HIGGSPLSION

Valentin V Khoze

IPPP Durham

- VVK & Michael Spannowsky 1704.03447, 1707.01531
- VVK 1705.04365
- VVK, J Reiness, M Spannowsky, P Waite 1709.08655
- & with Jakub Scholtz & Michael Spannowsky — to appear
100th Anniversary of Russian Revolution

7 November 2017
Now: an altogether different radical idea
• Before the Higgs discovery, massive Yang-Mills theory violated perturbative unitarity — problem with high-energy growth of 2 -> 2 processes

• Discovery of the (elementary) Higgs made the SM theory self-consistent

• The Higgs brings in the Hierarchy problem: radiative corrections push the Higgs mass to the new physics (high) scale: \( m_h^2 \simeq m_0^2 + \delta m_{\text{new}}^2 \)

• In this talk: consider \( n \sim 100 \)s of Higgs bosons produced in the final state \( n \lambda \) >> 1. Investigate scattering processes at ~ 100 TeV energies.

• HIGGSPLOSION: \( n \)-particle rates computed in a weakly-coupled theory can become unsuppressed above critical values of \( n \) and \( E \). Perturbative and non-perturbative semi-classical calculations. \( n! \sim \text{exponential growth with } n \text{ or } E \). (Scale \( n \sim E/m \)).

• A new unitarity problem — caused by the elementary Higgs bosons — appears to occur (?) for processes with large final state multiplicities \( n >> 1 \)

• HIGGSPLOSION offers a solution to both problems: it restores the unitarity of high-multiplicity processes and dynamically cuts off the values of the loop momenta contributing to the radiative corrections to the Higgs mass.
Compute 1 \rightarrow n amplitudes @LO with non-relativistic final-state momenta:

This classical equation for \( \eta(x) = h(x) \) determines directly the structure of the recursion relation for tree-level scattering amplitudes:

\[
\mathcal{A}_n(p_1, \ldots p_n) = n! \left( \frac{\lambda}{2M_h^2} \right)^{n-1} \exp \left[ -\frac{7}{6} n \varepsilon \right], \quad n \rightarrow \infty, \; \varepsilon \rightarrow 0, \; n \varepsilon = \text{fixed}
\]

factorial growth

amplitude on the n-particle threshold

In the large-n-non-relativistic limit the result is

see classic 1992-1994 papers: Brown; Voloshin; Argyres, Kleiss, Papodopoulos Libanov, Rubakov, Son, Troitski

more recently: VVK 1411.2925

prototype of the SM Higgs in the unitary gauge

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu h)^2 - \frac{\lambda}{4} (h^2 - v^2)^2
\]
Can now integrate over the \( n \)-particle phase-space

The cross-section and/or the \( n \)-particle partial decay \( \Gamma_n \)

\[
\Gamma_n(s) = \int d\Phi_n \frac{1}{n!} |A_{h^* \rightarrow n \times h}|^2
\]

The \( n \)-particle Lorentz-invariant phase space volume element

\[
\int d\Phi_n = (2\pi)^4 \delta^{(4)}(P_{in} - \sum_{j=1}^n p_j) \prod_{j=1}^n \int \frac{d^3 p_j}{(2\pi)^3 2p_j^0},
\]

in the large-\( n \) non-relativistic limit with \( n \varepsilon_h \) fixed becomes,

\[
\Phi_n \simeq \frac{1}{\sqrt{n}} \left( \frac{M_h^2}{2} \right)^n \exp \left[ \frac{3n}{2} \left( \log \frac{\varepsilon_h}{3\pi} + 1 \right) + \frac{n\varepsilon_h}{4} + O(n\varepsilon_h^2) \right]
\]

We find:

\[
\Gamma_n^{\text{tree}}(s) \sim \exp \left[ n \left( \log \frac{\lambda n}{4} - 1 \right) + \frac{3n}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} n\varepsilon + O(n\varepsilon^2) \right]
\]

Son 1994;
Libanov, Rubakov, Troitskii 1997; more recently: VVK 1411.2925
• The $n!$ growth of perturbative amplitudes is not entirely surprising: the number of contributing Feynman diagrams is known to grow factorially with $n$. [In scalar QFT there are no partial cancellations between individual diagrams (unlike QCD).]

