Perturbativity Constraints Beyond 4π

based on arXiv:1709.03501

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Non-SUSY models with extended Higgs sectors

Spectrum calculation at tree-level/OS

- Use physical observables to define interesting parameter regions: masses and mixing angles
- Use tree-level relations between observables and Lagrangian parameters: parameters are output

  Example: THDM, benchmark of LHC Higgs XS Working Group

m_H[GeV]	m_A[GeV]	$m_{H^\pm}$[GeV]	t_β	$c_{\beta-\alpha}$	M_{12}^2[GeV^2]	λ_1	λ_2	λ_3	λ_4	λ_5
500	500	$\sqrt{m_H^2 - v^2}$	2	0	-9887.6	7.49	0.59	1.96	-1.70	-3.68

- Problem: Handle of the model parameters is lost
- Assumption: On-shell renormalization can always be performed which keeps the spectrum as it is

can we be certain that the perturbative series is behaving well?

Perturbativity

How to ensure that the perturbative expansion of a model works?

Easy requirements at tree level:

- ✓ In a model with quartic couplings λ_i, require $\lambda_i < 4\,\pi$
- ✓ Compute the $2 \to 2$ scalar field scattering amplitudes, require perturbative unitarity: $|\mathcal{M}| < 8\,\pi$

However

already for the SM these conditions fail: $\lambda \lesssim 2.2 \ll 4\,\pi$

[Nierste, Riesselmann '95]

In principle, one would need to calculate processes at different loop orders for being able to judge whether perturbation theory works or not...

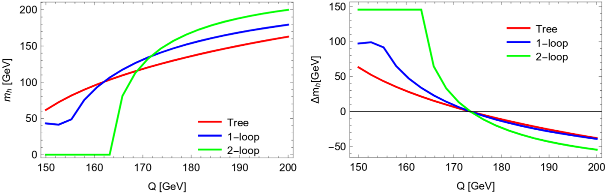
... obviously that's not what one wants to do for studying a BSM model

Perturbativity at loop order

Braathen, Goodsell, Staub '17:

↪ automated two-loop mass corrections in generic (also non-SUSY) models

↪ Corrections to the Higgs mass do not always converge.



Requiring $|(m_\phi^2)_{2L} - (m_\phi^2)_{1L}| < |(m_\phi^2)_{\text{Tree}} - (m_\phi^2)_{1L}|$ leads to non-trivial constraints on studied models: THDM and Georgi-Machacek model

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Can we define a set of loop-level but still easily-accessible rules for deciding/indicating whether a parameter point is perturbative or not?

Testing different conditions

- **$\overline{\text{MS}}$ scheme:** λ_i input, $m^2_{\phi_i} \to m^2_{\phi_i} + \Delta^{(1)} m^2_{\phi_i} + \Delta^{(2)} m^2_{\phi_i}$
- **On-Shell scheme:** masses input, $\lambda_i \to \lambda_i + \delta\lambda_i$

What can we do?

- promote the tree-level rules to the counterterms/renormalized couplings
- set up rules for the size of the loop corrections to parameters/masses

$$
\begin{aligned}
&(1) \qquad |\delta\lambda_i| < c \cdot \pi\,, \quad c = 1 \ldots 4 \\
&(2) \qquad |\delta\lambda_i| < |\lambda_i| \\
&(3) \qquad |\mathcal{M}(\lambda_i \to \lambda_i + \delta\lambda_i)| < 8\,\pi \\
&(4) \qquad |\Delta^{(2)} m^2_{\phi_i}| < |\Delta^{(1)} m^2_{\phi_i}|
\end{aligned}
$$

Applying the constraints: the Georgi-Machacek model

Triplets contribute to sizeably to EWSB, doubly-charged scalars

$$\eta = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta^0 & -\sqrt{2}\,(\eta^-)^* \\ -\sqrt{2}\,\eta^- & -\eta^0 \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^- & \sqrt{2}(\chi^0)^* \\ -\sqrt{2}\,\chi^{--} & -\chi^- \end{pmatrix}$$

