$gg \to hh$ in the high energy limit

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work in progress

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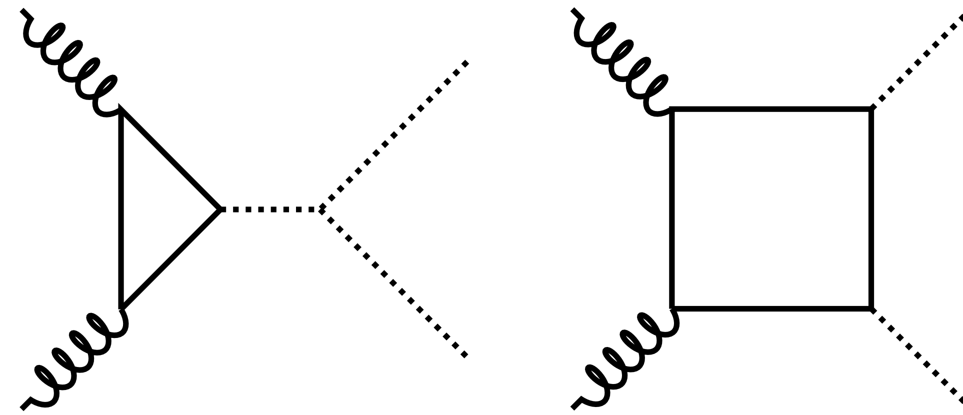
Institute for Theoretical Particle Physics

$gg \to hh$: previous works

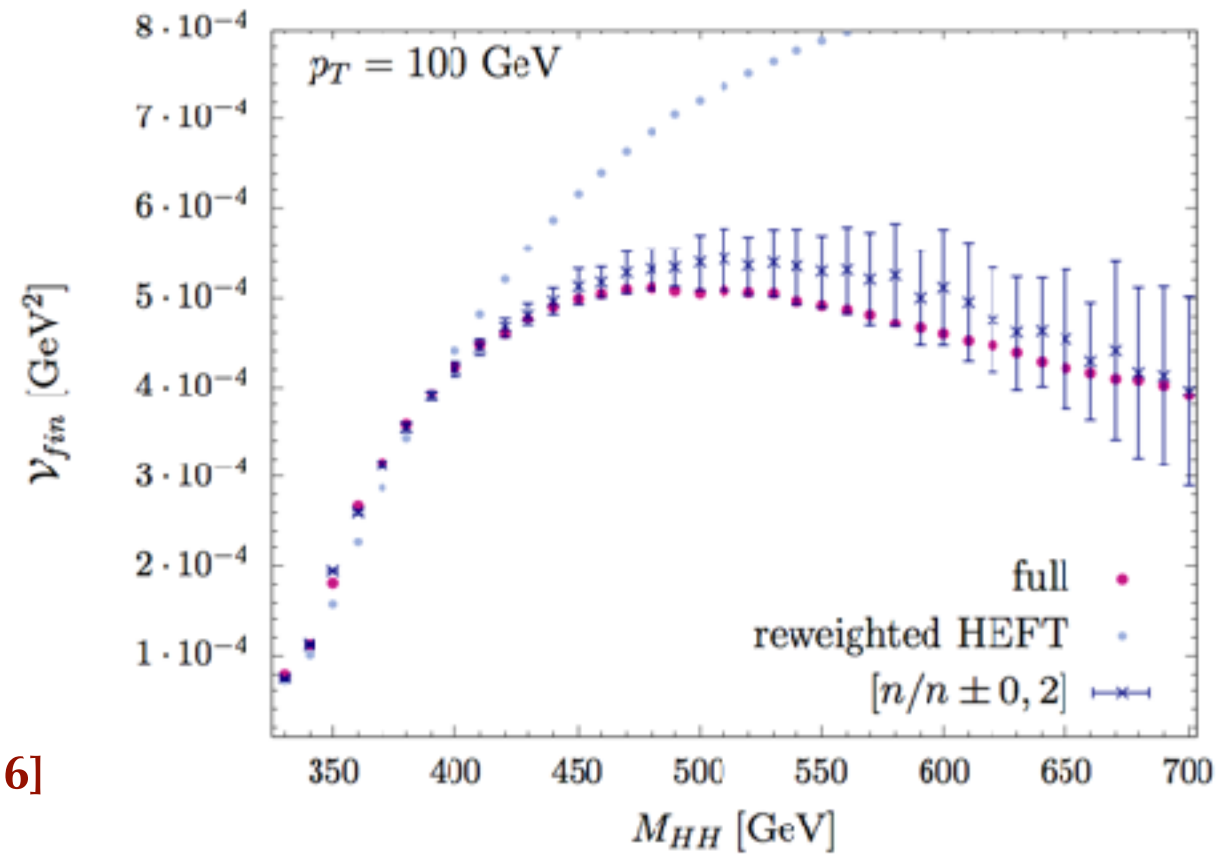


exact analytic@LO
[Eboli, Marques, Novaes, Natale, '87, Glover, van der Bij '88]

Born-improved HEFT@NLO
[Dawson, Dittmaier Spira, '98]

$\mathrm{FT}_{\mathrm{approx}}$, $\mathrm{FT}'_{\mathrm{approx}}$
[Maltoni, Vryonidou, Zaro, '14]

HEFT@NNLO with 1/mt corr.
[Grigo, Hoff, Melnikov, Steinhauser, '13,
Grigo, Melnikov, Steinhauser, '14,
Grigo, Hoff, Steinhauser, '15, Degrassi, Giardino, Gröber, '16]



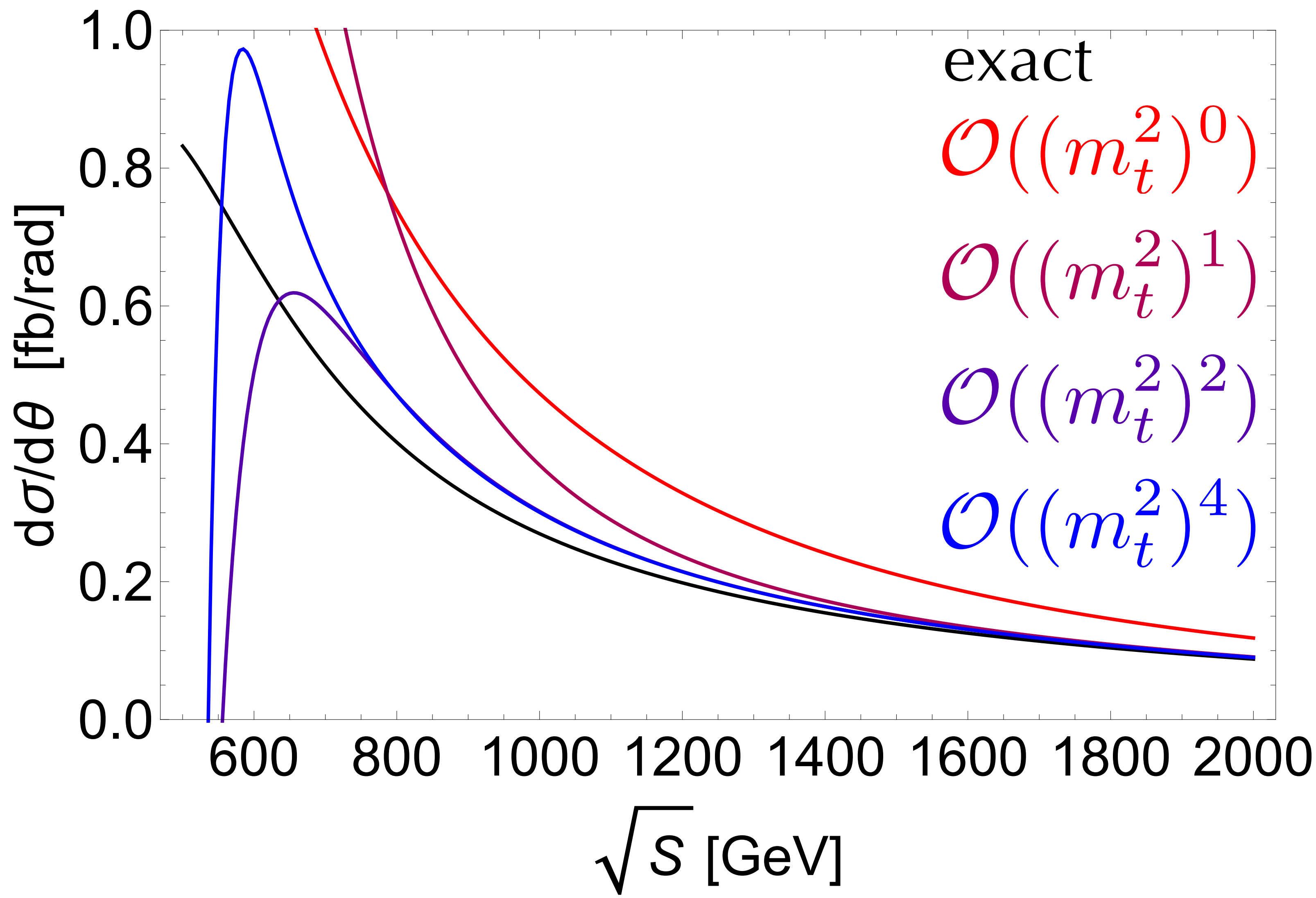
exact numerical@NLO (14TeV, 100TeV)
[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zicke, '16]

Padé approximation using the large top-mass and the threshold expansion@NLO
[Gröber, Maier Rauh, '17]

We supplement these works by providing the high energy expansion@NLO.

