropic spaces
- quantifying the perturbative domain of heterotic string theory
- explicit string-orbifold realizations
- an alternative: conventional orbifold GUTs as highly anisotropic string models

Preliminary remarks

- SUSY-GUTs remain one of the best-motivated options for beyond-the-SM physics
- the high scale of new physics is strongly supported by the values of neutrino masses
- heterotic string theory provides a UV-realization of this szenario with high aesthetic appeal
- it is worthwhile to make these models as realistic as possible (including the numerical values of M_P, M_{GUT}, α_{GUT})

The string scale / GUT scale problem

see, e.g., Caceres, Kaplunovsky, Mandelberg, '96

$$\mathcal{L}_{(10)} \sim e^{-2\phi} \left(\frac{R_{(10)}}{\alpha'^4} + \frac{F_{(10)}^2}{\alpha'^3} \right)$$

use: $g \sim e^{\phi}$, $m_H = 2\alpha'^{-1/2}$

string coupling, lightest massive state

$$\boxed{\alpha_{GUT} \sim \frac{g^2}{(R m_H)^6} \quad ; \quad M_P^2 \alpha_{GUT} \sim m_H^2 \sim (2 \cdot 10^{18}\,\mathrm{GeV})^2}$$

basic fact: $\alpha_{GUT} \ll 1$

① $R \sim m_H^{-1} \Rightarrow g \ll 1$

but unification scale too high

② $g \sim 1 \Rightarrow R \gg m_H$

but still, $R^{-1} \sim M_{GUT}$ is too high

(recall $M_{GUT,\,phen.} \sim 2 \cdot 10^{16}\,\mathrm{GeV}$)

Way out: (cf. Witten, '96)

Anisotropic models

$$R^6 \longrightarrow (R_\ell)^d (R_s)^{6-d}$$

large radii small radii

- break GUT geometrically at scale R_ℓ^{-1} (e.g. by Wilson-line)

- $$\frac{\alpha_{GUT}}{2} = \frac{g^2}{(R_\ell m_H)^d (R_s m_H)^{6-d}}$$

let $g \sim 1$, $R_s \sim m_H^{-1}$

$d=1$ $\Rightarrow$ $R_\ell^{-1} \sim 3\cdot 10^{16}$ GeV ; Great!

but: S^1 or S^1/Z_2-geometry too simple

$d=2$ $\Rightarrow$ $R_\ell^{-1} \sim 2\cdot 10^{18}$ GeV

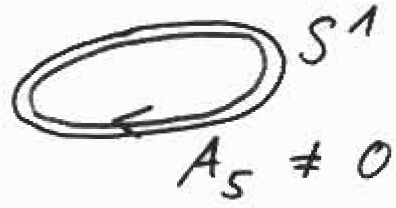
already too large

$\Rightarrow$ need to face either $g \gg 1$ or $R_s \ll m_H^{-1}$ (loss of perturbativity!)

Non-local gauge-symmetry breaking

(ignore stringy non-perturbativity for the moment and focuss on $d=2$ extra-dim. field theory)

- usual Wilson line:



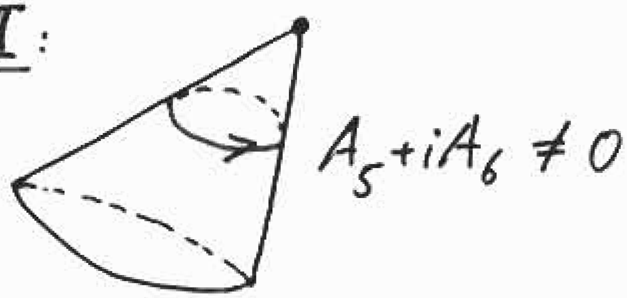
  - A_5 is a modulus,
    i.e., we break by the VEV of a massless scalar field (with all the usual problems of 4d GUTs, e.g., 2-3-splitting, unknown origin of GUT-Higgs-potential, ...)