$\Big((\eta^0)^* = \eta^0\Big)$; Assumption: Custodial symmetry

$$\langle\eta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\eta & 0 \\ 0 & -v_\eta \end{pmatrix}, \quad \langle\chi\rangle = \begin{pmatrix} 0 & v_\chi \\ 0 & 0 \end{pmatrix}, \quad v_\eta = v_\chi$$

$\Rightarrow \rho = 1$ at tree level

$$s_H \equiv \sin\theta_H = 2\sqrt{2}\,\frac{v_\chi}{v}$$

Define:

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \chi^{0*} & \eta^+ & \chi^{++} \\ \chi^- & \eta^0 & \chi^+ \\ \chi^{--} & \eta^- & \chi^0 \end{pmatrix}$$

The Georgi-Machacek model

Potential with custodial symmetry:

$$
\begin{aligned}
V(\Phi,\Delta) = &\frac{\mu_2^2}{2}\mathrm{Tr}\Phi^\dagger\Phi + \frac{\mu_3^2}{2}\mathrm{Tr}\Delta^\dagger\Delta + \lambda_1\left[\mathrm{Tr}\Phi^\dagger\Phi\right]^2 + \lambda_2\mathrm{Tr}\Phi^\dagger\Phi\,\mathrm{Tr}\Delta^\dagger\Delta \\
&+ \lambda_3\mathrm{Tr}\Delta^\dagger\Delta\Delta^\dagger\Delta + \lambda_4\left[\mathrm{Tr}\Delta^\dagger\Delta\right]^2 - \lambda_5\mathrm{Tr}\left(\Phi^\dagger\sigma^a\Phi\sigma^b\right)\mathrm{Tr}\left(\Delta^\dagger t^a\Delta t^b\right) \\
&- M_1\mathrm{Tr}\left(\Phi^\dagger\tau^a\Phi\tau^b\right)(U\Delta U^\dagger)_{ab} - M_2\mathrm{Tr}\left(\Delta^\dagger t^a\Delta t^b\right)(U\Delta U^\dagger)_{ab}
\end{aligned}
$$

Potential without custodial symmetry:

$$
\begin{aligned}
V =&\mu_2^2(\phi^\dagger\phi) + \mu_\chi^2\mathrm{Tr}(\chi^\dagger\chi) + \frac{1}{2}\mu_\eta^2\mathrm{Tr}(\eta^\dagger\eta) + 4\lambda_1(\phi^\dagger\phi)^2 + 2(\phi^\dagger\phi)\left(2\lambda_{2a}\mathrm{Tr}(\chi^\dagger\chi) + \lambda_{2b}\mathrm{Tr}(\eta^\dagger\eta)\right) \\
&+ 2\lambda_{3a}\mathrm{Tr}\left((\eta^\dagger\eta)^2\right) + 2\lambda_{3b}\left(\mathrm{Tr}\left((\chi^\dagger\chi)^2\right) + 3\mathrm{Tr}(\chi^\dagger\chi\chi\chi^\dagger)\right) + 4\lambda_{3c}\mathrm{Tr}(\chi^\dagger\chi\eta^\dagger\eta + \chi^\dagger\eta^\dagger\chi\eta) + \\
&+ \lambda_{4a}\left(\mathrm{Tr}(\eta^\dagger\eta)\right)^2 + 4\lambda_{4b}\left(\mathrm{Tr}(\chi^\dagger\chi)\right)^2 + 4\lambda_{4c}\mathrm{Tr}(\eta^\dagger\eta)\mathrm{Tr}(\chi^\dagger\chi) - \lambda_{5a}(\phi^\dagger\chi^\dagger\chi\phi - \phi^\dagger\chi\chi^\dagger\phi) \\
&+ \sqrt{2}\lambda_{5b}(\phi^\dagger\eta\chi^\dagger\phi^c + \mathrm{h.c.}) + \frac{1}{\sqrt{2}}M_{1a}\phi^\dagger\eta\phi + \frac{1}{2}M_{1b}\left(\phi^T(i\tau_2)\chi\phi + \mathrm{h.c.}\right) + 3\sqrt{2}M_2\left(\mathrm{Tr}(\chi^\dagger\chi\eta) + \mathrm{h.c.}\right)
\end{aligned}
$$