$gg \to hh$: our aim is to obtain the high energy expansion@NLO.

LO $@T = -S/2$, $(\theta = \pi/2,\ p_T = \sqrt{S}/2)$



Outline

(1) Introduction

(2) high energy expansion of Feynman integral

(3) calculation of the two-loop gg->hh amplitude (reduction)

(4) analytic result of the two-loop massive double box diagram

in the high energy limit (preliminary)

(5) summary and outlook

asymptotic expansion of Feynman integral

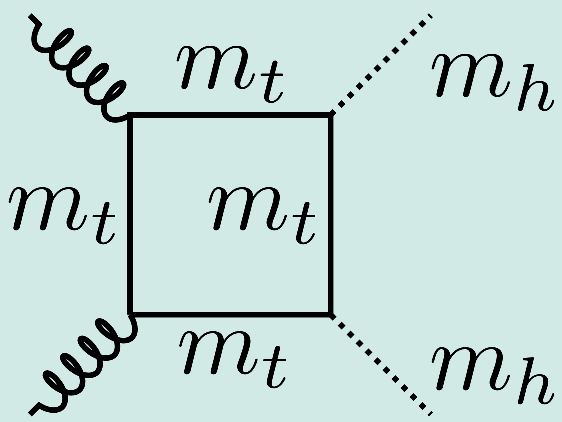
[Smirnov `90, Beneke, Smirnov '97, Smirnov `02, Jantzen `11]

is useful when (i) the integral is hard to solve due to multi-scale complexity
(ii) certain hierarchy in dimensionful parameters makes sense

In our case, we assume $m_h^2 < m_t^2 \ll |S| \sim |T| \sim |U|$

Then,
(1) Expansion in m_h is the Taylor expansion.

$$I(m_h^2) = I(0) + m_h^2 I'(0) + \cdots$$

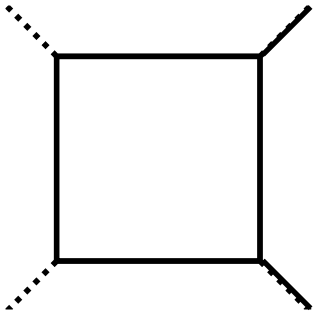
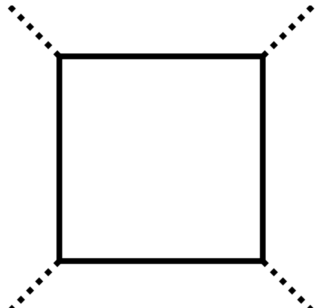
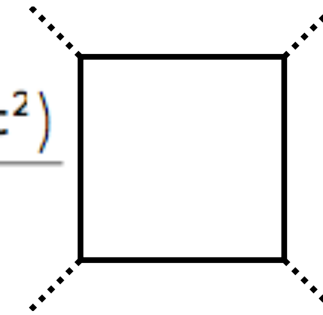


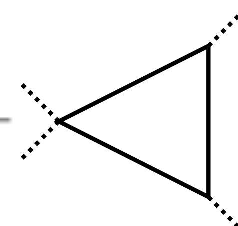
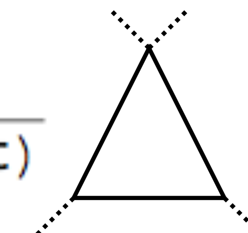
(2) Expansion in m_t is **not** the Taylor expansion.

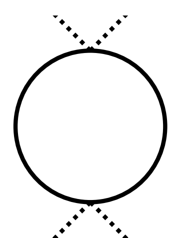
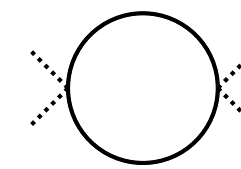
$$I(m_t) = \sum_n (m_t)^n f_n(S, T, \log m_t)$$

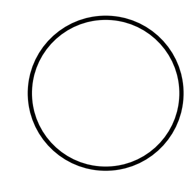
We use "asy.m" to perform the asymptotic expansion.
[Pak, Smirnov '10, Jantzen, Smirnov, Smirnov '12]

Expansion in m_h (Taylor expansion)

 $=$  $+m_h^2 \Bigg(\frac{s\,(4\,mts\,s + 4\,mts\,t - 6\,s\,t + d\,s\,t - 10\,t^2 + 2\,d\,t^2)}{t\,(s+t)\,(-4\,mts\,s - 4\,mts\,t + s\,t)}$ 

$+ \frac{2\,(-4+d)\,s\,(s+2\,t)}{t\,(s+t)\,(-4\,mts\,s - 4\,mts\,t + s\,t)}$  $+ \frac{2\,(-4+d)\,(s+2\,t)}{(s+t)\,(-4\,mts\,s - 4\,mts\,t + s\,t)}$ 

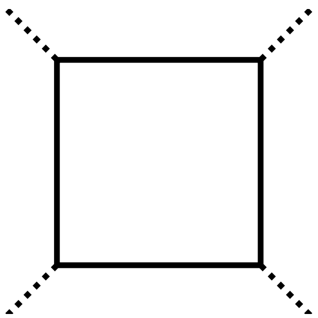
$- \frac{4\,(-3+d)\,(s+2\,t)}{s\,t\,(-4\,mts\,s - 4\,mts\,t + s\,t)}$  $+ \frac{8\,(-3+d)\,(2\,mts - t)}{(4\,mts - t)\,t\,(4\,mts\,s + 4\,mts\,t - s\,t)}$ 

$+ \frac{(-2+d)\,(-48\,mts^2\,s + 16\,d\,mts^2\,s - 48\,mts^2\,t + 16\,d\,mts^2\,t + 26\,mts\,s\,t - 8\,d\,mts\,s\,t + 12\,mts\,t^2 - 4\,d\,mts\,t^2 - 4\,s\,t^2 + d\,s\,t^2)}{mts^2\,s\,(4\,mts - t)\,t\,(4\,mts\,s + 4\,mts\,t - s\,t)}$ 

$+\mathcal{O}(m_h^4) \Bigg)$

The massive-higgs diagram can be expressed as an infinite sum of the massless-higgs diagrams.

Expansion in m_t



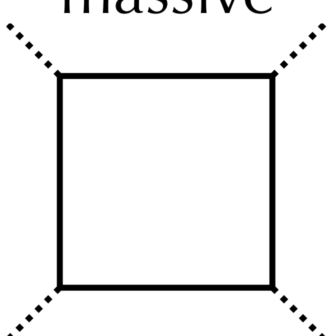
$$= \int Dk \frac{1}{k^2 - m_t^2} \frac{1}{(k+p_1)^2 - m_t^2} \frac{1}{(k+p_1+p_2)^2 - m_t^2} \frac{1}{(k+p_3)^2 - m_t^2}$$

$$= \sum_{n=0}^{\infty} (m_t^2)^n f_n(S, T, \log m_t)$$

Naive expansion of the integrand like

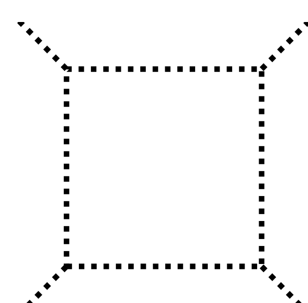
$$\frac{1}{k^2 - m_t^2} = \frac{1}{k^2} + \frac{m_t^2}{(k^2)^2} + \cdots$$ gives wrong result.

massive



is finite.

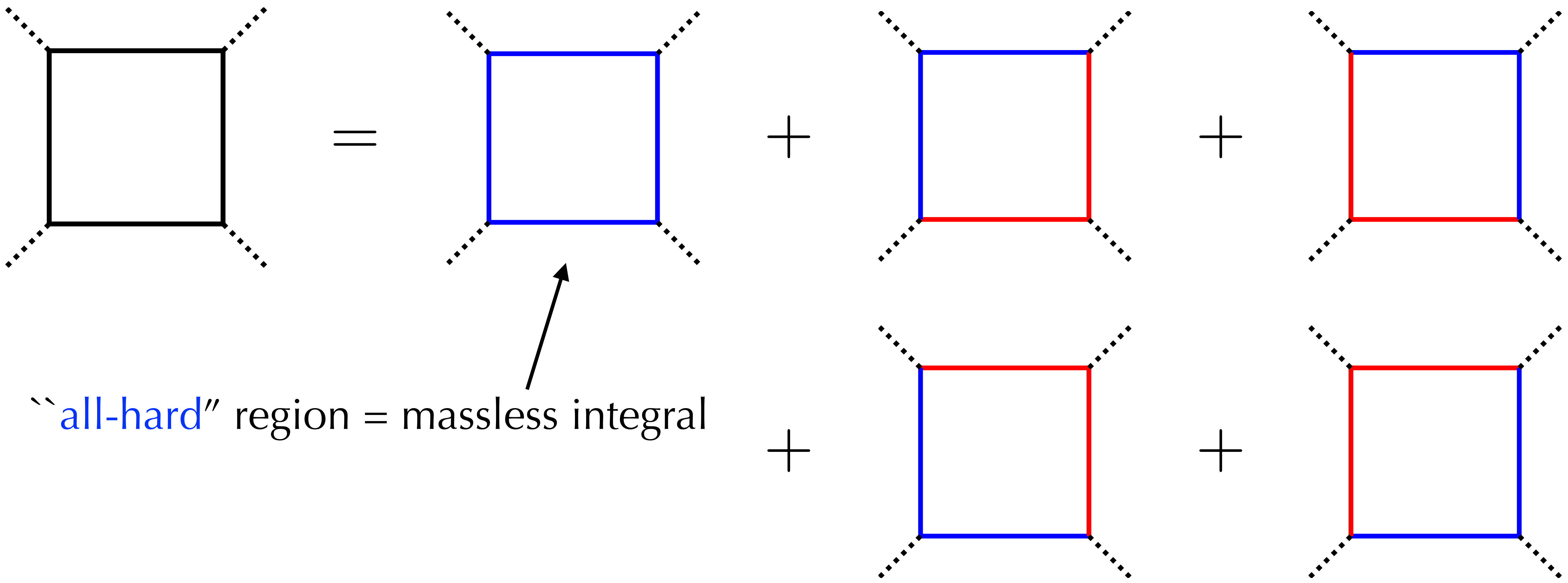
massless



$$= \frac{1}{st}\left(\frac{4}{\varepsilon^2} - \frac{2\log st}{\varepsilon} + 2\log s \log t - \frac{4\pi^2}{3}\right) + \mathcal{O}(\epsilon)$$