- Wilson line on orbifold I:

  - same problems as above

  (known as "continuous Wilson line" in heter. models)

- Wilson line on orbifold II:



  Wilson line can be shrunk to length $= 0$

  $\Rightarrow$ symm. breaking at fixed point, "true" GUT scale = string scale

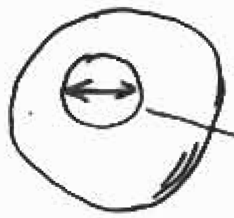
(known as "quantized Wilson line" in heter. models)

Better possibility:

Non-local quantized Wilson line

(cf. Witten '85)

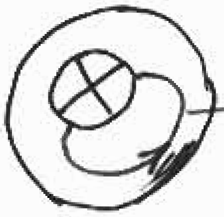
Example: $\rightarrow$ Hall, Murayama, Nomura, '01



S^2 with a hole

identify opposite points

$\downarrow$



Wilson line

$$U = P\exp \int A_\mu dx^\mu$$

$\downarrow$

the double - loop can be contracted to zero

$$\Downarrow$$

$$\boxed{U^2 = 1}$$

$\Rightarrow$ The Wilson line is quantized

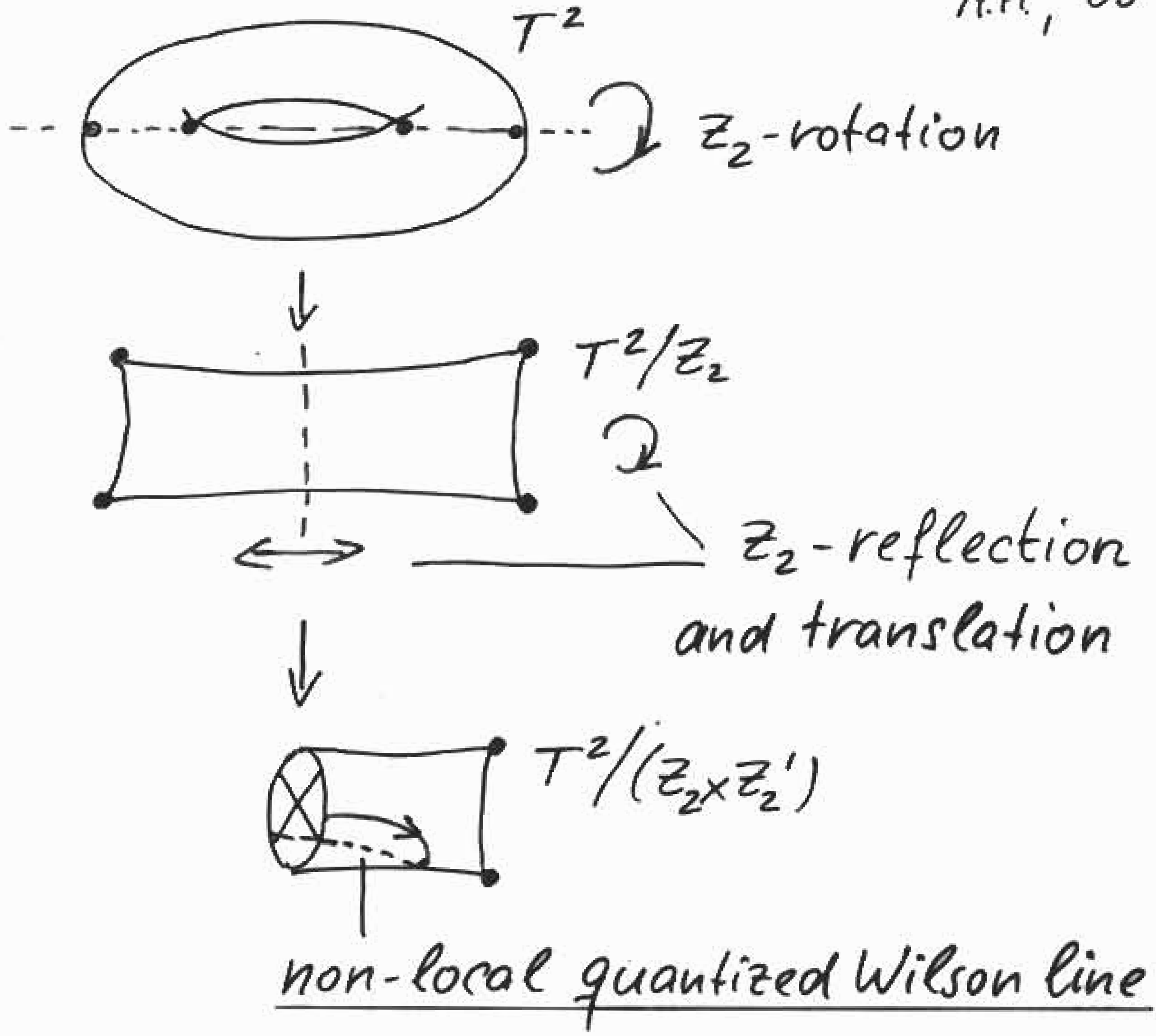
(Mathematically: manifold has homotopy $\mathbb{Z}_n$

