Yang-Mills thermodynamics

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“Delta Meeting”

ITP Heidelberg 2007

December 14, 2007

Universität Karlsruhe (TH)
Forschungsuniversität • gegründet 1825

plan

- brief motivation and preview on phase diagram
  [hep-th/0504064 and 0710.0962]
- deconfining ground-state physics:
  coarse-grained, interacting calorons
- coarse-grained excitations:
  Legendre-trafos and loop expansion
- preconfinement:
  cond. magn. monopoles, dual Meissner effect
- low temperatures:
  Hagedorn, flip of statistics, Borel summation
- summary, conclusions, mention of applications

Why nonpert. YMTD?

- infrared instability of PT even for $T \gg \Lambda$ in magnetic sector
  [Linde 1980]

- highly nonpert. ground-state physics even for $T \gg \Lambda$:
  - $\theta_{\mu\mu} \propto T$, magnetic charge, ...
    [Miller 1998, Ilgenfritz et al., Muller-Preussker et al., Gattringer, Bruckmann & van Baal, Garcia Perez et al., Bruckmann, Nogradi, van Baal... 1999-2007]
  - spatial string tension: $\sigma \propto T^2$
    [Philipsen 1998, Korthals-Altes 1998, Laine et al. 2004 ...]

- limited lattice control of thermodynamical quantities at low temperature

preview: phase diagram SU(2)

confining

ground state:

Cooper−pair condensate of single center−vortex loops, pressure precisely zero

excitations:

massless (single) and massive (self−intersecting) center−vortex loops (spin−1/2 fermions)

preconfining

ground state:

condensate of magnetic monopoles, collapsing center−vortex loops, negative pressure

excitations:

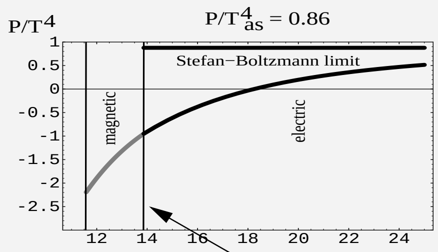
massive dual gauge modes

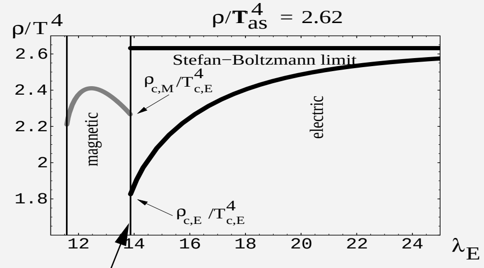
deconfining

ground state: interacting calorons and anticalorons, negative pressure

excitations: massless and massive gauge modes

power−like approach to Stefan Boltzmann limit





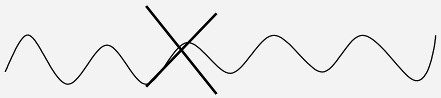
decoupling of V^+, V^-, no (anti) screening

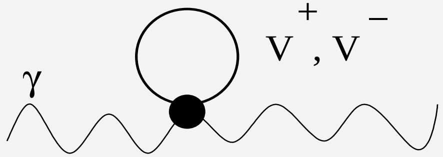
T_c

Hagedorn

2nd order like

T





(anti) screening

deconfining ground state

- coarse-grained (anti)calorons of $|Q| = 1$
  $\Rightarrow$ adjoint scalar field ϕ^a, $|\phi|$ spatially homogeneous
- strategy:

  – thermodynamics $\Rightarrow$ ϕ^a periodic in eucl. time
  in any admissible gauge $\Rightarrow$
  phase $\hat{\phi}^a$ determined by *classical* configs.

  – stable configs.: $|Q| = 1$ HS (anti)calorons (BPS)
  of trivial holonomy
  (no EXPLICIT holonomy in deconfining phase!)
  [Gross, Pisarski, Yaffe 1981]

– compute $\hat{\phi}^a \in$(Kernel of $\mathcal{D}$)$\equiv \mathcal{K}$ respecting isotropy and $S_{\rm HS} = \frac{8\pi^2}{g^2} \neq f(T,\Lambda)$ in *inf.*-vol. average over magnetic-magnetic correlation mediated by *single* (anti)caloron