• Important to distinguish between the two types of large-$n$ corrections:

(a) *higher-order* perturbative corrections to some leading-order quantities

(b) our case where the *leading-order* tree-level contribution to the $1^* \to n$ Amplitude grows factorially with the particle multiplicity $n$ of the final state.

• This was studied in the 90s in scalar QFTs

• But now realised that the characteristic energy scale for EW applications starts in the 50-100 TeV range. FCC would provide an exciting challenge to realise this in the context of the multi- Higgs and Massive Vector bosons production in the SM.
Contrast asymptotic growth of higher-order corrections in perturbation theory with the $\sim n!$ contributions to $\Gamma_n(s)$

Not the same types of beasts
Contrast asymptotic growth of higher-order corrections in perturbation theory with the \( \sim n! \) contributions to \( \Gamma_n(s) \).

For \( \Gamma_n(E) \) we’ll find this

Not the same types of beasts
Perturbative as well as semi-classical calculations result in the exponential form for the n-particle width $\Gamma \sim \exp[F_{\text{holy grail}}]$

- Libanov, Rubakov, Son, Troitsky; Son: 1994-1995

In the non-rel. limit for perturbative Higgs bosons only production we obtained:

$$\sigma_n \propto \exp\left[n\left(\log \frac{\lambda n}{4} - 1\right) + \frac{3n}{2}\left(\log \frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}n\varepsilon\right]$$

More generally, in the large-$n$ limit with $\lambda n = \text{fixed}$ and $\varepsilon = \text{fixed}$, one expects

$$\sigma_n \propto \exp\left[\frac{1}{\lambda} F_{\text{h.g.}}(\lambda n, \varepsilon)\right]$$

where the holy grail function $F_{\text{h.g.}}$ is of the form,

$$\frac{1}{\lambda} F_{\text{h.g.}}(\lambda n, \varepsilon) = \frac{\lambda n}{\lambda} (f_0(\lambda n) + f(\varepsilon))$$

In our higgs model, i.e. the scalar theory with SSB,

$$f_0(\lambda n) = \log \frac{\lambda n}{4} - 1 \quad \text{at tree level}$$

$$f(\varepsilon) \to \frac{3}{2}\left(\log \frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\varepsilon \quad \text{for } \varepsilon \ll 1$$
Can also include *loop corrections* to amplitudes on thresholds:

The 1-loop corrected threshold amplitude for the pure $n$ Higgs production:

$$
\phi^4 \text{ with SSB : } A_{1 \to n}^{\text{tree} + \text{1loop}} = n! (2v)^{1-n} \left( 1 + n(n-1) \frac{\sqrt{3}\lambda}{8\pi} \right)
$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

*Libanov, Rubakov, Son, Troitsky 1994*

$$
A_{1 \to n} = A_{1 \to n}^{\text{tree}} \times \exp \left[ B \lambda n^2 + O(\lambda n) \right]
$$

in the limit $\lambda \to 0$, $n \to \infty$ with $\lambda n$ fixed. Here $B$ is determined from the 1-loop calculation (as above) – *Smith; Voloshin 1992*: $B = + \lambda n \frac{\sqrt{3}}{4\pi}$

$$
f_0(\lambda n) = \log \frac{\lambda n}{4} - 1 + \lambda n \frac{\sqrt{3}}{4\pi} + O(\lambda n)^2
$$

$$
f(\varepsilon) \to \frac{3}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \text{ for } \varepsilon \ll 1
$$
Multi-particle decay rates $\Gamma_n$ can also be computed using an alternative semi-classical method. This is an intrinsically non-perturbative approach, with no reference in its outset made to perturbation theory.

The path integral is computed in the steepest descent method, controlled by two large parameters, $1/\lambda \to \infty$ and $n \to \infty$.

$$\lambda \to 0, \quad n \to \infty, \quad \text{with} \quad \lambda n = \text{fixed}, \quad \varepsilon = \text{fixed}.$$ 

The semi-classical computation in the regime where,

$$\lambda n = \text{fixed} \ll 1, \quad \varepsilon = \text{fixed} \ll 1,$$

reproduces the tree-level perturbative results for non-relativistic final states. Remarkably, this semi-classical calculation also reproduces the leading-order quantum corrections arising from resumming one-loop effects.
The semiclassical approach is equally applicable and more relevant to the realisation of the non-perturbative Higgsbwn case where,

\[ \lambda n = \text{fixed} \gg 1, \quad \varepsilon = \text{fixed} \ll 1. \]