$$\Downarrow$$

$\mathbb{Z}_n$-Wilson lines possible

On a $d=2$ (supersymmetric) orbifold

A.H., '03



This can be part of a consistent string model! (see below)

cf: 'freely acting' orbifolds explored in SUSY breaking

To quantify this idea, let us return to

<u>string theory:</u>

($g \underset{=}{\sim} e^{\phi}$ is not good enough!)

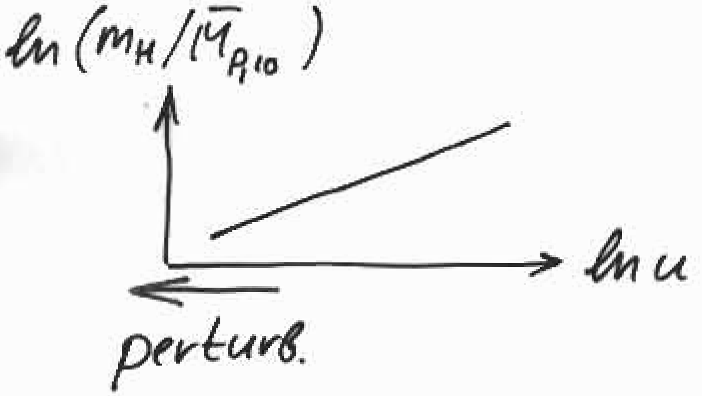
$$\mathcal{L}_{(10)} = \frac{1}{2}\bar{M}_{P,10}^{8} R_{(10)} + \frac{1}{4} M_{YM,10}^{6}\, \mathrm{tr}\, F_{\mu\nu}F^{\mu\nu}$$

(SO_{32})