Expansion by region [Beneke, Smirnov '97, Smirnov `02, Jantzen `11]

blue: hard-scaling propagator
red: soft-scaling propagators



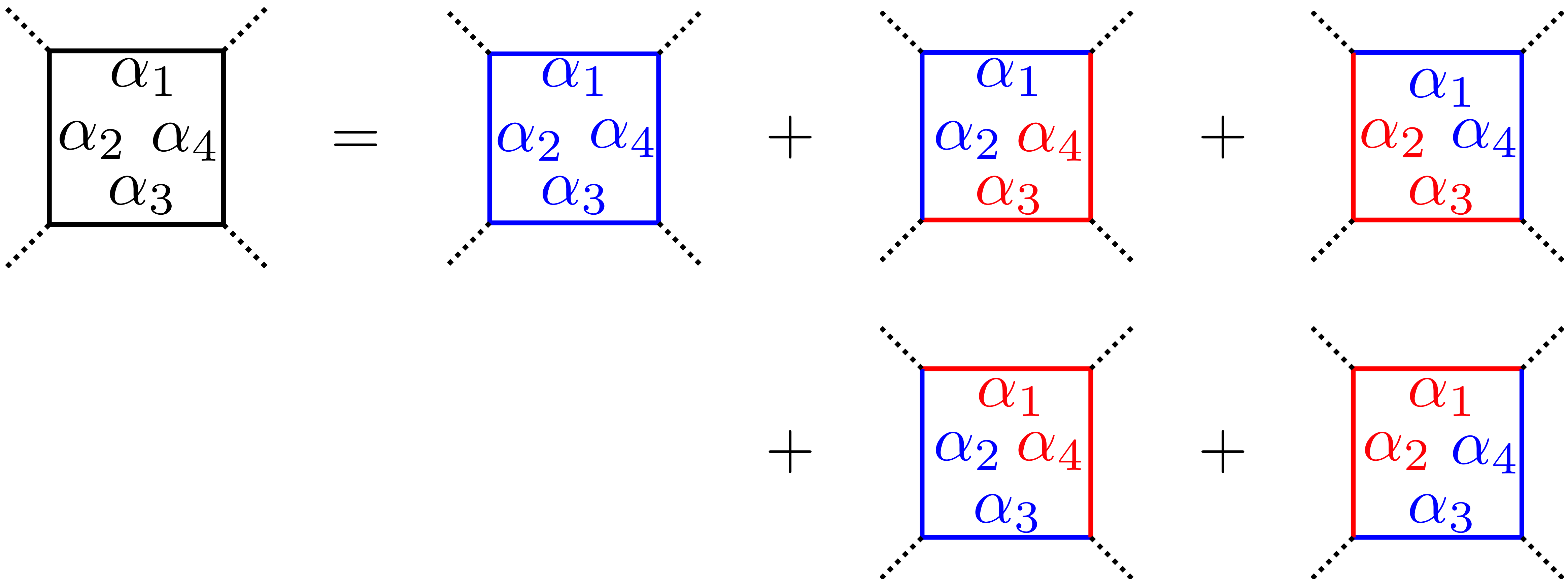
the scaling of propagators in terms of alpha-parameter representation

$$\int_0^\infty \left(\prod_{n=1}^{4} d\alpha_n \right) \alpha_{1234}^{-d/2} \; \mathrm{e}^{-m^2 \alpha_{1234} - (s\alpha_1\alpha_3 + t\alpha_2\alpha_4)/\alpha_{1234}}$$

$$\alpha_{1234} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

Expansion by region [Beneke, Smirnov ’97, Smirnov `02, Jantzen `11]

blue: hard-scaling propagator
red: soft-scaling propagators



the scaling of propagators in terms of alpha-parameter representation

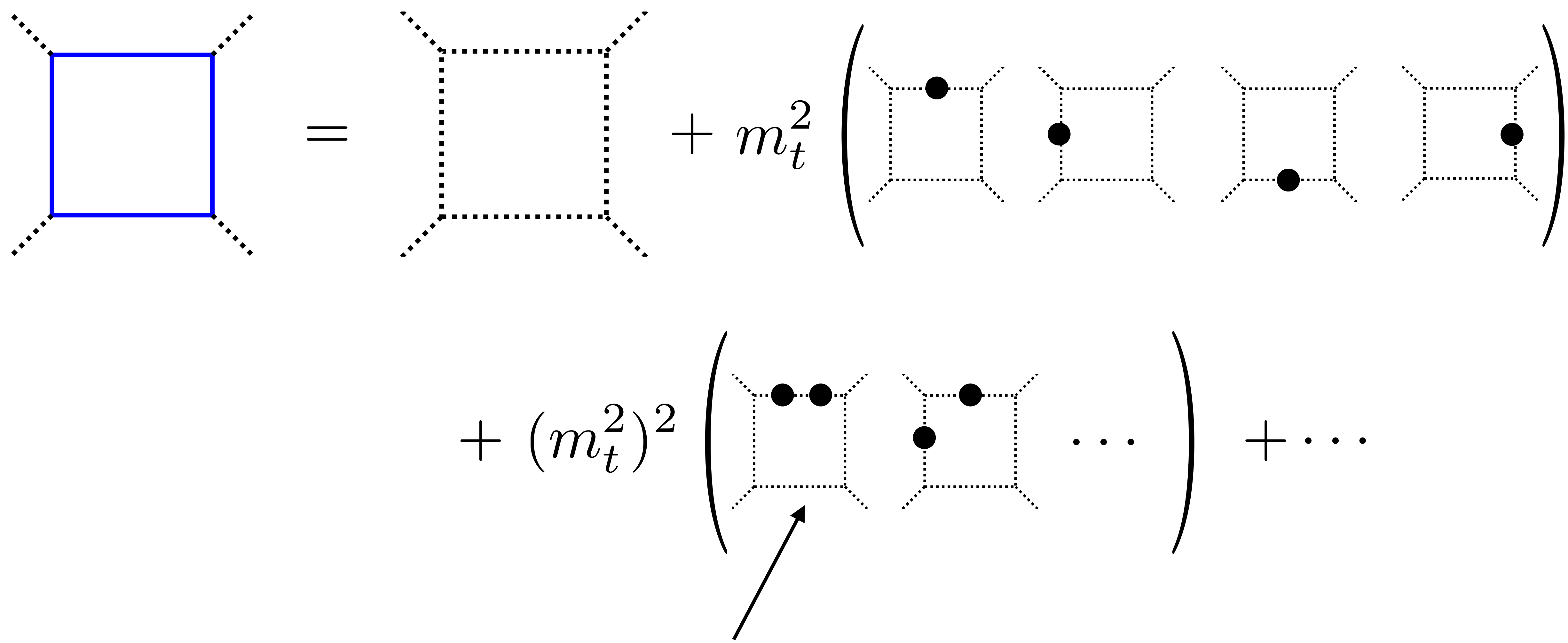
$$\int_0^\infty \left(\prod_{n=1}^{4} d\alpha_n \right) \alpha_{1234}^{-d/2} \, \mathrm{e}^{-m^2 \alpha_{1234} - (s\alpha_1\alpha_3 + t\alpha_2\alpha_4)/\alpha_{1234}}$$

$$\alpha_{1234} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

Expansion by region: ``all-hard" region

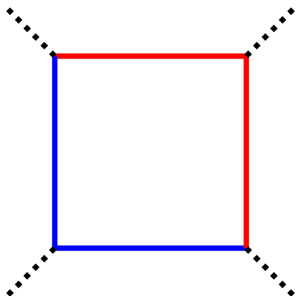
In our case, the expansion in this region corresponds to the naive Taylor expansion.

The right hand side consists of massless diagrams with dots.



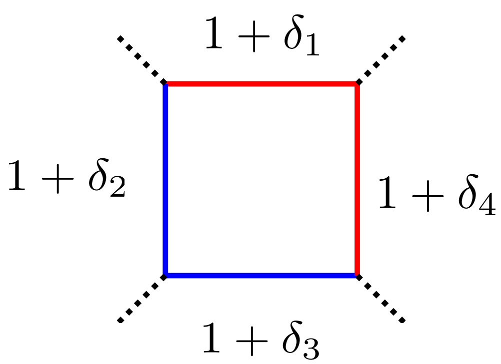
We can apply the integration by parts (IBP) reduction.