– fixes $\mathcal{D}$ uniquely $\Rightarrow$ winding number

– pick max. resolution μ such that $\mu = |\phi| = \text{const}$

– consistency of BPS and Euler-Lagrange

$\Rightarrow V(\phi)$ and Λ as constant of integration

$\Rightarrow \mu = |\phi|$ consistent with IR saturation of $\mathcal{K}$

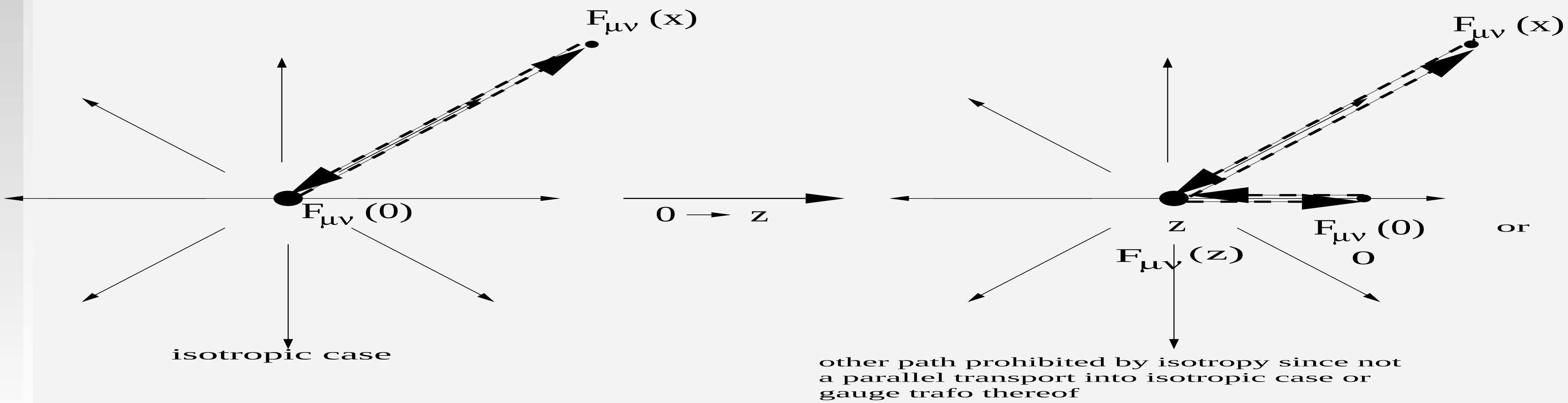
technically:

(integration over $S_3^{R=\infty}$)

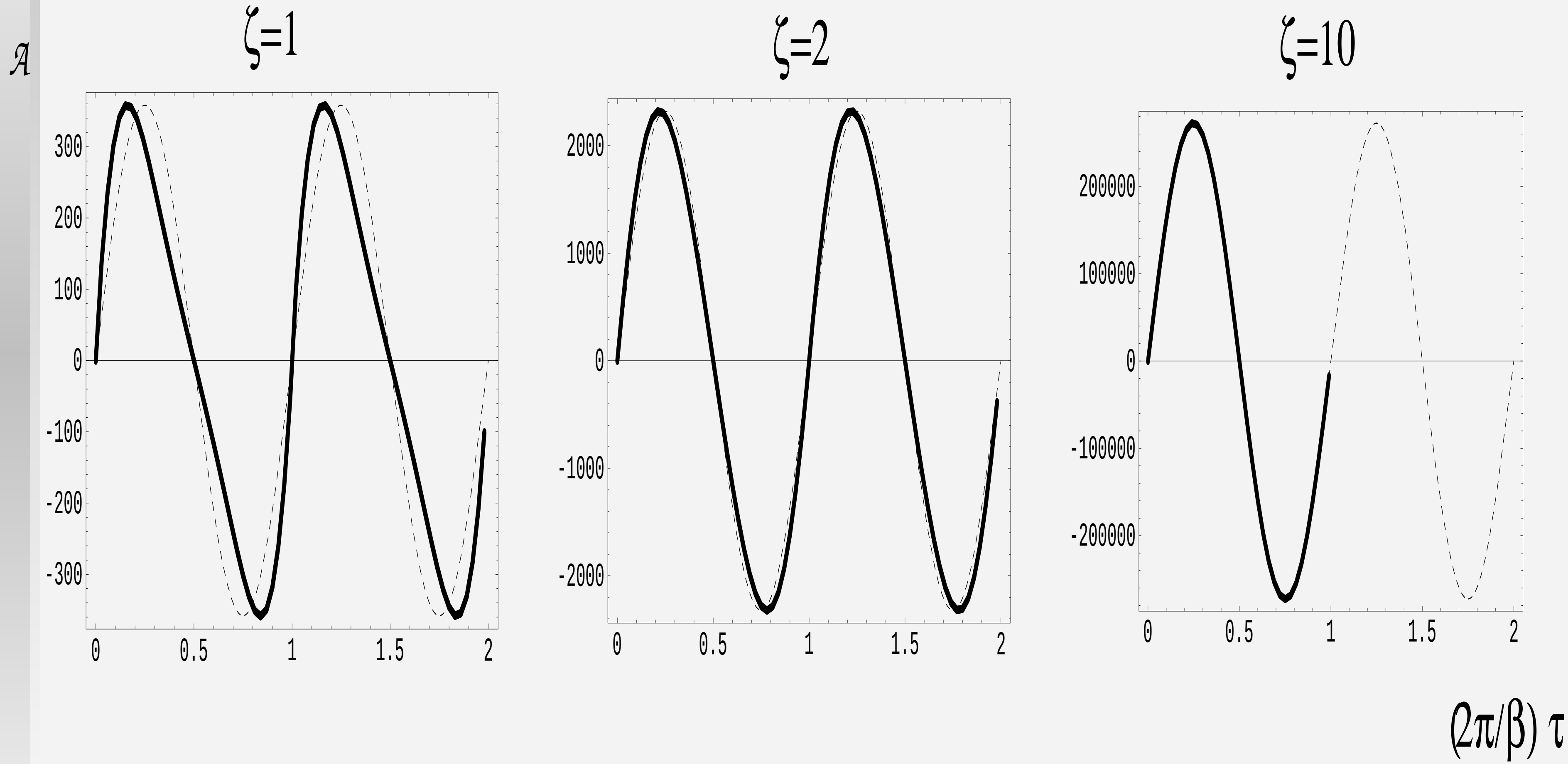
$$\hat{\phi}^a(\tau) \in \sum_{\text{HS (anti)caloron}} \text{tr} \int d^3x \int d\rho \, \frac{\lambda^a}{2} \times$$