The Georgi-Machacek model

Mass spectrum (at tree level):

- 3 CP-even neutral scalars. Masses m_h, m_H, m_5
- 1 physical CP-odd neutral scalar. Mass m_3
- 2 physical singly-charged scalars. Masses m_3 and m_5
- 1 doubly-charged scalar. Mass m_5

Input parametrization:

$$m_h,\ m_H,\ \alpha,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ s_H$$

- λ_1, M_1, M_2 are derived from tree-level relations
- μ_2^2 and μ_3^2 are derived from tadpole equations

At one-loop

not enough parameters in custodial case to renormalise scalar sector on-shell!

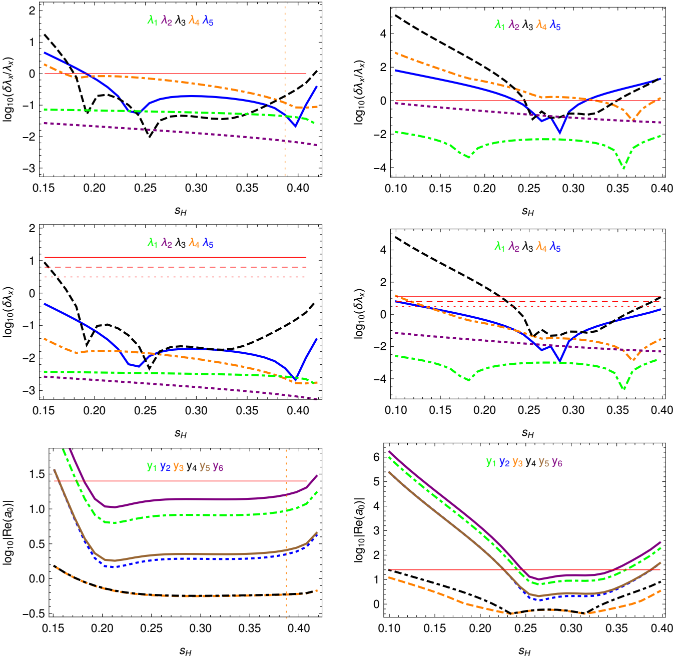
$$x \to x + \delta_x$$

$$x \in \{\lambda_1, \lambda_{2a}, \lambda_{2b}, \lambda_{3a}, \lambda_{3b}, \lambda_{3c}, \lambda_{4a}, \lambda_{4b}, \lambda_{4c}, \lambda_{5a}, \lambda_{5b}, M_{1a}, M_{1b}, M_2, \mu_2^2, m_\eta^2, m_\chi^2\}$$

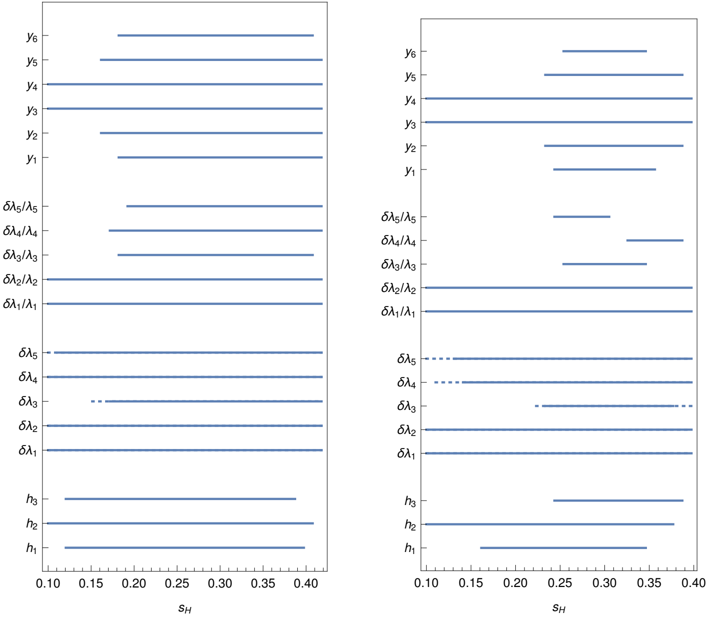
For instance, e.g. $\delta\lambda_{3b} - \delta\lambda_{3c} \simeq \frac{g_1^2}{16\pi^2}\frac{m_5^2}{s_H^2 v^2}$ for $m_5 \gg v$ and s_H small