Expansion by region: soft regions



$$= \int_0^\infty \left(\prod_{n=1}^{4} d\alpha_n\right) \begin{array}{l} \alpha_{12}^{-d/2}\ \mathrm{e}^{-m^2\alpha_{12}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{12}} \\ -\,\alpha_{12}^{-d/2-2}(\alpha_3+\alpha_4)((d/2)\alpha_{12}+m^2(\alpha_{12})^2-s\alpha_1\alpha_3-t\alpha_2\alpha_4) \\ \times\,\mathrm{e}^{-m^2\alpha_{12}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{12}} \\ +\cdots \end{array}$$

Usual momentum representation is not always possible…



$$f_0^{(2)} = \frac{(m^2)^{-\varepsilon}}{st}\left[\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3}-\frac{1}{\delta_4}+\log st\right)\right]$$

$$f_0^{(3)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{1}{\delta_3}-\frac{1}{\delta_4}+\log t/m^2\right)+\frac{\pi^2}{12}\right]$$

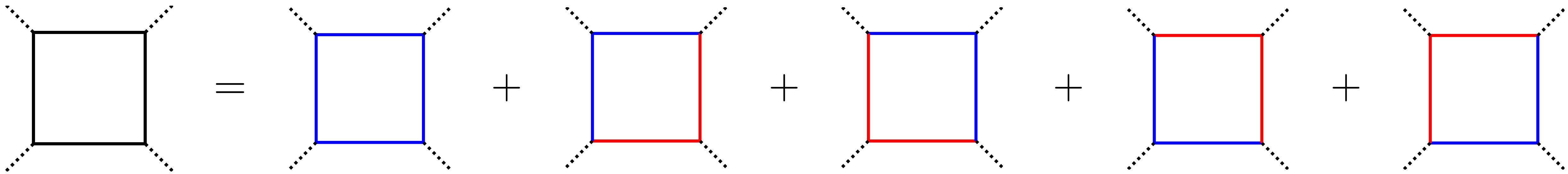
$$f_0^{(4)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3}+\frac{1}{\delta_4}+\log s/m^2\right)+\frac{\pi^2}{12}\right]$$

$$f_0^{(5)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{2}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{1}{\delta_3}+\frac{1}{\delta_4}-2\log m^2\right)+\frac{\pi^2}{6}\right]$$

The integrals are ill-defined, so we have to introduce analytic regularization of the exponent of propagators.

Cancellation of auxiliary parameters between soft regions occurs.

Expansion by region: total



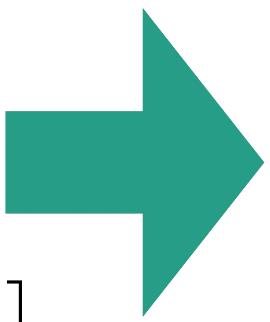
$$f_0^{(1)} = \frac{1}{st}\left(\frac{4}{\varepsilon^2} - \frac{2\log st}{\varepsilon} + 2\log s \log t - \frac{4\pi^2}{3}\right)$$

$$f_0^{(2)} = \frac{(m^2)^{-\varepsilon}}{st}\left[\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3} - \frac{1}{\delta_4} + \log st\right)\right]$$

$$f_0^{(3)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{1}{\delta_3} - \frac{1}{\delta_4} + \log t/m^2\right) + \frac{\pi^2}{12}\right]$$

$$f_0^{(4)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{1}{\delta_3} + \frac{1}{\delta_4} + \log s/m^2\right) + \frac{\pi^2}{12}\right]$$

$$f_0^{(5)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{1}{\delta_3} + \frac{1}{\delta_4} - 2\log m^2\right) + \frac{\pi^2}{6}\right]$$



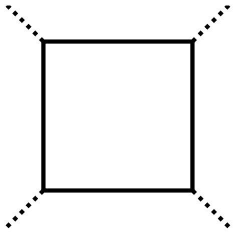
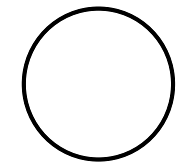
$$I = \sum_{n=0}^{\infty}(m^2)^n f_n$$

$$f_0 = \frac{1}{st}\left(2\log\frac{s}{m^2}\log\frac{t}{m^2} - \pi^2\right)$$

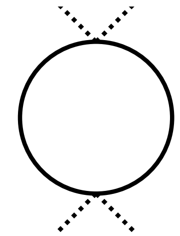
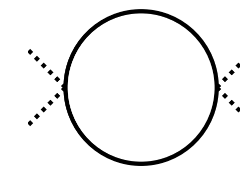
Cancellation of auxiliary parameters between soft regions occurs.

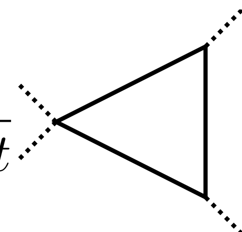
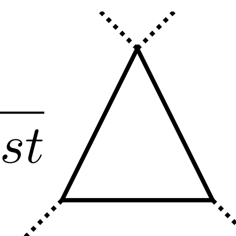
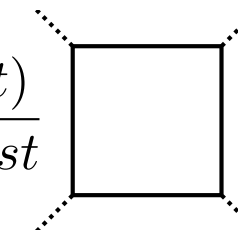
Expansion in m_t : using differential equation

[Kotikov '91]

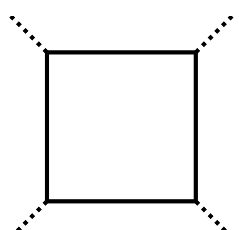
$$\frac{\partial}{\partial(m_t^2)}$$  $$= -\frac{2(d-2)\left(2m^2(s+t)+st\right)}{m^4\,(4m^2+s)\,(4m^2+t)\,(4m^2(s+t)+st)}$$ 

We used LiteRed [Lee '13] for obtaining the diff.-eq.

$$-\frac{2(d-3)t}{m^2\,(4m^2+t)\,(4m^2(s+t)+st)}$$  $$-\frac{2(d-3)s}{m^2\,(4m^2+s)\,(4m^2(s+t)+st)}$$ 

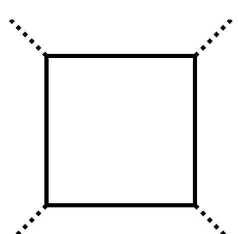
$$-\frac{(d-4)s}{4m^4(s+t)+m^2st}$$  $$-\frac{(d-4)t}{4m^4(s+t)+m^2st}$$  $$-\frac{2(d-5)(s+t)}{4m^2(s+t)+st}$$ 

Substituting the form,

 $$= \sum_{n_1,n_2} c_{n_1,n_2}(m_t^2)^{n_1}(\log m_t)^{n_2}$$

we obtain recursive relations of C_n's.

See also [Melnikov, Tancredi, Wever '16]

 $$= (m_t^2)^0 f_0 + (m_t^2)^1 f_1 + (m_t^2)^2 f_2 + \cdots$$

setup to calculate the two-loop amplitude

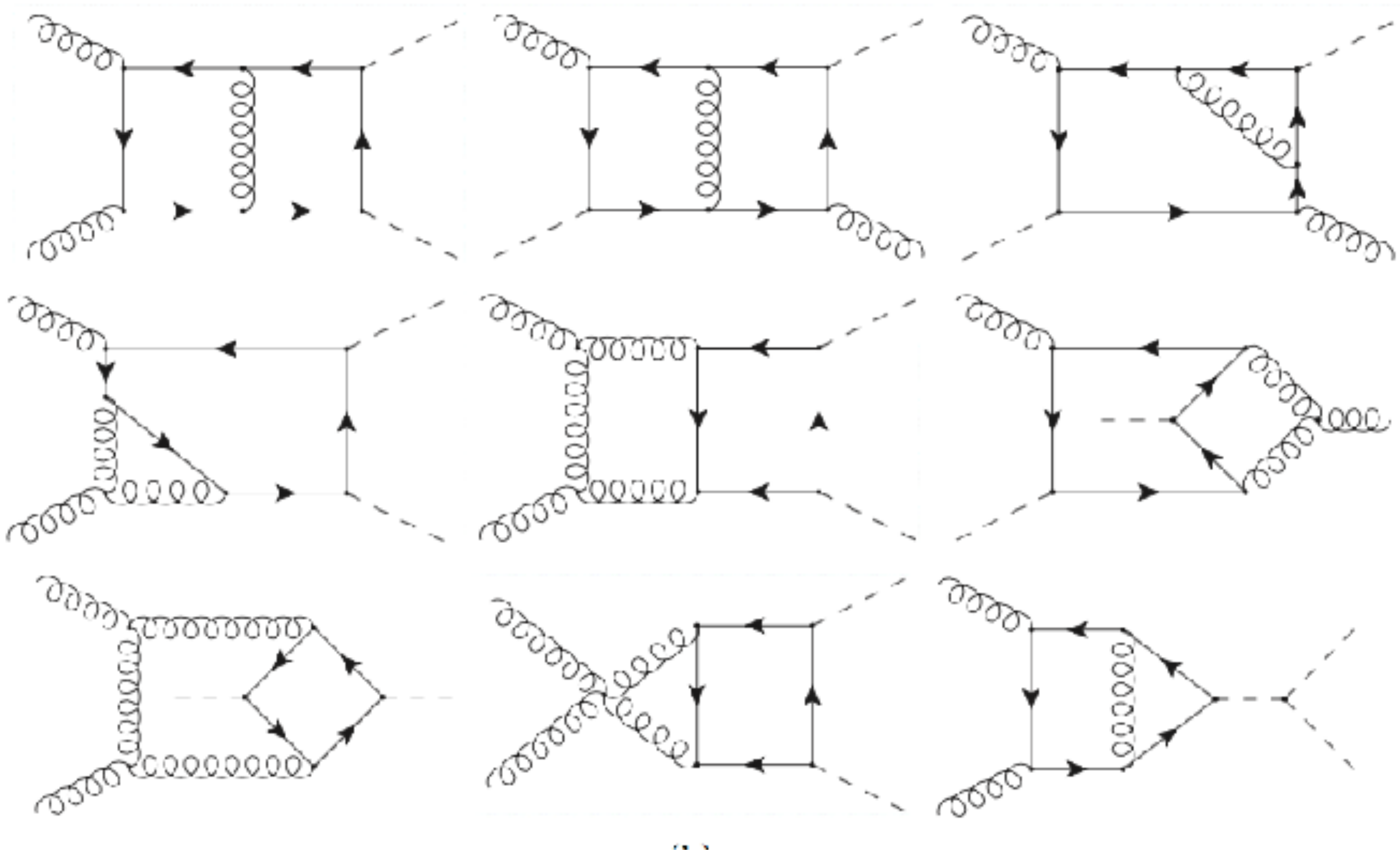
qgraf [Nogueira, '93] : generate amplitudes

q2e/exp [Harlander, Seidensticker, Steinhauser, '98, Seidensticker, '99] : rewrite output to FORM notation