$$F_{\mu\nu}((\tau,0)) \, \{(\tau,0),(\tau,\vec{x})\} \times$$

$$F_{\mu\nu}((\tau,\vec{x})) \, \{(\tau,\vec{x}),(\tau,0)\} \, .$$



saturation:



𝒜
ζ=1
ζ=2
ζ=10
300
200
100
0
-100
-200
-300
0
0.5
1
1.5
2
2000
1000
0
-1000
-2000
200000
100000
0
-100000
-200000
(2π/β) τ

$\Longrightarrow$

$$\mathcal{D} = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$$

$$\Rightarrow \partial_{|\phi|^2} V(|\phi|^2) = -\frac{V(|\phi|^2)}{|\phi|^2} \quad \Rightarrow \quad V(\phi) = \mathrm{tr}\,\Lambda^6 \phi^{-2}$$

$$\Rightarrow |\phi| = \sqrt{\frac{\Lambda^3}{2\pi T}}$$

$\Rightarrow$ unique, coarse-grained action for noninteracting $|Q| = 1$ HS (anti)calorons

$\Rightarrow$ ϕ's inertness for $\lambda \equiv \frac{2\pi T}{\Lambda} \geq 13.87 = \lambda_c$ (as we shall see)

What about $Q = 0$?

- perturbative renormalizability:
  ['t Hooft, Veltman 1971-73]

  $\Rightarrow$ coarse-graining over $Q = 0$ yields
  same form as fundamental action
- gauge invariance glues $Q = 0$ to $|Q| = 1 \Rightarrow$

$$S = \mathrm{tr} \int_0^\beta d\tau \int d^3x \left(\tfrac{1}{2} G_{\mu\nu} G_{\mu\nu} + D_\mu \phi D_\mu \phi + \Lambda^6 \phi^{-2}\right)$$

- subject to offshellness constraints in unitary-Coulomb gauge (coarse-graining down to resolution $|\phi|$)

full ground state

- from $D_\mu G_{\mu\nu} = ie[\phi, D_\nu \phi]$:
  – pure gauge $a_\mu^{bg} = \frac{\pi}{e} T \delta_{\mu 4} \lambda_3$
  $\Rightarrow$ ground-state energy-density and pressure
  $$\rho^{g.s} = 4\pi \Lambda^3 T = -P^{g.s} \neq 0$$
- rotation to unitary gauge $a_\mu^{bg} = 0$:
  – gauge transformation singular but admissible
  (does not affect periodicity of fluct. δa_μ)
  – but: $\text{Pol}[a^{bg}] = -\mathbf{1} \stackrel{GT}{\rightarrow} \text{Pol}[a^{bg}] = +\mathbf{1}$
  $\Rightarrow$ Z_2^{el} degeneracy
  $\Rightarrow$ deconfinement