This calculation was carried out for the spontaneously broken theory with the result given by,

\[ \mathcal{R}_n(\lambda; n, \varepsilon) = \exp \left[ \frac{\lambda n}{\lambda} \left( \log \frac{\lambda n}{4} + 0.85 \sqrt{\lambda n} + \frac{1}{2} + \frac{3}{2} \log \frac{\varepsilon}{3\pi} - \frac{25}{12} \varepsilon \right) \right], \]

Higher order corrections are suppressed by \( O(1/\sqrt{\lambda n}) \) and powers of \( \varepsilon \).
Thus we have computed the rate $R$ in the large lambda $n$ limit:

using the semi-classical approach and the thin-wall approximation

$$R = \exp \left[ \frac{\lambda n}{\lambda} \left( \log \frac{\lambda n}{4} + 3.02 \sqrt{\frac{\lambda n}{4\pi}} - 1 + \frac{3}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right) \right]$$

$\lambda n \gg 1$  small $\varepsilon$

Higgsplasion realised at large lambda $n$

$R$ vs. $n$ for different $E/M$ values.
HIGGSPLION and HIGGSPERSION

\[
\mathcal{M}_{gg\rightarrow h^*} \times \frac{i}{p^2 - m_h^2 - Re \Sigma(p^2) + i m_h \Gamma(p^2)} \times \mathcal{M}_{h^*\rightarrow n \times h}
\]

Include self-energy

\[
\sigma_{gg\rightarrow n \times h} \sim y_t^2 \frac{m_t^2}{m_h} \log^4 \left( \frac{m_t}{\sqrt{s}} \right) \times \frac{1}{(s - Re \Sigma(s))^2 + m_h^2 \Gamma^2(s)} \times \Gamma_n(s)
\]

VVK & Spannowsky 1704.0344

- Unitarity restored!
Summary of the main idea

The Dyson propagator (continued to Euclidean space) is,

\[
\Delta_R(x_1, x_2) = \langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + m^2 + \Sigma_R(p^2)} e^{ip_0 \Delta \tau + i\vec{p} \Delta \vec{x}}.
\]

When the theory enters the Higgspllosion regime, the self-energy undergoes a sharp exponential growth,

\[
\Sigma_R(p^2) \sim \begin{cases} 
0 & \text{: for } p^2 < E_*^2 \\
\infty & \text{: for } p^2 \geq E_*^2
\end{cases}
\]

The loop momentum integral becomes cut off by \( \Sigma \) outside the ball of radius \( E_* \)

\[
\Delta_R(x_1, x_2) = \int_{p^2 \leq E_*^2} \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + m^2} e^{ip_0 \Delta \tau + i\vec{p} \Delta \vec{x}}
\]

\[
\sim \begin{cases} 
1/|\Delta x|^2 & \text{: for } 1/E_* \ll |\Delta x| \ll 1/m \\
E_*^2 & \text{: for } |\Delta x| \lesssim 1/E_*
\end{cases}
\]
Summary of the main idea

A conventional wisdom: in the description of nature based on a local QFT, one should always be able to probe shorter and shorter distances with higher and higher energies.

Higgspslosion is a dynamical mechanism, or a new phase of the theory, which presents an obstacle to this principle at energies above $E_*$. 

$E_*$ is the new dynamical scale of the theory, where multi-particle decay rates become unsuppressed.

Schematically, $E_* = C \frac{m}{\lambda}$, where $C$ is a model-dependent constant of $\mathcal{O}(100)$. This expression holds in the weak-coupling limit $\lambda \to 0$. 
Higgsplosion

At energy scales above $E_\ast$ the dynamics of the system is changed:

1. Distance scales below $|x| \lesssim 1/E_\ast$ cannot be resolved in interactions;
2. UV divergences are regulated;
3. The theory becomes asymptotically safe;
4. And the Hierarchy problem of the Standard Model is therefore absent.

Consider the scaling behaviour of the propagator of a massive scalar particle

$$\Delta(x) := \langle 0 | T(\phi(x) \phi(0)) | 0 \rangle \sim \begin{cases} m^2 e^{-m|x|} : & \text{for } |x| \gg 1/m \\ 1/|x|^2 : & \text{for } 1/E_\ast \ll |x| \ll 1/m \\ E_\ast^2 : & \text{for } |x| \lesssim 1/E_\ast \end{cases}$$

where for $|x| \lesssim 1/E_\ast$ one enters the Higgsplosion regime.