FIRE [Smirnov, '14] (with LiteRed rules [Lee, '13]) : reduction to master integrals

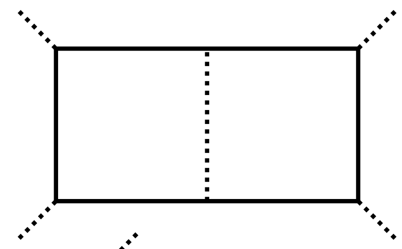
tsort [Smirnov, Pak] : minimization of master integrals

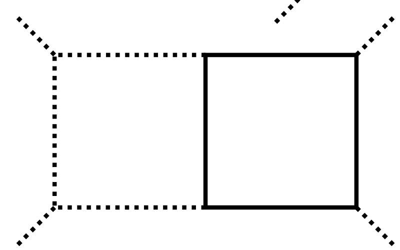
Up to this point, we retain the full top mass dependence.

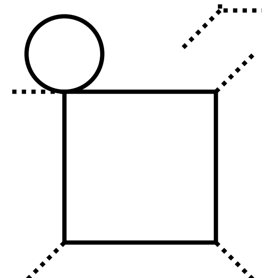


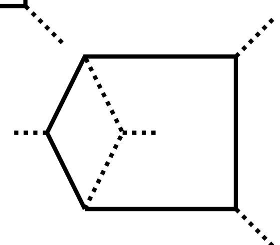
Master integrals at 2 loop

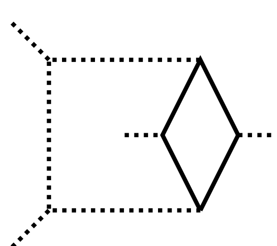
167 =135(planar&crossing)+32(nonplanar&crossing)

=29+(20+15+19+11+9)  +crossing

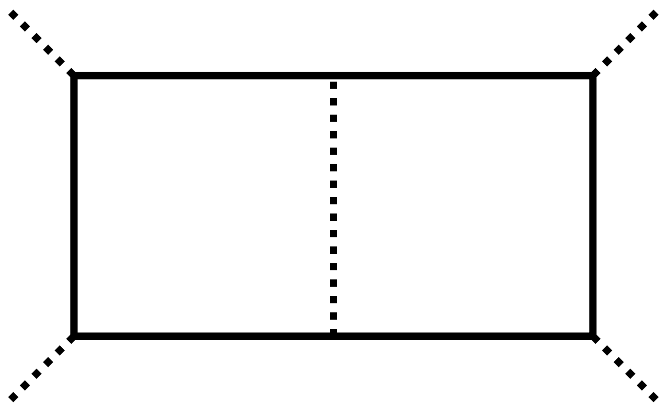
+17+(9+3)  +crossing

+1+(2)  +crossing

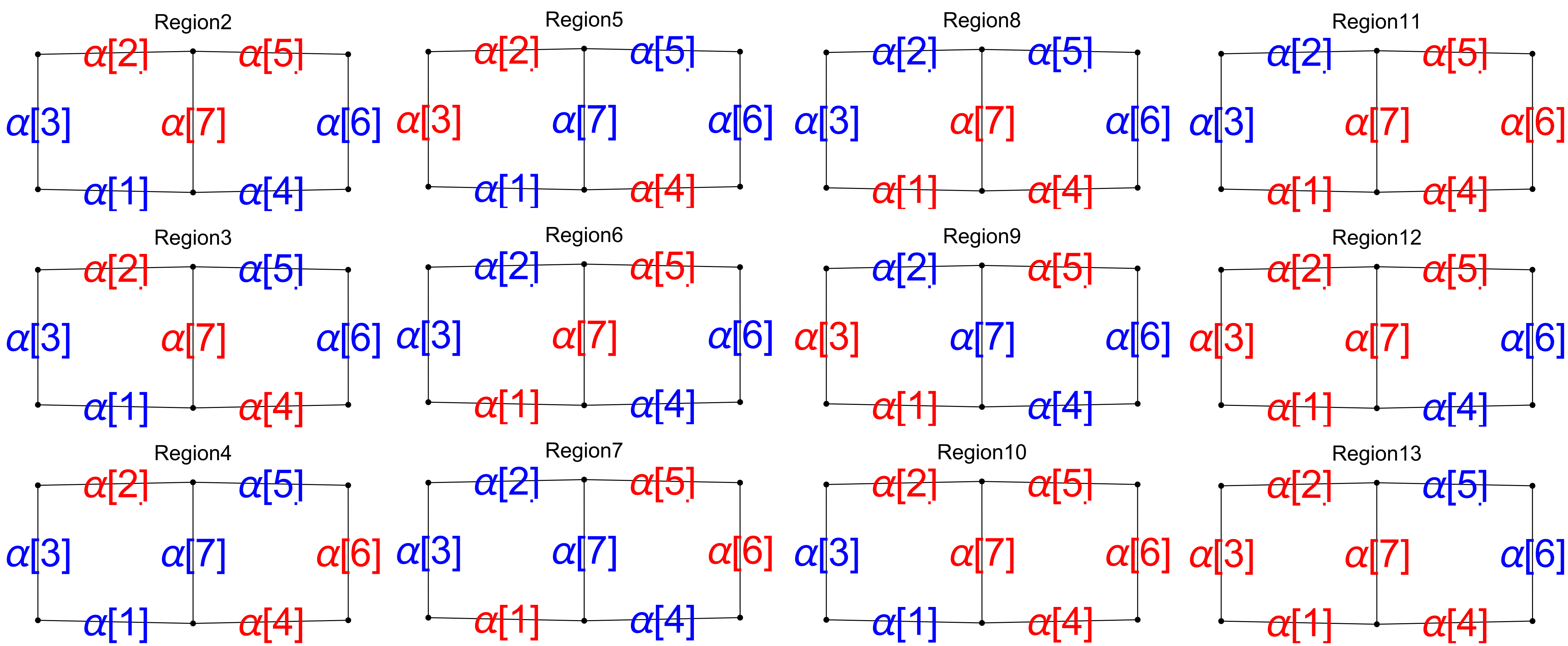
+11+(6+6)  +crossing

+9 

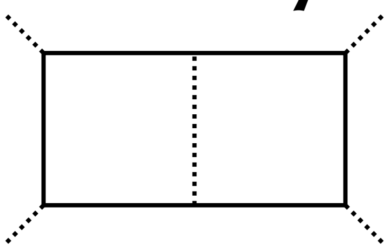
high energy expansion of massive double box



There are 13 regions. (all-hard region + 12 soft regions)



Analytic result of massive double box diagram



$$=$$

$$\tau = T/S, \quad l_t = \log\tau, \quad l_m = \log(-m_t^2/S)$$

$$\frac{1}{S^3}\left(\frac{1}{\tau}\left(-\frac{7\pi^4}{15} - 4\pi^2\,\mathrm{HPL}[\{2\},-\tau] - 24\,\mathrm{HPL}[\{4\},-\tau] + l_m^4 + 16\,\mathrm{HPL}[\{3\},-\tau]\,l_t - \right.\right.$$

$$\frac{8}{3}l_m^3 l_t - 2\pi^2 l_t^2 - 4\,\mathrm{HPL}[\{2\},-\tau]\,l_t^2 - \frac{l_t^4}{3} + l_m^2\left(-\frac{2\pi^2}{3} + 2 l_t^2\right) + l_m\left(\frac{8\pi^2 l_t}{3} - 4\,\mathrm{Zeta}[3]\right) + 4 l_t\,\mathrm{Zeta}[3]\Bigg) + \frac{1}{\tau}$$

$$\mathrm{ep}\left(\frac{23}{30}\pi^4\,\mathrm{HPL}[\{1\},-\tau] - 2 i\pi^3\,\mathrm{HPL}[\{1\},-\tau]^2 - 4\pi^2\,\mathrm{HPL}[\{1\},-\tau]\,\mathrm{HPL}[\{2\},-\tau] - \frac{4}{3}\pi^2\,\mathrm{HPL}[\{3\},-\tau] + 84\,\mathrm{HPL}[\{5\},-\tau] + \right.$$