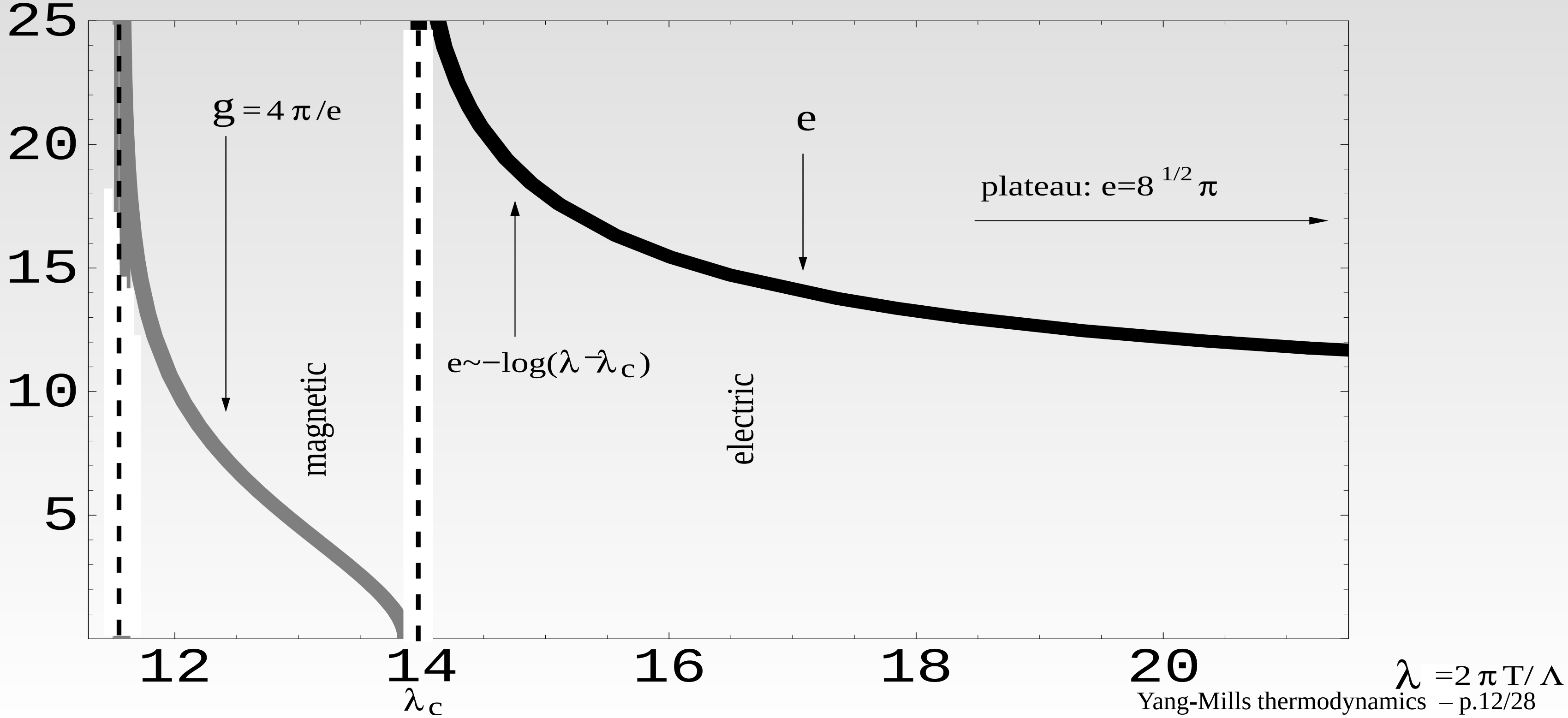
excitations and loop expansion

- adjoint Higgs mechanism:

  2 out of 3 directions massive with $m = e\sqrt{\frac{\Lambda_E^3}{2\pi T}}$

- evol. of eff. coupl. e: Legendre trafos betw. P, ρ, ...

  $\Rightarrow e = \sqrt{8}\pi$ almost everywhere



- trace anomaly: $\frac{\theta_{\mu\mu}}{\Lambda^4} = 12\lambda$ for $\lambda \gg \lambda_c$
  [Giacosa, RH 2007]
- counting of d.o.f.:

  fundamentally:

  3 species (gluons)$\times$2 pols.+
  1 species (monop)$\times$2 charges =8

  after coarse-graining:

  2 species (gluons)$\times$3 pols.+
  1 species (gluon)$\times$2 pols.=8

  $\Rightarrow$ 8 (fund)=8 (coarse-grained).