$\lambda_2 = 0.1,\ \lambda_3 = 0.5,\ \lambda_4 = -0.02,\ \lambda_5 = 0.1,\ \alpha = 0.35$

$m_H = 300\,\text{GeV};\ \lambda_1 < 0.1$ $\qquad$ $m_H = 800\,\text{GeV};\ \lambda_1 \sim 0.2$



Comparison



y_6
y_5
y_4
y_3
y_2
y_1
$\delta\lambda_5/\lambda_5$
$\delta\lambda_4/\lambda_4$
$\delta\lambda_3/\lambda_3$
$\delta\lambda_2/\lambda_2$
$\delta\lambda_1/\lambda_1$
$\delta\lambda_5$
$\delta\lambda_4$
$\delta\lambda_3$
$\delta\lambda_2$
$\delta\lambda_1$
h_3
h_2
h_1
0.10 0.15 0.20 0.25 0.30 0.35 0.40
s_H
y_6
y_5
y_4
y_3
y_2
y_1
$\delta\lambda_5/\lambda_5$
$\delta\lambda_4/\lambda_4$
$\delta\lambda_3/\lambda_3$
$\delta\lambda_2/\lambda_2$
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$\delta\lambda_5$
$\delta\lambda_4$
$\delta\lambda_3$
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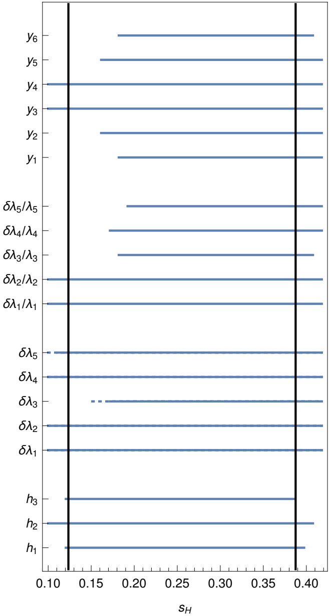
$\delta\lambda_1$
h_3
h_2
h_1
0.10 0.15 0.20 0.25 0.30 0.35 0.40
s_H

Comparison



y_6
y_5
y_4
y_3
y_2
y_1
$\delta\lambda_5/\lambda_5$
$\delta\lambda_4/\lambda_4$
$\delta\lambda_3/\lambda_3$
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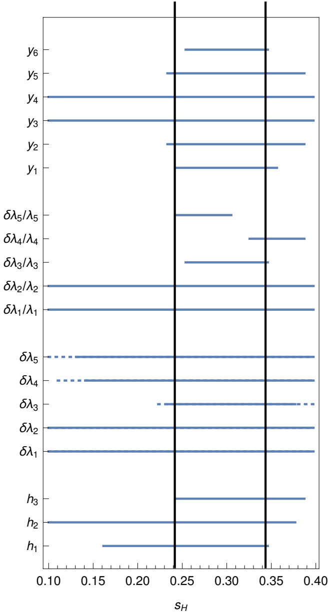
h_3
h_2
h_1
0.10 0.15 0.20 0.25 0.30 0.35 0.40
s_H