$$14\pi^2\,\mathrm{HPL}[\{1,2\},-\tau] + 36\,\mathrm{HPL}[\{1,4\},-\tau] + 28\pi^2\,\mathrm{HPL}[\{2,1\},-\tau] + 56\,\mathrm{HPL}[\{2,3\},-\tau] + 76\,\mathrm{HPL}[\{3,2\},-\tau] +$$

$$96\,\mathrm{HPL}[\{4,1\},-\tau] + 4\pi^2\,\mathrm{HPL}[\{1,1,0\},-\tau] + \frac{83\pi^4 l_t}{90} - 2\pi^2\,\mathrm{HPL}[\{1\},-\tau]^2 l_t + 10\pi^2\,\mathrm{HPL}[\{2\},-\tau]\,l_t - 10\,\mathrm{HPL}[\{2\},-\tau]^2 l_t -$$

$$24\,\mathrm{HPL}[\{1,3\},-\tau]\,l_t - 24\,\mathrm{HPL}[\{2,2\},-\tau]\,l_t - 24\,\mathrm{HPL}[\{3,1\},-\tau]\,l_t + \frac{19}{6}l_m^4 l_t + \frac{5}{3}\pi^2\,\mathrm{HPL}[\{1\},-\tau]\,l_t^2 -$$

$$2 i\pi\,\mathrm{HPL}[\{1\},-\tau]^2 l_t^2 - 4\,\mathrm{HPL}[\{1\},-\tau]\,\mathrm{HPL}[\{2\},-\tau]\,l_t^2 - 20\,\mathrm{HPL}[\{3\},-\tau]\,l_t^2 + 14\,\mathrm{HPL}[\{1,2\},-\tau]\,l_t^2 + 28\,\mathrm{HPL}[\{2,1\},-\tau]\,l_t^2 +$$

$$4\,\mathrm{HPL}[\{1,1,0\},-\tau]\,l_t^2 + \frac{25}{9}\pi^2 l_t^3 - 2\,\mathrm{HPL}[\{1\},-\tau]^2 l_t^3 + 6\,\mathrm{HPL}[\{2\},-\tau]\,l_t^3 + \frac{1}{2}\mathrm{HPL}[\{1\},-\tau]\,l_t^4 + \frac{l_t^5}{2} + l_m^3\left(-\frac{4\pi^2}{9} - 2 l_t^2\right) +$$

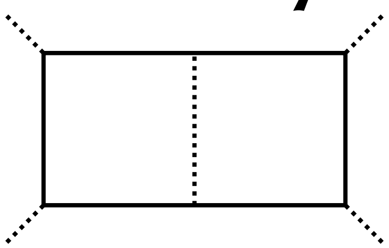
$$l_m^2\left(\pi^2\,\mathrm{HPL}[\{1\},-\tau] + 2\,\mathrm{HPL}[\{3\},-\tau] + \pi^2 l_t - 2\,\mathrm{HPL}[\{2\},-\tau]\,l_t + \mathrm{HPL}[\{1\},-\tau]\,l_t^2 + \frac{l_t^3}{3} - 4\,\mathrm{Zeta}[3]\right) + 6\pi^2\,\mathrm{Zeta}[3] -$$

$$56\,\mathrm{HPL}[\{2\},-\tau]\,\mathrm{Zeta}[3] + 24\,\mathrm{HPL}[\{1\},-\tau]\,l_t\,\mathrm{Zeta}[3] - 4 l_t^2\,\mathrm{Zeta}[3] + l_m\left(-\frac{11\pi^4}{45} - \frac{8}{3}\pi^2\,\mathrm{HPL}[\{2\},-\tau] - 16\,\mathrm{HPL}[\{4\},-\tau] - \right.$$

$$\left.\left.\left.\frac{4}{3}\pi^2\,\mathrm{HPL}[\{1\},-\tau]\,l_t + 8\,\mathrm{HPL}[\{3\},-\tau]\,l_t - \frac{10}{3}\pi^2 l_t^2 - \frac{4}{3}\mathrm{HPL}[\{1\},-\tau]\,l_t^3 - \frac{2 l_t^4}{3} + 8 l_t\,\mathrm{Zeta}[3]\right) - 52\,\mathrm{Zeta}[5]\right)\right) + \mathcal{O}(m_t, \mathrm{ep}^2)$$

We can evaluate this expression with the package HPL.m [Maitre '05].