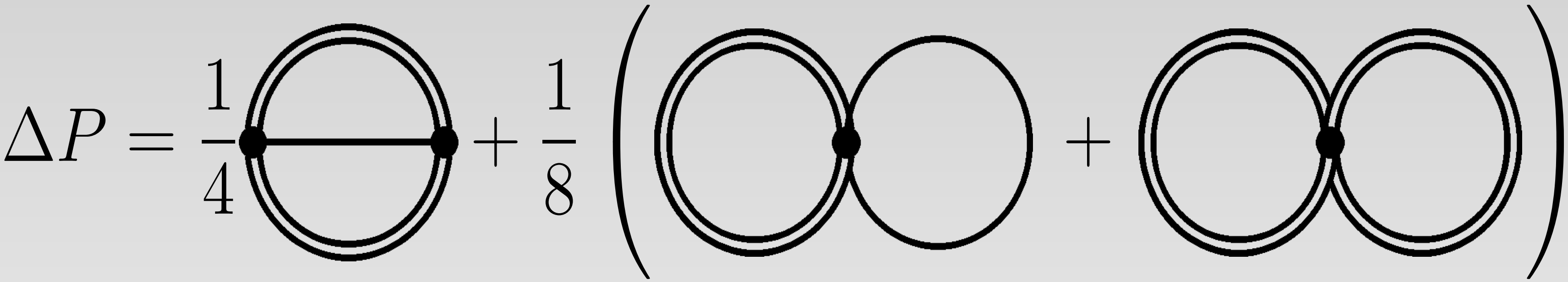
  same way for SU(3):

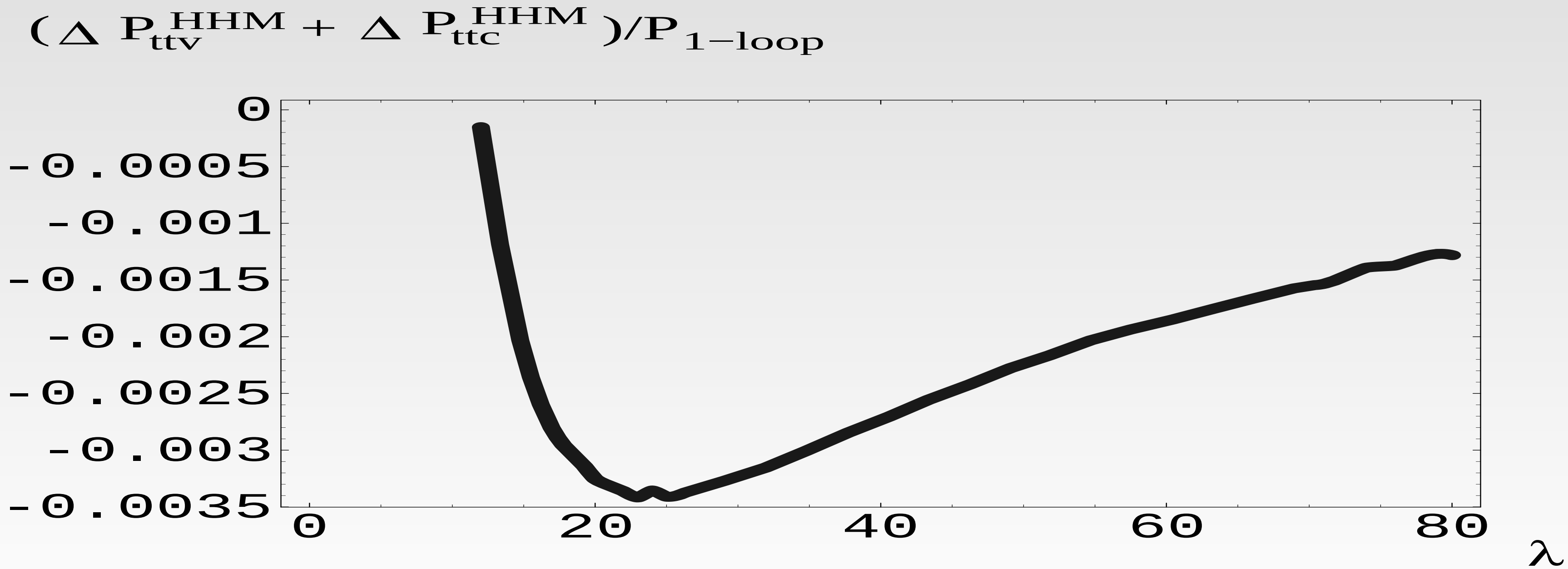
  $\Rightarrow$ 22 (fund)=22 (coarse-grained)

- **loop expansion:**

  2-loop:

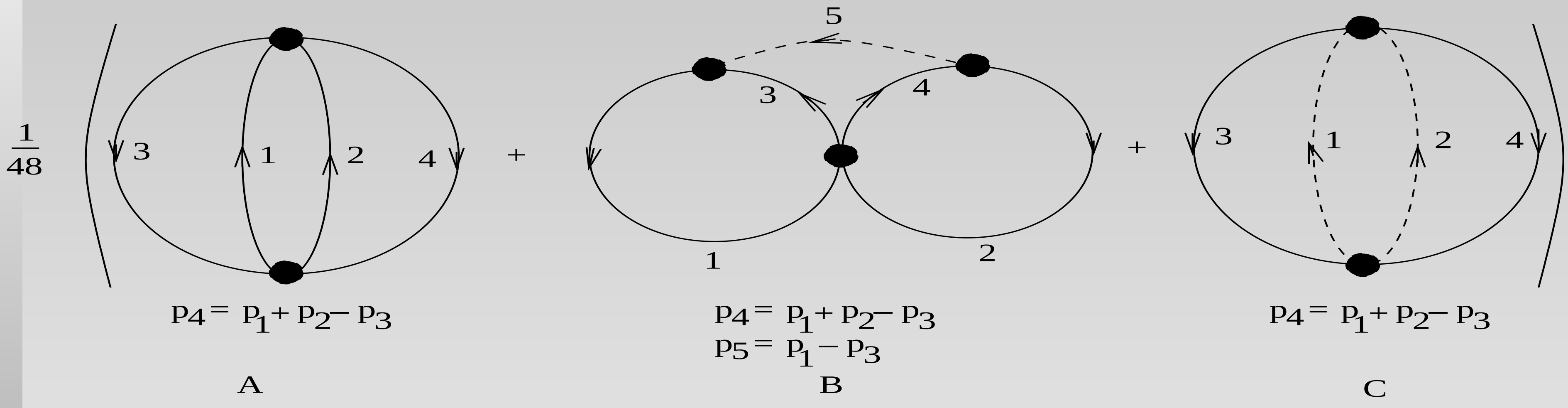
  [Rohrer,Herbst,RH 2004; Schwarz,RH,Giacosa 2006]

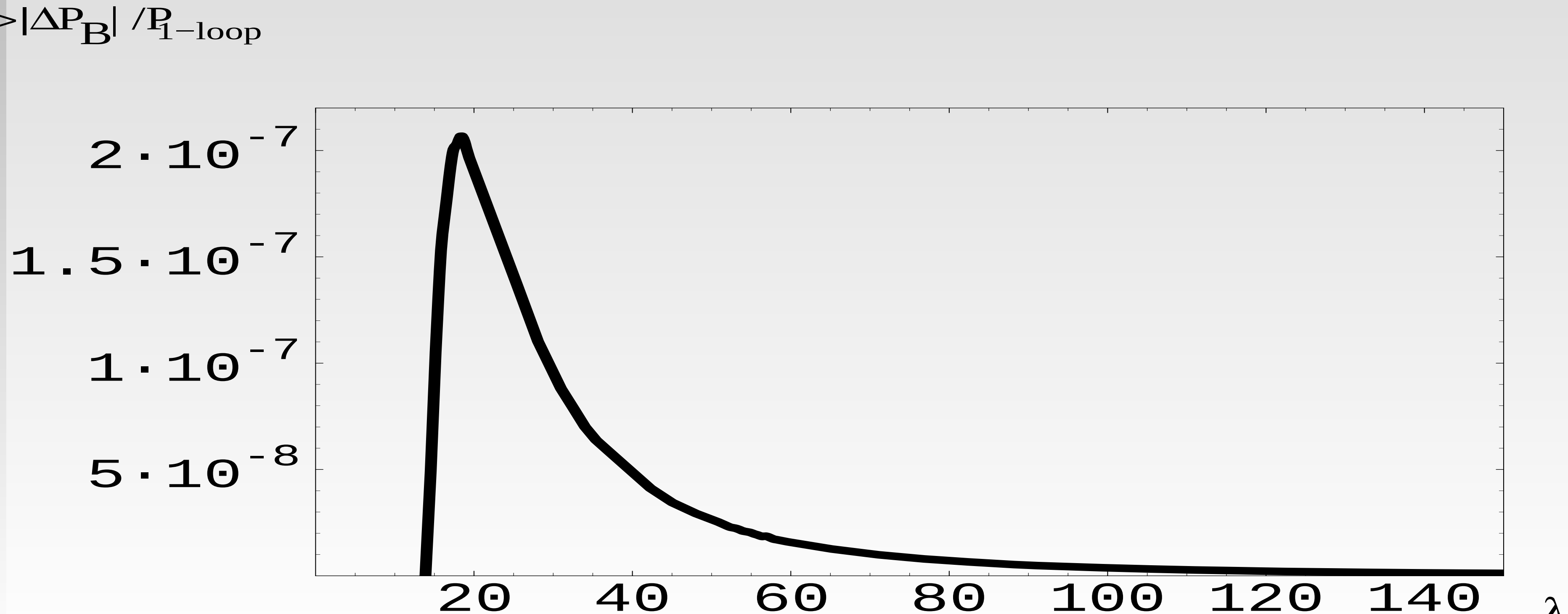




irreducible 3-loop:

[Kaviani,RH 2007]





arguments on loop expansion in general:
[RH 2006]