y_6
y_5
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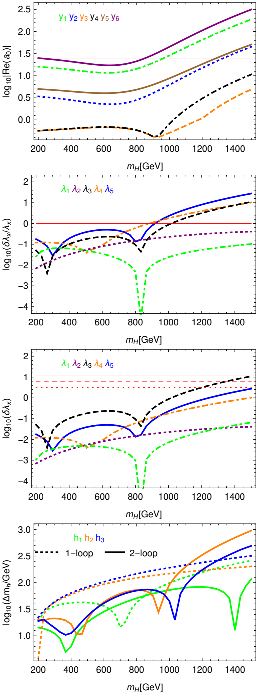
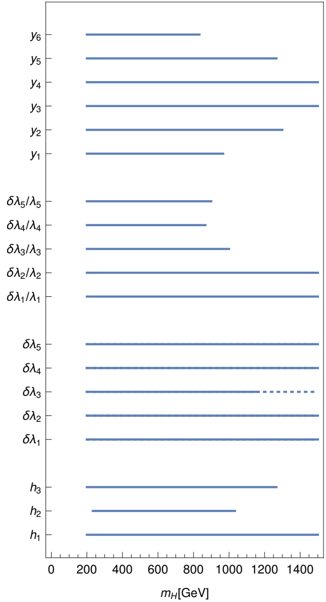
$\delta\lambda_5/\lambda_5$
$\delta\lambda_4/\lambda_4$
$\delta\lambda_3/\lambda_3$
$\delta\lambda_2/\lambda_2$
$\delta\lambda_1/\lambda_1$
$\delta\lambda_5$
$\delta\lambda_4$
$\delta\lambda_3$
$\delta\lambda_2$
$\delta\lambda_1$
h_3
h_2
h_1
0.10 0.15 0.20 0.25 0.30 0.35 0.40
s_H

$\lambda_2 = 0.1,\ \lambda_3 = 1,\ \lambda_4 = -0.1,\ \lambda_5 = 0.1,\ \alpha = 0.35,\ s_H = 0.25$

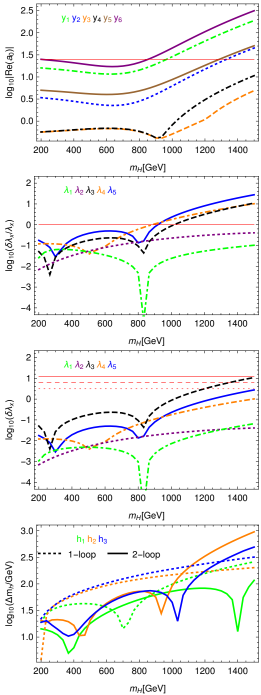


Dependence on heavy scalar mass:

$$\Delta m_5^2 \sim m_5^2 \left(1 + C\,\frac{m_5^2}{v^2}\right)$$

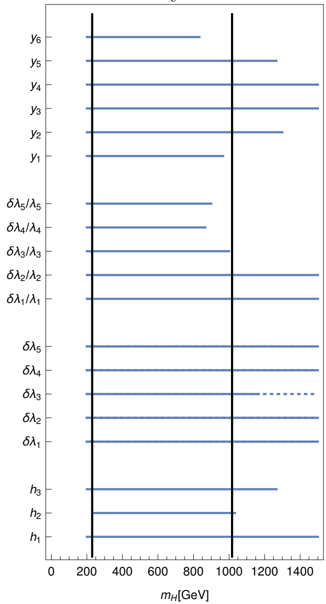


$\lambda_2 = 0.1,\ \lambda_3 = 1,\ \lambda_4 = -0.1,\ \lambda_5 = 0.1,\ \alpha = 0.35,\ s_H = 0.25$

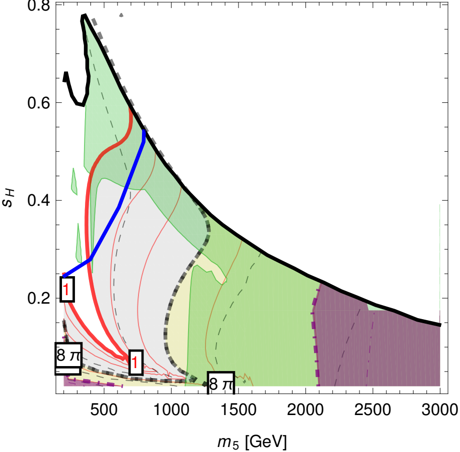


Dependence on heavy scalar mass:

$$\Delta m_5^2 \sim m_5^2 (1 + C \frac{m_5^2}{v^2})$$



"H5plane" e.g. 1610.07922, 1709.01883



$$m_h = 125 \text{ GeV}$$
$$\lambda_3 = -0.1$$
$$\lambda_4 = 0.2$$
$$\lambda_2 = 0.4 \frac{m_5}{1 \text{ TeV}}$$
$$M_1 = \frac{\sqrt{2}\, s_H}{v}(m_5^2 + v^2)$$
$$M_2 = \frac{M_1}{6}$$

