Infrared-Safe Correlation Functions from Inflation

Byrnes, Gerstenlauer, A.H., Nurmi, Tasinato (1005.3307)
Gerstenlauer, A.H., Tasinato (1102.0560)

Outline

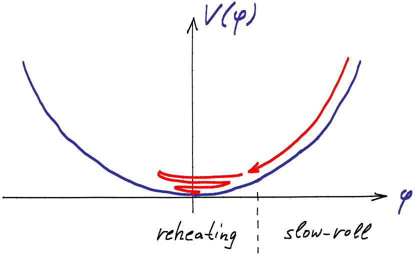
- IR divergences in the curvature correlator
- Defining an IR-safe correlator
- Tensor modes / Higher correlators / Explicit calculability
- Implications for late geometry

Introduction

- Single-field slow-roll inflation with potential $V(\varphi)$

$$V \ll 1 \qquad V^{(n)}/V \ll 1 \qquad (\overline{M}_P = 1)$$

- The geometry is almost-de-Sitter: $3H^2 = V$
- Equation-of-motion: $\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$



Introduction (continued)

- It is sufficient to treat $\delta\varphi$ as a massless scalar in the geometry

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

- With the ansatz

$$\delta\varphi(x) \sim \int d^3k \left\{ a_{\vec{k}} f_{\vec{k}}(x) + a^\dagger_{\vec{k}} f^*_{\vec{k}}(x) \right\}$$

and $f_{\vec{k}} \sim -\frac{iH}{\sqrt{k^3}}\, e^{i\vec{k}\vec{x}}$ at $t \to \infty$ one finds:

$$\delta\varphi(x) \sim \int d^3k \frac{iH}{\sqrt{k^3}}\, e^{i\vec{k}\vec{x}} \left(a^\dagger_{-\vec{k}} - a_{\vec{k}}\right)$$

- In other words, outside the horizon $\delta\varphi$ is a classical Gaussian random variable:

$$\delta\varphi(x) \sim \int d^3k \frac{H}{\sqrt{k^3}} e^{i\vec{k}\vec{x}} b_{\vec{k}}$$

where $\langle b_{\vec{k}} b_{\vec{q}} \rangle \sim \delta^3(\vec{k} + \vec{q})$.

- Its expectation value is logarithmically divergent:

$$\langle \delta\varphi(x)^2 \rangle \sim H^2 \int \frac{d^3k}{k^3}$$

... Starobinsky '85 ... Allen, Folacci '87 .... Weinberg '05
......
Burgess, Leblond, Holman, Shandera '10
Rajaraman, Kumar, Leblond '11
......

Introduction (continued)

- The UV divergence is easy to understand and remove

What is the origin of the IR divergence?

Allen, Folacci '87
Kirsten, Garriga '93

- de Sitter space is 'effectively' compact
- Hence, the zero mode of $\delta\varphi$ is dynamical
- It 'diffuses' like a QM-particle without potential
- However, this is irrelevant since reheating 'measures' the value of the zero-mode
- But: The effect 'returns' through loop corrections

IR divergences in δN formalism

Starobinsky '85, Sasaki/Stewart '95
....
Lyth/Rodriguez '05

- Consider some late, constant-energy-density surface (reheating surface):

$$ds^2 = e^{2\zeta} \left(e^{\gamma}\right)_{ij} dx^i dx^j \,.$$

- Ignoring γ_{ij} for the moment, one has

$$\zeta(x) = N(\varphi + \delta\varphi(x)) - N(\varphi)$$

where

$$N(\varphi) = \int^{\varphi} d\tilde{\varphi} \, \frac{V(\tilde{\varphi})}{V'(\tilde{\varphi})}$$

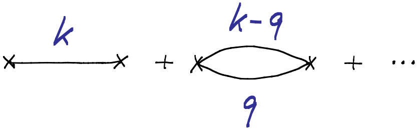
IR divergences in δN formalism (continued)

- Expanding in $\delta\varphi$ we have

$$\zeta(x) = N_\varphi \delta\varphi(x) + \frac{1}{2} N_{\varphi\varphi} \delta\varphi(x)^2 + \cdots$$

and, for the curvature correlator:

$$\langle \zeta_k \zeta_p \rangle = N_\varphi^2 \langle \delta\varphi_k \delta\varphi_p \rangle + \frac{1}{4} N_{\varphi\varphi}^2 \langle (\delta\varphi^2)_k (\delta\varphi^2)_p \rangle + \cdots$$

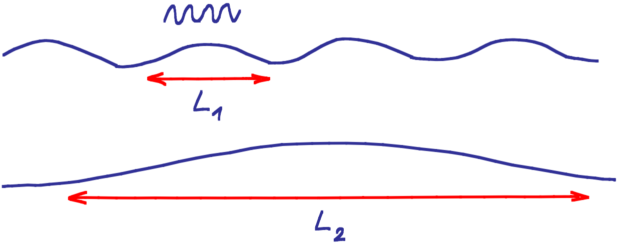


- IR-divergent corrections $\sim \int d^3q/q^3$ result

Intuitive physical picture:

- Long-wavelength modes affect measured short-wavelength fluctuations (e.g. L_1).
- Modes outside the 'box size' can be absorbed in constant ζ-background and are irrelevant (e.g. L_2).