<u>define:</u> $u = \left(\frac{\bar{M}_{P,10}}{M_{YM,10}}\right)^{12}$

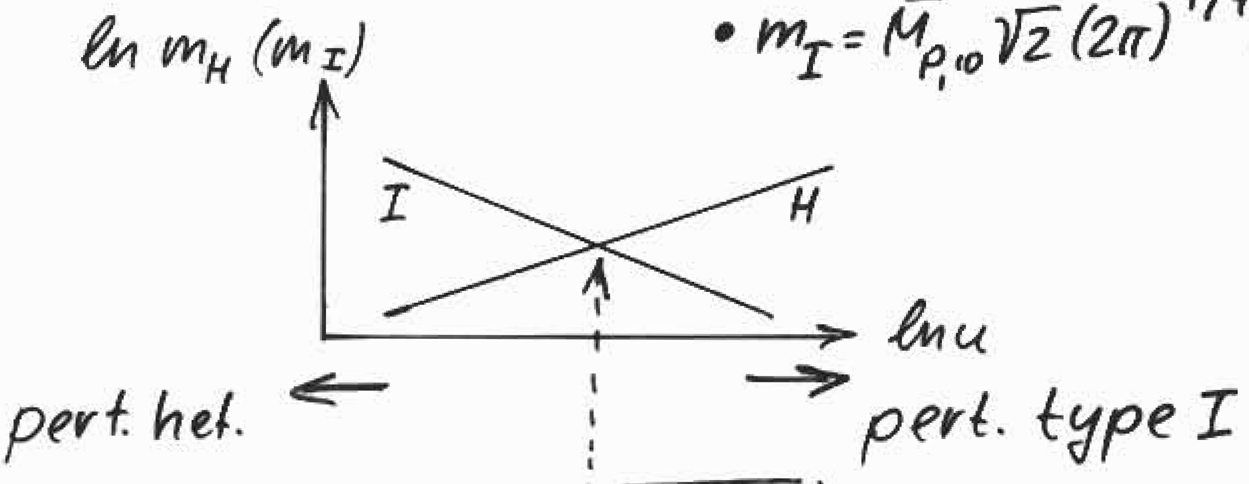
$$\Downarrow$$

$$m_H = \bar{M}_{P,10} \cdot u^{1/4}$$



<u>Type I – SO(32) – dual:</u>

- same action
- $m_I = \bar{M}_{P,10}\sqrt{2}\,(2\pi)^{7/4} u^{-1/4}$



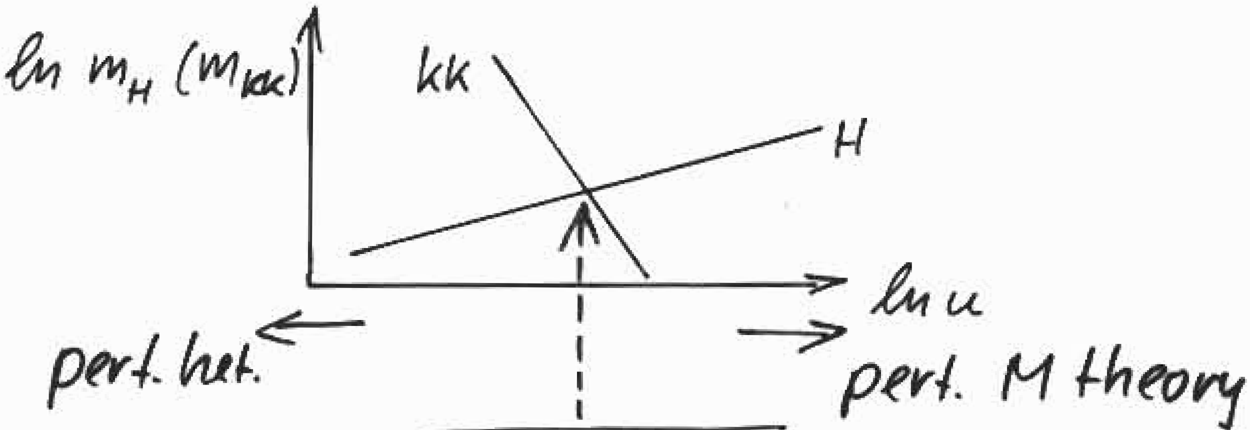
$$\boxed{u_c^2 = 4(2\pi)^7}$$

analogously:

<u>het. $E_8 \times E_8$ — het. M-theory</u>

(11d SUGRA on S^1/Z_2 with E_8 at boundaries)

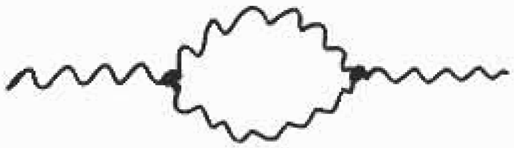
$m_H = \bar{M}_{P,10}\, u^{1/4}$ $\qquad$ $m_{KK} = \bar{M}_{P,10}\, 2(2\pi)^{7/2}\, u^{-3/4}$



$$\boxed{u_c^2 = 4(2\pi)^7}$$

exactly as before!