red contours: $\text{Max}(|\delta\lambda_x/\lambda_x|) = 1,\, 2,\, 4,\, 16,\, 64$

black dashed: scattering eigenvalues of $4\pi,\, 8\pi,\, 16\pi,\, 64\pi$

green: $\overline{\text{MS}}$ constraints from $|(m^2_{\phi_i})_{2\text{L}} - (m^2_{\phi_i})_{1\text{L}}| \geq |(m^2_{\phi_i})_{\text{Tree}} - (m^2_{\phi_i})_{1\text{L}}|$

purple: $\text{Max}(|\delta\lambda_x|) = 2\pi$ and 4π

Conclusions

Promoting non-SUSY models to higher loop orders:

- perturbative series can break down well before naive tree-level estimates
- set of loop-level perturbativity checks to improve on tree-level rules
- $\overline{\text{MS}}$ condition yields similar constraints as using $|\delta\lambda_i|$ or $|\mathcal{M}(\lambda_i \to \lambda_i + \delta\lambda_i)|$

Caution:

Neither of the defined rules should be considered a strict bound on a model – but they should help indicate when problems with the perturbative expansion are expected

Thank You

INPUT PARAMETRIZATION

$$M_1 = \frac{3 s_H \sqrt{2 - 2 s_H^2}\left(t_\alpha^2 + 1\right) v^2 (2\lambda_2 - \lambda_5) - 2\sqrt{3} m_h^2 t_\alpha + 2\sqrt{3} m_H^2 t_\alpha}{3\sqrt{1 - s_H^2}\left(t_\alpha^2 + 1\right) v}\,,$$

$$\begin{aligned} M_2 = & \frac{1}{9 s_H^2 \sqrt{1 - s_H^2}\left(t_\alpha^2 + 1\right) v^2} \times \Big[v \Big(m_h^2 t_\alpha \left(-3\sqrt{2 - 2 s_H^2}\, s_H t_\alpha + 2\sqrt{3} s_H^2 - 2\sqrt{3} \right)\,, \\ & - 3 s_H \sqrt{2 - 2 s_H^2}\left(t_\alpha^2 + 1\right) v^2 \left(2\lambda_2 \left(s_H^2 - 1\right) + s_H^2 (-(\lambda_3 + 3\lambda_4)) - \lambda_5 s_H^2 + \lambda_5 \right) \Big) \\ & + m_H^2 v \left(-2\sqrt{3} s_H^2 t_\alpha - 3\sqrt{2 - 2 s_H^2}\, s_H + 2\sqrt{3} t_\alpha \right) \Big]\,, \end{aligned}$$

$$\lambda_1 = -\frac{m_h^2 + m_H^2 t_\alpha^2}{8\left(s_H^2 - 1\right)\left(t_\alpha^2 + 1\right) v^2}\,,$$

where $t_\alpha = \tan\alpha$. The full list of input parameters for this choice is

$$m_h,\ m_H,\ \alpha,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ s_H\,.$$

Testing the perturbativity

$x \to x + \delta_x$

$x \in \{\lambda_1, \lambda_{2a}, \lambda_{2b}, \lambda_{3a}, \lambda_{3b}, \lambda_{3c}, \lambda_{4a}, \lambda_{4b}, \lambda_{4c}, \lambda_{5a}, \lambda_{5b}, M_{1a}, M_{1b}, M_2, \mu_2^2, m_\eta^2, m_\chi^2\}$