Lyth '07



Fluctuations of the Hubble scale

- Obviously, the technical origin of the effect is the dependence of $N_\varphi(\varphi)$ on $\delta\varphi_q$ with $q \ll k$.
- Hence, the Hubble scale H should be modified analogously:

$$\delta\varphi(x) \sim \int_k \frac{e^{-ikx}}{\sqrt{k^3}} \, H(\varphi(t_k) + \delta\bar{\varphi}(x)) \, b_k \,,$$

where

$$\delta\bar{\varphi}(x) \sim \int_{q \ll k} \frac{e^{-iqx}}{q^{3/2}} \, b_q \,.$$

Fluctuations of the Hubble scale (continued)

- Collecting all subleading terms one finds

$$\mathcal{P}_\zeta(k) \;\sim\; N_\varphi^2 H^2 \;+\; \frac{1}{2}\langle\delta\bar{\varphi}^2\rangle\,\frac{d^2}{d\varphi^2}(N_\varphi^2 H^2)$$

- or, equivalently,

$$\mathcal{P}_\zeta(k) \;=\; \left(1 \;+\; \frac{1}{2}\langle\bar{\zeta}^2\rangle\frac{d^2}{d(\ln k)^2}\right)\,\mathcal{P}_\zeta^0(k)$$

see also Giddings, Sloth '10
Senatore '10
Giddings, Sloth '11

IR-safe correlation functions

- Recall our gauge choice

$$ds^2 = e^{2\zeta}\,(e^{\gamma})_{ij}\,dx^i dx^j\,.$$

- The conventional power spectrum can be defined as

$$\mathcal{P}_\zeta(k) \;\sim\; k^3 \int_y e^{iky}\,\langle\, \zeta(x)\zeta(x+y)\,\rangle\,.$$

- This is sensitive to the box-size L since the physical meaning of y depends on the (strongly varying) background $\bar\zeta$.
- To avoid this, use invariant distance $z = y\, e^{\bar\zeta}$.
  The z-dependence of the correlator

$$\langle\, \zeta(x)\zeta(x + ze^{-\bar\zeta(x)})\,\rangle$$

  is then a background-independent and hence IR-safe object.

related to Urakawa/Tanaka '10 ?

- Its Fourier transform is our desired IR-safe power spectrum:

$$\mathcal{P}^0_\zeta(k) \ \sim \ k^3 \int_z e^{ikz} \, \langle \, \zeta(x)\zeta(x + ze^{-\bar{\zeta}(x)}) \, \rangle \, .$$

- The original IR-sensitive power spectrum follows as

$$\begin{aligned}
\mathcal{P}_\zeta(k) \ &\sim \ k^3 \int_y e^{iky} \, \langle \, \zeta(x) \, \zeta(x + y) \, \rangle \\
&\sim \ k^3 \int_y e^{iky} \, \langle \, \zeta(x) \, \zeta(x + (ye^{\bar{\zeta}})e^{-\bar{\zeta}}) \, \rangle \\
&\sim \ \langle \, (ke^{-\bar{\zeta}})^3 \int_z \exp(i ke^{-\bar{\zeta}} z) \, \zeta(x) \, \zeta(x + ze^{-\bar{\zeta}}) \, \rangle \\
&\sim \ \langle \, \mathcal{P}^0_\zeta(ke^{-\bar{\zeta}}) \, \rangle
\end{aligned}$$

in agreement with our previous result.

Tensor modes / Higher correlators

- Our IR-safe power spectrum immediately generalizes to the case of background tensor modes,

$$\mathcal{P}^0_\zeta(k) \sim k^3 \int_z e^{ikz} \langle \zeta(x)\zeta(x + e^{-\bar{\zeta}}(e^{-\bar{\gamma}/2} z))\rangle \,,$$

and to higher correlation functions,

$$P^0_{(n)}(k_1 \ldots k_n) \sim \int_{z_1} \cdots \int_{z_n} e^{i(k_1 z_1 + \cdots + k_n z_n)} \langle \zeta(x)\zeta(x+y_1) \cdots \zeta(x+y_n)\rangle \,,$$

where

$$y_i = y_i(z, \bar{\zeta}, \bar{\gamma}) = e^{-\bar{\zeta} - \bar{\gamma}/2} z \,.$$

Explicit averaging over the background

- For scalar modes, the IR-enhancement can be worked out explicitly:

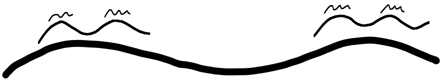
$$\mathcal{P}_\zeta(k) = \frac{1}{\sigma\sqrt{2\pi}} \int d\bar{\zeta} e^{-\bar{\zeta}^2/2\sigma^2} \mathcal{P}_\zeta^0(ke^{-\bar{\zeta}}) \,,$$

with

$$\sigma^2 \equiv \langle \bar{\zeta}^2 \rangle \sim \int_{1/L \ll q \ll k} (N_\varphi H)^2(q) \, \frac{d^3 q}{q^3} \,.$$

- The breakdown of convergence at large L can be analytically understood

- This breakdown implies a peculiar geometry of the reheating surface at large length scales:



- It is ‘locally’ approximately flat, but deviates from flatness if one looks at very large regions with very high resolution.

Summary

- An interesting class of IR divergences in correlation functions comes from long-wavelength background modes.
- This can be quantified in an (appropriately modified) δN formalism
- One can define IR-safe correlators.
- One can return to usual correlators and recover their IR-corrections.
- Are there observable effects (given our relatively small L)?
- Are there interesting implications for quantum gravity in de Sitter space? (cf. Arkani-Hamed et al., Giddings, ...)