Effective field theory

 $\sim \int^{\Lambda} \frac{d^{10}k/(k^2)^2}{(2\pi)^{10}}$

$$\frac{\text{one loop}}{\text{tree level}} \simeq g_{YM}^2 \frac{\Lambda^{d-4}}{(d-4)2^d \pi^{d/2}\Gamma(d/2)} \cdot \frac{C_A\, d(A)}{C_F\, d(F)}$$

$d=10$, $\Lambda = m_H$

$$\frac{\text{one loop}}{\text{tree level}} = 1 \quad \text{at} \quad u^2 = u_c'^2 = \frac{3}{5}\cdot 2^8 (2\pi)^5$$

$$\boxed{u_c'^2/u_c^2 \simeq 0.97}$$

Better agreement than one could hope for!

$\Rightarrow$ Definition: $\boxed{g = u/u_c}$

($g<1 \Leftrightarrow$ perturbative)

- returning to $d=2$ large radii, we now fix

$$M_{GUT} = 2\cdot 10^{16}\,\text{GeV}$$

$$\alpha_{GUT} = \frac{1}{25}$$

$$(\rightarrow m_H \text{ fixed})$$

- g and R_s are now constrained by

$$\boxed{g_H = 12\,(R_s\, m_H)^2}$$

- $g < 1$ & $R_s > m_H^{-1}$ impossible
- search for pert. UV-completion by
  1) S-duality $\longrightarrow$ type I
  2) T-duality $\longrightarrow$ type IIB w/ D5-branes

$$(12)^3 / g_{I} = (R_{s,I}\, m_H)^2$$

$$\Downarrow$$

lightest "non-pert." states at $R_{s,I}^{-1} \simeq 2\, R_\ell^{-1}$

marginally O.K.

(further improvement using Wilson lines)

(recall: heavy states decouple exponentially!)

Explicit example:

geometry: $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$

$\mathbb{Z}_2$: $z_1 \to z_1 + \pi R_1$, $z_2 \to -z_2$, $z_3 \to -z_3$

$\mathbb{Z}_2'$: $z_1 \to -z_1$, $z_2 \to -z_2 + \pi R_2$ $z_3 \to z_3$

free

group-theoretical:

$$SO_{32} \longrightarrow SO_{10} \times \cdots \longrightarrow SU_4 \times SU_2 \times SU_2 \times \cdots$$

local breaking, including Wilson-lines (leading to a 6d SO_{10}-GUT)

non-local GUT-breaking associated with large radii R_1, R_2

... much more remains to be done ...

A closely related possibility:

conventional 5d orbifold GUTs

- consider $d=1$ large radius
  (with local breaking at boundary of $S^1/Z_2 \times Z_2'$)

- modified logarithmic running above
  $$M_c = R_e^{-1}$$
  can be used to delay unification to $M > M_{GUT}$

- precision unification affected by non-pert. boundary effects

- our analysis: optimal values are
  $$M_c \simeq 2.6 \cdot 10^{15}\,\text{GeV}, \quad M \simeq 20 M_c$$
  scale of non-pert. string-theoretic effects

- this is marginally consistent with the orbifold GUT idea

- fundamental disadvantage: model-dependence of the running above M_c

Conclusions

- the string-scale/GUT scale problem can be addressed by
  - GUT breaking through non-local quantized Wilson lines
  - increasing the relevant large radii to $R_\ell \sim (M_{GUT})^{-1} \sim (2 \cdot 10^{16}\,\mathrm{GeV})^{-1}$
- simple heterotic orbifold models with this feature are available
- no doublet-triplet splitting problem as in 4d models
- no tunable large thresholds or modified logarithmic running required
- need to explore this new class of heterotic orbifolds