Tree-level unitarity conditions: Eigenvalues of scattering matrix

$$\pm 8(2\lambda_{3b} + \lambda_{4b}),$$
$$\pm 4(2\lambda_{4b} + \lambda_{3b}),$$
$$\pm 2[2\lambda_{3a} + 2(\lambda_{4a} + \lambda_{4b}) + 3\lambda_{3b} \pm \sqrt{(2\lambda_{3a} - 3\lambda_{3b} + 2\lambda_{4a} - 2\lambda_{4b})^2 + 8\lambda_{3c}^2}],$$
$$\pm 8(2\lambda_{3c} + \lambda_{4c}),$$
$$\pm 4(2\lambda_{4c} + \lambda_{3c}),$$
$$\pm(4\lambda_{2a} - \lambda_{5a}),$$
$$\pm 2(2\lambda_{2a} + \lambda_{5a}),$$
$$\pm 2[2\lambda_1 + 2\lambda_{4b} - \lambda_{3b} \pm \sqrt{(2\lambda_1 + \lambda_{3b} - 2\lambda_{4b})^2 + \lambda_{5a}^2}],$$
$$\pm 2[2\lambda_1 + 2\lambda_{4c} - \lambda_{3c} \pm \sqrt{(2\lambda_1 + \lambda_{3c} - 2\lambda_{4c})^2 + \lambda_{5b}^2}],$$
$$\pm\frac{1}{2}[-4\lambda_{2a} - 4\lambda_{2b} - \lambda_{5a} \pm \sqrt{(4\lambda_{2a} - 4\lambda_{2b} + \lambda_{5a})^2 + 8\lambda_{5b}^2}],$$
$$\pm[2\lambda_{2a} + 2\lambda_{2b} - \lambda_{5a} \pm \sqrt{(-2\lambda_{2a} + 2\lambda_{2b} + \lambda_{5a})^2 + 8\lambda_{5b}^2}].$$

+ 3 more complicated eigenvalues

Vacuum Stability

Criterion for exclusion: Potential is unbounded from below.

Custodial case:

$$\lambda_1 > 0\,,$$

$$\lambda_4 > \begin{cases} -\frac{1}{3}\lambda_3\,, & \lambda_3 \geq 0\,, \\ -\lambda_3\,, & \lambda_3 < 0\,, \end{cases}$$

$$\lambda_2 > \begin{cases} \frac{1}{2}\lambda_5 - 2\sqrt{\lambda_1\left(\frac{1}{3}\lambda_3 + \lambda_4\right)}\,, & \lambda_5 \geq 0 \wedge \lambda_3 \geq 0\,, \\ \omega_+(\zeta)\lambda_5 - 2\sqrt{\lambda_1(\zeta\lambda_3 + \lambda_4)}\,, & \lambda_5 \geq 0 \wedge \lambda_3 < 0\,, \\ \omega_-(\zeta)\lambda_5 - 2\sqrt{\lambda_1(\zeta\lambda_3 + \lambda_4)}\,, & \lambda_5 < 0\,, \end{cases}$$

$$\omega_\pm(\zeta) = \frac{1}{6}(1 - B(\zeta)) \pm \frac{\sqrt{2}}{3}\left[(1 - B(\zeta))\left(\frac{1}{2} + B(\zeta)\right)\right]^{1/2}\,,$$

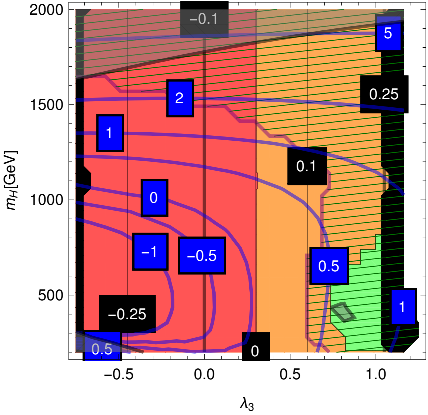
$$B(\zeta) = \sqrt{\frac{3}{2}\left(\zeta - \frac{1}{3}\right)}\,.$$

Broken custodial symmetry:

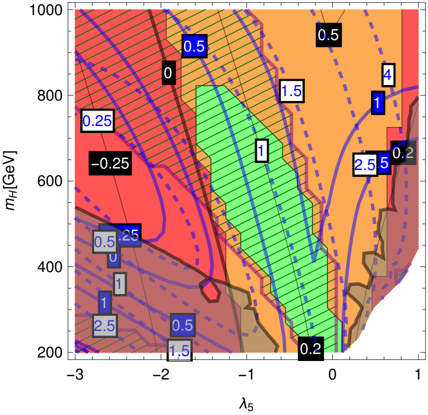
$$\begin{aligned}
\lambda_1 &> 0\,, \\
\lambda_{3a} + \lambda_{4a} &> 0\,, \\
\lambda_{3b} + 2\lambda_{4b} &> 0\,, \\
\lambda_{3b} + \lambda_{4b} &> 0\,, \\
\lambda_{3c} + \sqrt{2}\sqrt{(\lambda_{3a} + \lambda_{4a})(\lambda_{3b} + 2\lambda_{4b})} + 2\lambda_{4c} &> 0\,, \\
\sqrt{2}\sqrt{(\lambda_{3a} + \lambda_{4a})(\lambda_{3b} + 2\lambda_{4b})} + 2\lambda_{4c} &> 0\,, \\
\sqrt{(\lambda_{3a} + \lambda_{4a})(\lambda_{3b} + \lambda_{4b})} + \lambda_{4c} &> 0\,, \\
\lambda_{3c} + \sqrt{(\lambda_{3a} + \lambda_{4a})(\lambda_{3b} + \lambda_{4b})} + \lambda_{4c} &> 0\,, \\
4\lambda_{2a} + 4\sqrt{2}\sqrt{\lambda_1(\lambda_{3b} + 2\lambda_{4b})} - \lambda_{5a} &> 0\,, \\
4\lambda_{2a} + 4\sqrt{2}\sqrt{\lambda_1(\lambda_{3b} + 2\lambda_{4b})} + \lambda_{5a} &> 0\,, \\
\lambda_{2a} + 2\sqrt{\lambda_1(\lambda_{3b} + \lambda_{4b})} &> 0\,, \\
\lambda_{2b} + 2\sqrt{\lambda_1(\lambda_{3a} + \lambda_{4a})} &> 0\,, \\
\lambda_{3a} + 2\lambda_{3b} + \lambda_{4a} + 4\lambda_{4b} + 4\lambda_{4c} &> 0\,, \\
2\lambda_{2a} + \lambda_{2b} + 2\sqrt{\lambda_1(\lambda_{3a} + 2\lambda_{3b} + \lambda_{4a} + 4(\lambda_{4b} + \lambda_{4c}))} & \\
-(\lambda_{5a}/2) - \lambda_{5b} &> 0\,.
\end{aligned}$$

Vacuum Stability

$\lambda_2 = -\lambda_4 = \lambda_5 = 0.1,\ \alpha = 0.26,\ s_H = 0.23$



$\lambda_2 = -\lambda_4 = 0.1,\ \lambda_3 = 0.5\,\alpha = 0.35,\ s_H = 0.33$



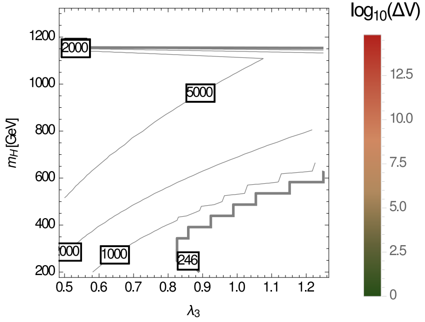
black contours: Min(TL UFB), blue: Min(UFB, $\lambda_i + \delta\lambda_i$)

green area: stable at tree-level, green hatched: stable at one-loop

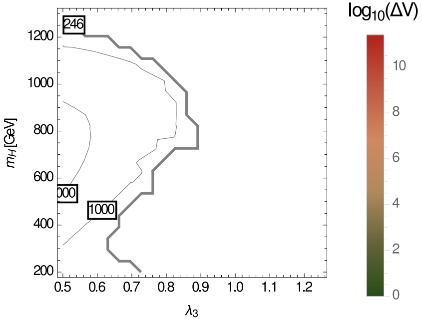
Vacuum Stability

$$\lambda_2 = -\lambda_4 = \lambda_5 = 0.1,\ \alpha = 0.35,\ s_H = 0.33$$

Tree-level potential:



One-loop potential:



Contour lines: $\sqrt{v_\phi^2 + 4(v_\eta^2 + v_\chi^2)}$