Analytic result of massive double box diagram

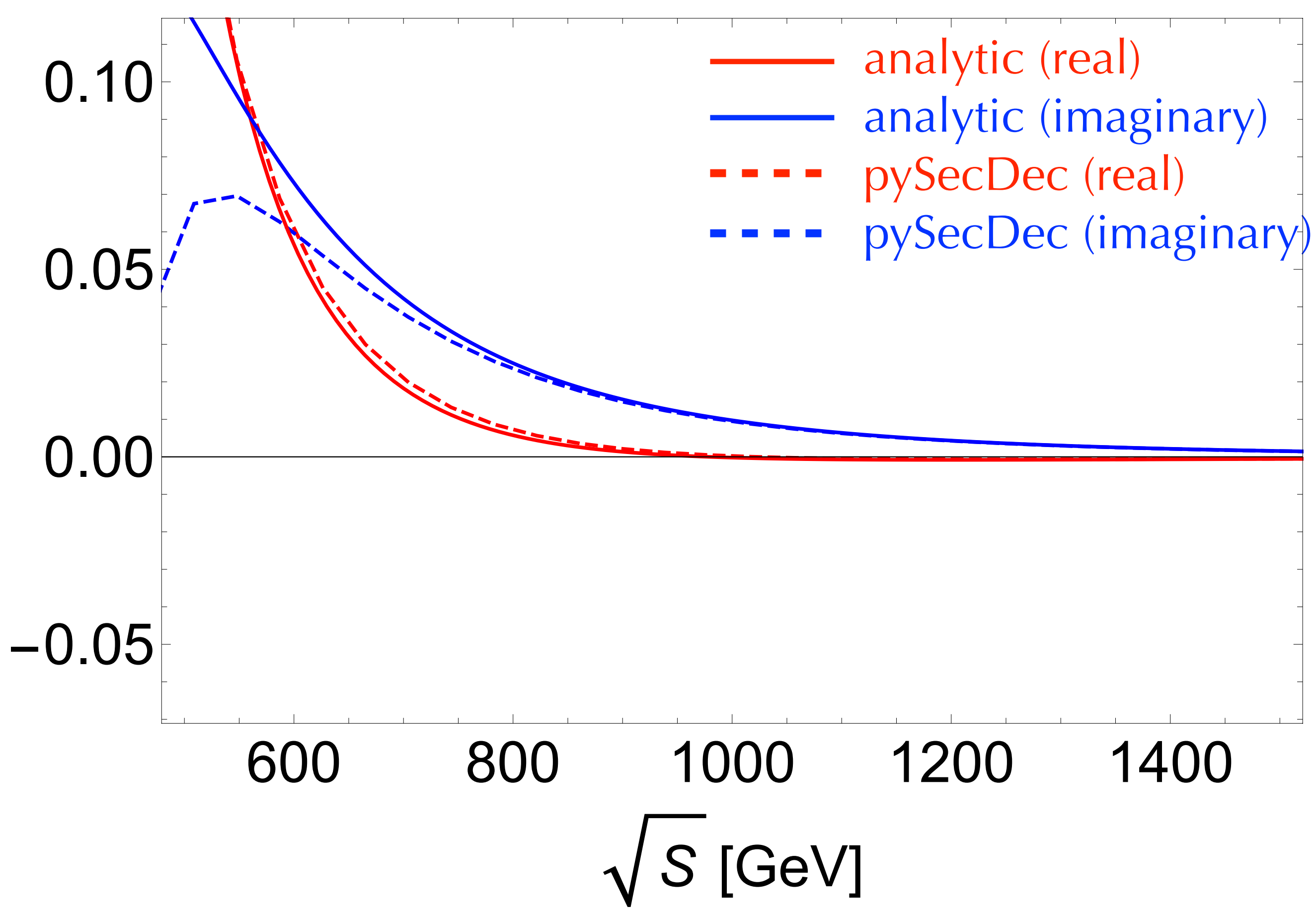


leading order in m_t $\quad @T = -S/2,\ (\theta = \pi/2,\ p_T = \sqrt{S}/2)$

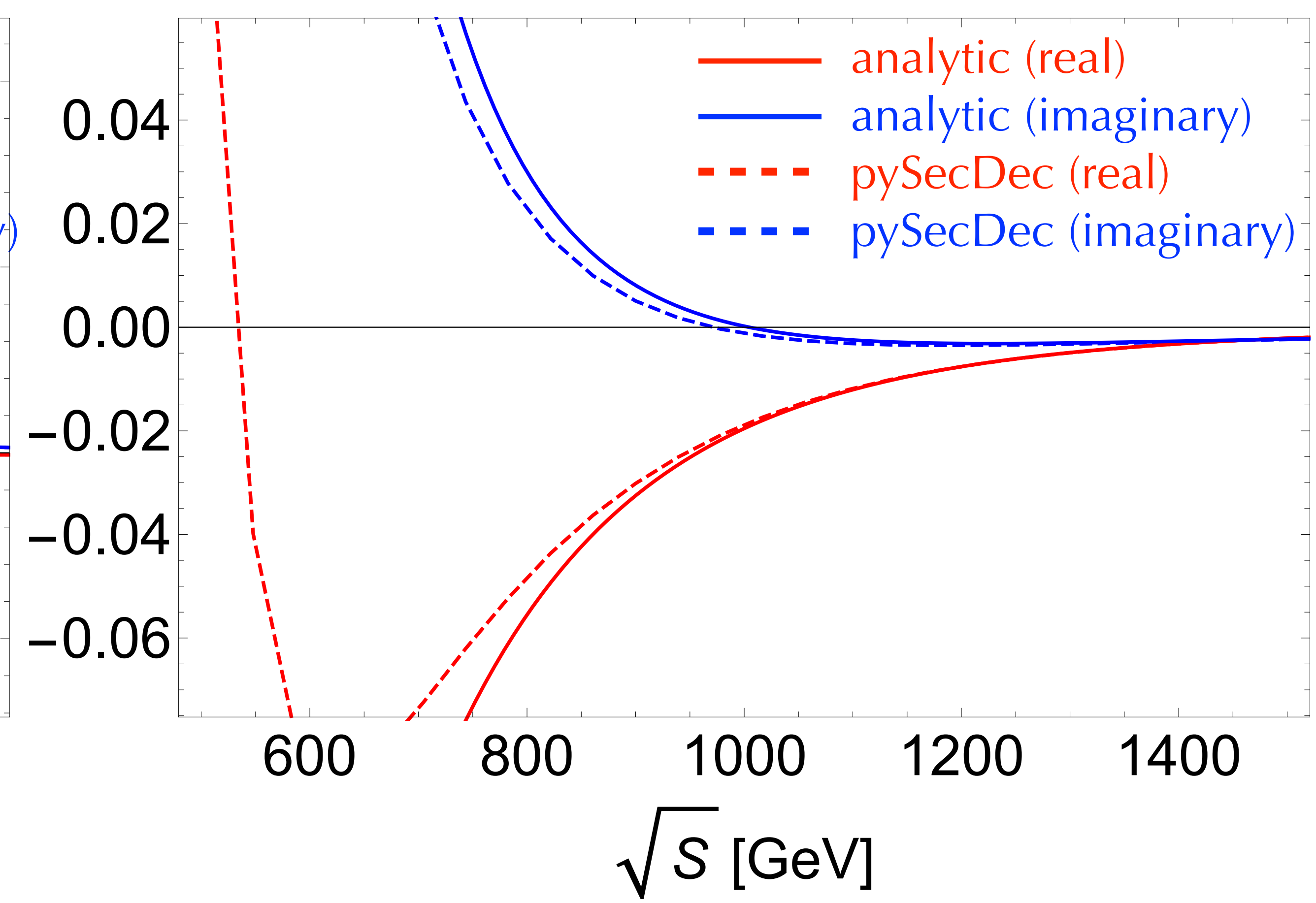
Comparison between our result and the numerical result from pySecDec.

[Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zicke, '16]

finite part



epsilon-linear part



We multiply the integral by m_t^6 to make it dimensionless.

Summary

We are calculating the two-loop gg->hh amplitude in the high energy approximation.

Reduction to the master integrals is done.

Some of the most complicated integrals are evaluated.

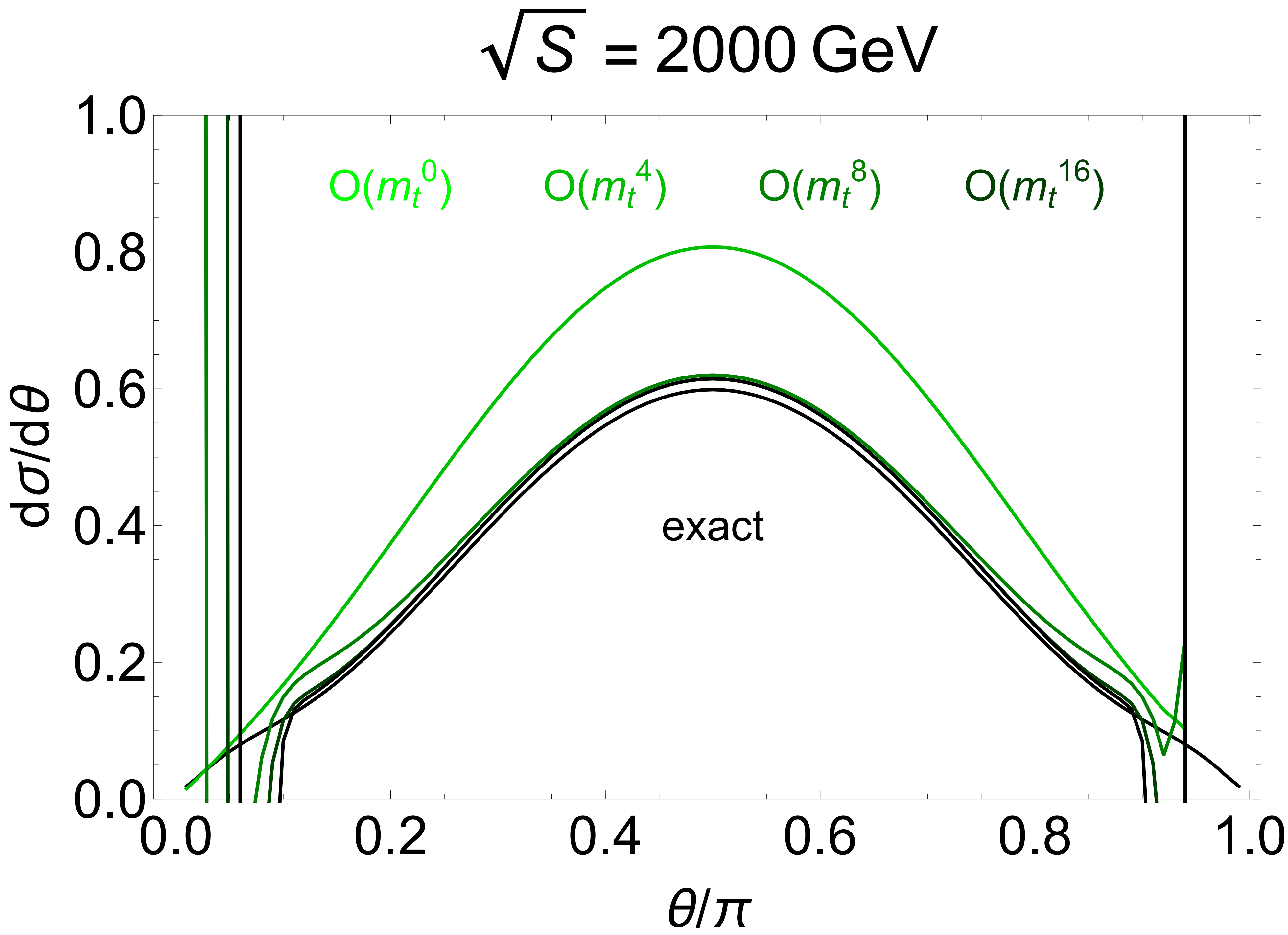
To Do

Complete the evaluation of the planar diagrams.

Including higher order of m_h

Non-planar diagrams

Application to physical process gg->HH @LO



higgs-top coupling

0.87 ± 0.15 [1606.02266] see also ATLAS-CONF-2017-077

$\rightarrow 7\%$ (prospect) [1710.08639]