This is a non-perturbative criterion. Can in principle be computed on a lattice.
Loop integrals are effectively cut off at $E_\ast$ by the exploding width $\Gamma(p^2)$ of the propagating state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta $k_i^2 \sim m^2 \ll E_\ast^2$.

The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the $n$ soft particle quanta of the same field $\phi$. 

VVK & Michael Spannowsky 1704.03447, 1707.01531
Asymptotic Safety

For all parameters of the theory (running coupling constants, masses, etc):

![Graph showing Higgsphlosion and No Higgsphlosion](image)

- **Higgsphlosion**
- **No Higgsphlosion**

**UV fixed point**
Higgsploding the Hierarchy problem

\[ \mathcal{L}_X = \frac{1}{2} \partial^\mu X \partial_\mu X - \frac{1}{2} M_X^2 X^2 - \frac{\lambda_P}{4} X^2 h^2 - \mu X h^2 \]

\[ \Delta M_h^2 \sim \lambda_P \int \frac{d^4 p}{16 \pi^4} \frac{1}{p^2 + M_X^2 + \Sigma_X(p^2)} \propto \lambda_P \frac{E^2_*}{M_X^2} E^2_* \ll \lambda_P M_X^2. \]

Due to Higgspllosion the multi-particle contribution to the width of \( X \) explode at \( p^2 = s_* \) where \( \sqrt{s_*} \simeq O(25) \text{ TeV} \)

It provides a sharp UV cut-off in the integral, possibly at \( s_* \ll M_X^2 \)

Hence, the contribution to the Higgs mass amounts to

For \( \Gamma(s_*) \simeq M_X \) at \( s_* \ll M_X^2 \) \( \implies \Delta M_h^2 \propto \lambda_P \frac{s_*}{M_X^2} s_* \ll \lambda_P M_X^2 \)

and thus mends the Hierarchy problem by \( \left( \frac{\sqrt{s_*}}{M_X} \right)^4 \simeq \left( \frac{25 \text{ TeV}}{M_X} \right)^4 \)
Prospects of direct observation of Higgs inflation

Vector boson fusion at high-energy pp colliders (FCC)

Propagator with Higgspersion at $\sqrt{s_*}$

$\begin{align*}
  s_* - m_h^2 &= Re\tilde{\Sigma}(s_*) + im_h\Gamma(s_*) \\
  \Gamma_n/M &= \exp[n\log n + 3.02n^{3/2}((\log 3)^{1/2} + 1)]
\end{align*}$

$\Gamma_n/M$ versus $n$ for different values of $E/M$.

Energy excess over $\sqrt{s_*}$ carried away by jets.

$n$ non-relativistic Higgses Higgsplosion at $\sqrt{s_*}$.
Vector boson fusion at high-energy pp colliders (FCC)

using $p_{\text{jet}} > 40$ GeV

preliminary: no Higgs decays into SM dofs included; & no vector bosons in final states yet

VVK, J Scholtz, M Spannowsky
Here focus on a class of observables which have no tree-level contributions.

At LHC energies effects of Higgs collision are small (next slide).

However O(1) effects can be achieved for these loop-induced processes if the interactions are probed close to $\sim 2E^*$. 

- VVK, J Reiness, M Spannowsky, P Waite 1709.08655
Effects of Higgsplosion on Precision Observables

\[ \frac{\hat{\sigma}_{gg \to h}}{\sigma_{gg \to h}^{\text{SM}}} - 1 \]

\[ 1 - \frac{\Gamma_{h \to XX}^*}{\Gamma_{h \to XX}^{\text{SM}}} \]

\[ \Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{theory}} \approx 2.90 \cdot 10^{-9} \]

\[ a_e^{\text{exp}} = 11596521807.3(2.8) \cdot 10^{-13} \]
Summary

• The Higgsplison / Higgspersion mechanism makes theory UV finite (all loop momentum integrals are dynamically cut-off at scales above the Higgsplison energy).

• UV-finiteness => all coupling constants slopes become flat above the Higgsplison scale => automatic asymptotic safety

• [Below the Higgsplison scale there is the usual logarithmic running]

• 1. Asymptotic Safety

• 2. No Landau poles for the U(1) and the Yukawa couplings

• 3. The Higgs self-coupling does not turn negative => stable EW vacuum

• No new physics degrees of freedom required — very minimal solution