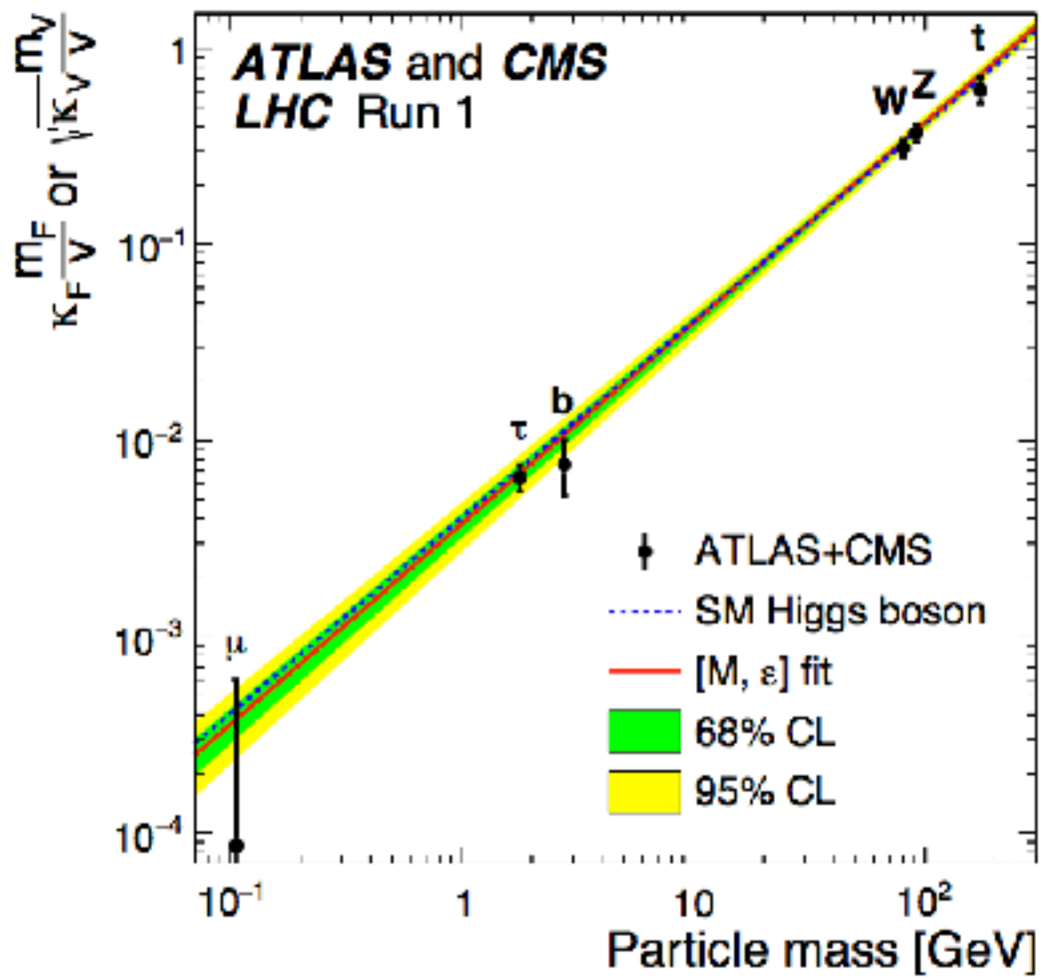
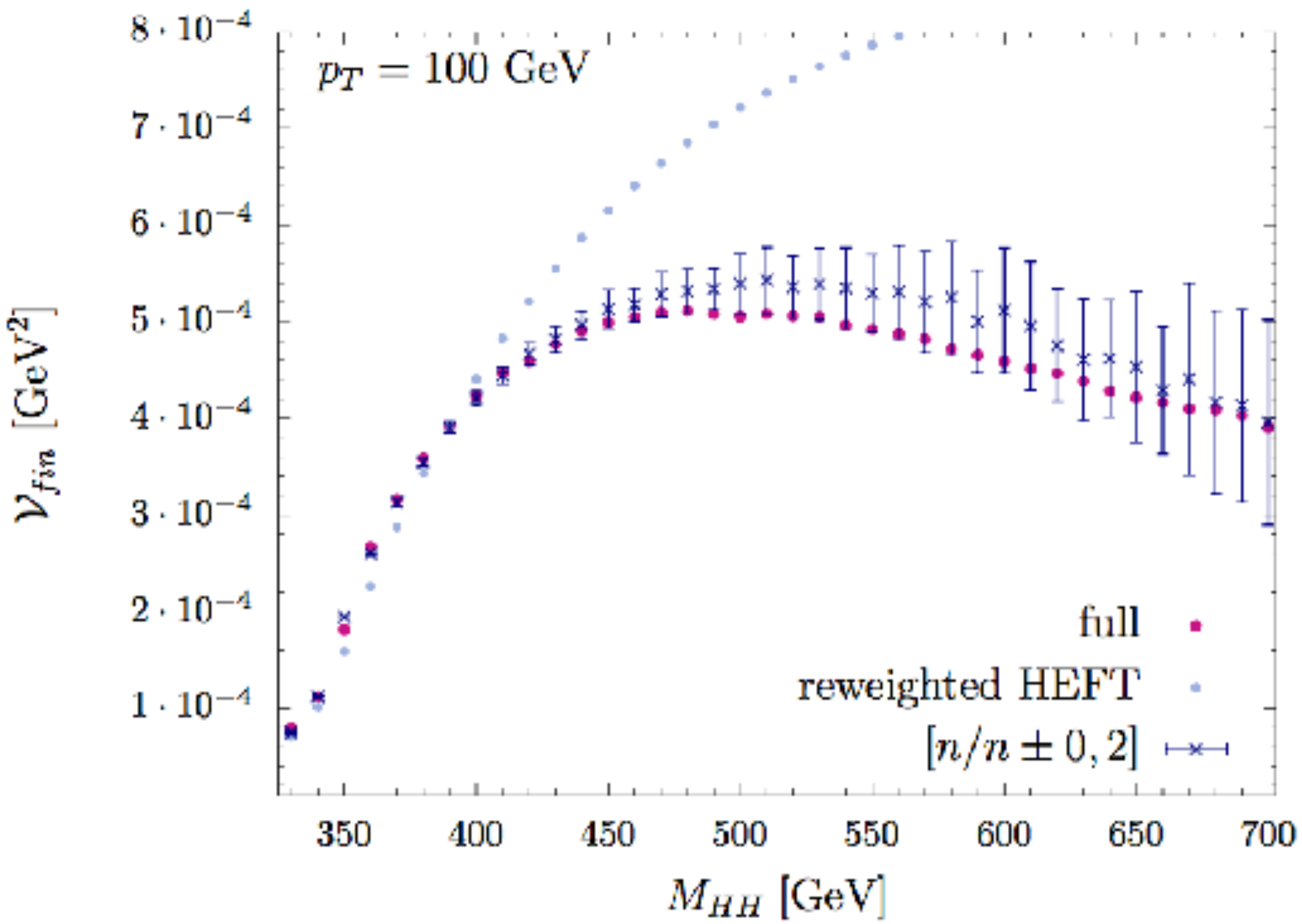


Figure 19: Best fit values as a function of particle mass for the combination of ATLAS and CMS data in the case of the parameterisation described in the text, with parameters defined as $\kappa_F \cdot m_F/v$ for the fermions, and as $\sqrt{\kappa_V} \cdot m_V/v$ for the weak vector bosons, where $v = 246$ GeV is the vacuum expectation value of the Higgs field. The dashed (blue) line indicates the predicted dependence on the particle mass in the case of the SM Higgs boson. The solid (red) line indicates the best fit result to the $[M, \epsilon]$ phenomenological model of Ref. [129] with the corresponding 68% and 95% CL bands.

definition of Vfin

$$\mathcal{V}_{fin} = \frac{\alpha_s^2(\mu_R)}{16\pi^2}\frac{\hat{s}^2}{128v^2}\Bigg[|\mathcal{M}_{born}|^2\left(C_A\pi^2 - C_A\log^2\left(\frac{\mu_R^2}{\hat{s}}\right)\right)$$

$$+2\left\{(F_1^{1l})^*\left(F_1^{2l,[n/m]} + F_1^{2\Delta}\right) + (F_2^{1l})^*\left(F_2^{2l,[n/m]} + F_2^{2\Delta}\right) + \text{h.c.}\right\}\Bigg]$$

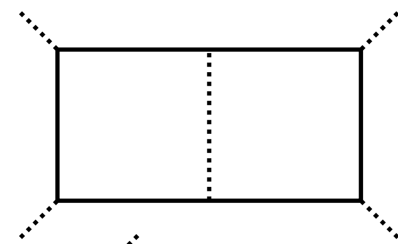


gg->hh in the high energy limit

Master integrals at 2 loop

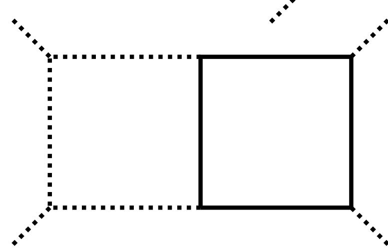
$167 = 135_{\text{(planar\&crossing)}} + 32_{\text{(nonplanar\&crossing)}}$

$= 29_{+(20+15+19+11+9)}$



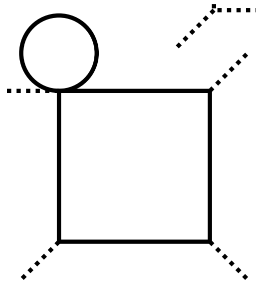
$+[s \leftrightarrow t]$
$+[t \to u] + [s \leftrightarrow t \& t \to u]$
$+[s \to u] + [s \leftrightarrow t \& s \to u]$

$+17_{+(9+3)}$



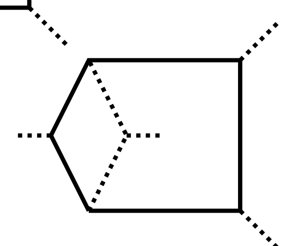
$+[t \to u] + [s \to u]$

$+1_{+(2)}$



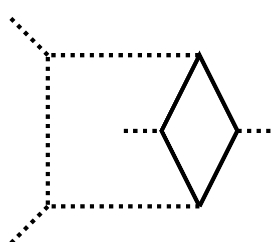
$+[t \to u] + [s \to u]$

$+11_{+(6+6)}$



$+[s \leftrightarrow t] + [s \leftrightarrow t \& s \to u]$

$+9$



High pT makes the previous approximation worse

Padé approximation using the large top-mass and the threshold expansion@NLO
[Gröber, Maier Rauh, '17]

		$\mathcal{V}_{fin}$ [GeV^2] $\times 10^4$			
M_{HH} [GeV]	p_T [GeV]	HEFT	$[n/m]$	$[n/n \pm 0, 2]$	full
336.85	37.75	0.912	0.997 ± 0.007	0.992 ± 0.007	0.996 ± 0.000
350.04	118.65	1.589	1.937 ± 0.011	1.946 ± 0.016	1.939 ± 0.061
411.36	163.21	4.894	4.356 ± 0.199	4.562 ± 0.110	4.510 ± 0.124
454.69	126.69	6.240	5.396 ± 0.219	5.181 ± 0.183	5.086 ± 0.060
586.96	219.87	7.797	5.030 ± 0.657	5.585 ± 0.574	4.943 ± 0.057
663.51	94.55	8.551	5.429 ± 1.197	4.392 ± 0.765	4.120 ± 0.018

Table 2: Numbers for the virtual corrections for some representative phase space points for the HEFT result reweighted with the full Born cross section (as in Ref. [78]), the Padé-approximated ones and the full calculation [85].