- resummation of 1PI diagrams
  $\Rightarrow$ **no pinch singularities**

- irreducible diagrams **terminate**
  at finite loop order

(Euler characteristics for spherical polyhedron,
constraints on loop momenta in effective theory
$\Rightarrow$
number of constraints **exceeds**
number of independent radial loop variables
at **sufficiently large number of loops**)

preconfining phase

- cond. of monop. (stable, nontrivial holonomy):
  [Diakonov et al. 2004; Gerhold, Ilgenfritz, Muller-Preussker 2006; Diakonov & Petrov 2007]
  - phase of complex scalar = magnetic flux through $S_2^{R=\infty}$ of M-A pair at rest ($e \longrightarrow \infty$)
  - modulus as in dec. phase
  - no change of form of action for free dual gauge modes by coarse-graining

  $\Rightarrow$ **unique effective action**
  - Polyakov loop always unique
  - pressure exact at one loop
  - evol. of dual coupling g (Legendre trafos)

- counting of d.o.f.:

  fundamentally:

  1 species ('photon')$\times$2 pols.+
  1 species (center-vortex loop) =3

  after coarse-graining:

  1 species (massive 'photon')$\times$3 pols.=3

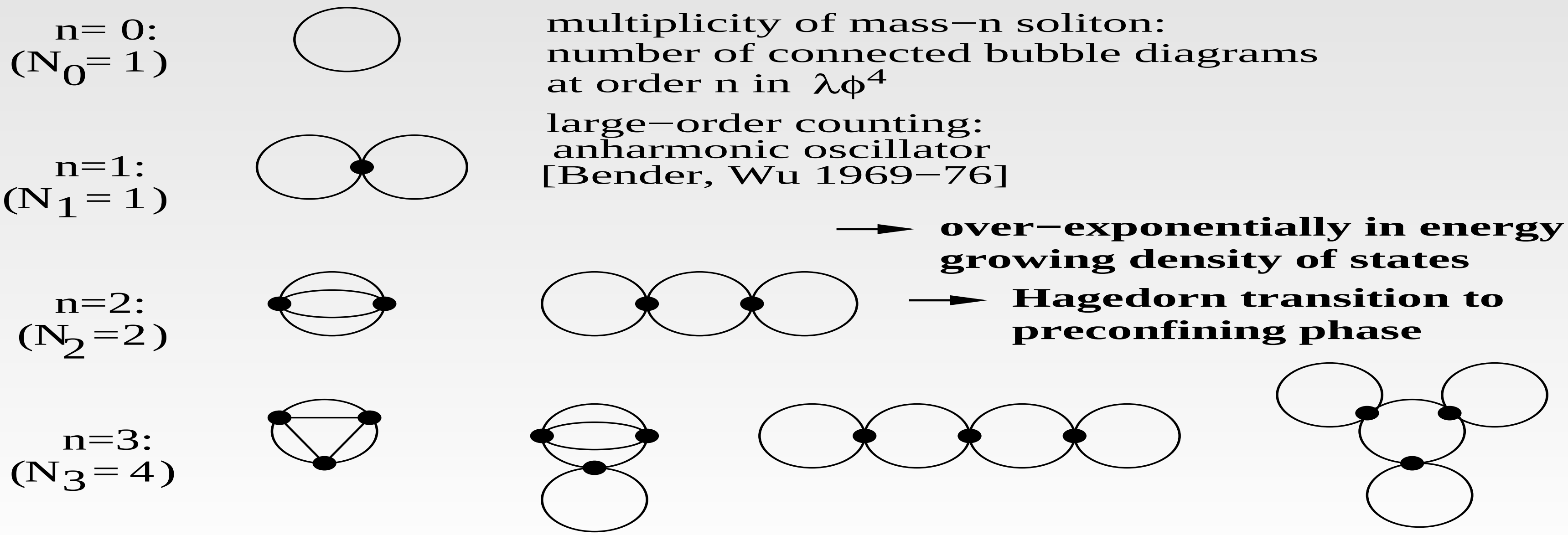
  $\Rightarrow$ 3 (fund)=3 (coarse-grained).

  same way for SU(3):

  $\Rightarrow$ 6 (fund)=6 (coarse-grained)

low temperature

- condensation of center-vortex loops (CVL's) ['t Hooft 1978, de Forcrand & Smekal 2002, de Forcrand & Noth 2005, ...]
  - discrete values of phase of complex scalar field = units of center flux through min. surface; spanned by $S_1^{R=\infty}$; modulus=condensate
  - spectrum: single and selfintersecting CVL's



- counting of d.o.f.:

fundamentally:

1 species ('very massive photon')$\times$3 pols.=3

after coarse-graining:

1 species (massless CVL)+
1 species (massive CVL)$\times$ 2 charges=3

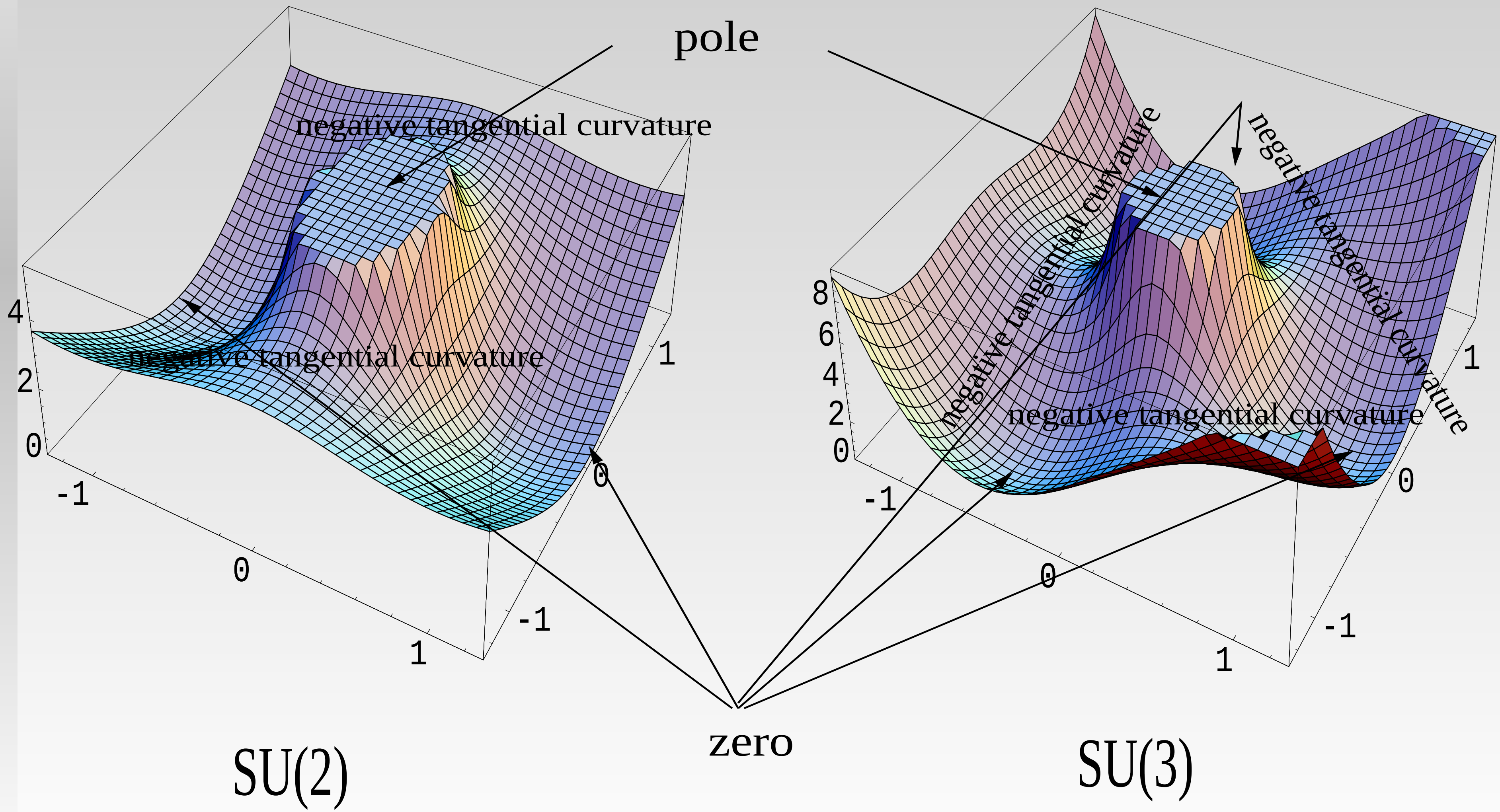
$\Rightarrow$ 3 (fund)=3 (coarse-grained).

same way for SU(3):

$\Rightarrow$ 6 (fund)=6 (coarse-grained)

– potential unique up to inessential, U(1) invariant rescaling

[lattice invest.: Scheffler, RH, Stamatescu 2007]



– **asymptotic-series** representation of pressure:

$$P_{\rm as} = \frac{\Lambda^4}{2\pi^2}\hat{\beta}^{-4}\times$$
$$\left(\frac{7\pi^4}{180} + \sqrt{2\pi}\,\hat{\beta}^{\frac{3}{2}}\sum_{l=0}^{L} a_l \sum_{n\geq 1}(32\lambda)^n\,n!\,n^{\frac{3}{2}+l}\right)\,,$$

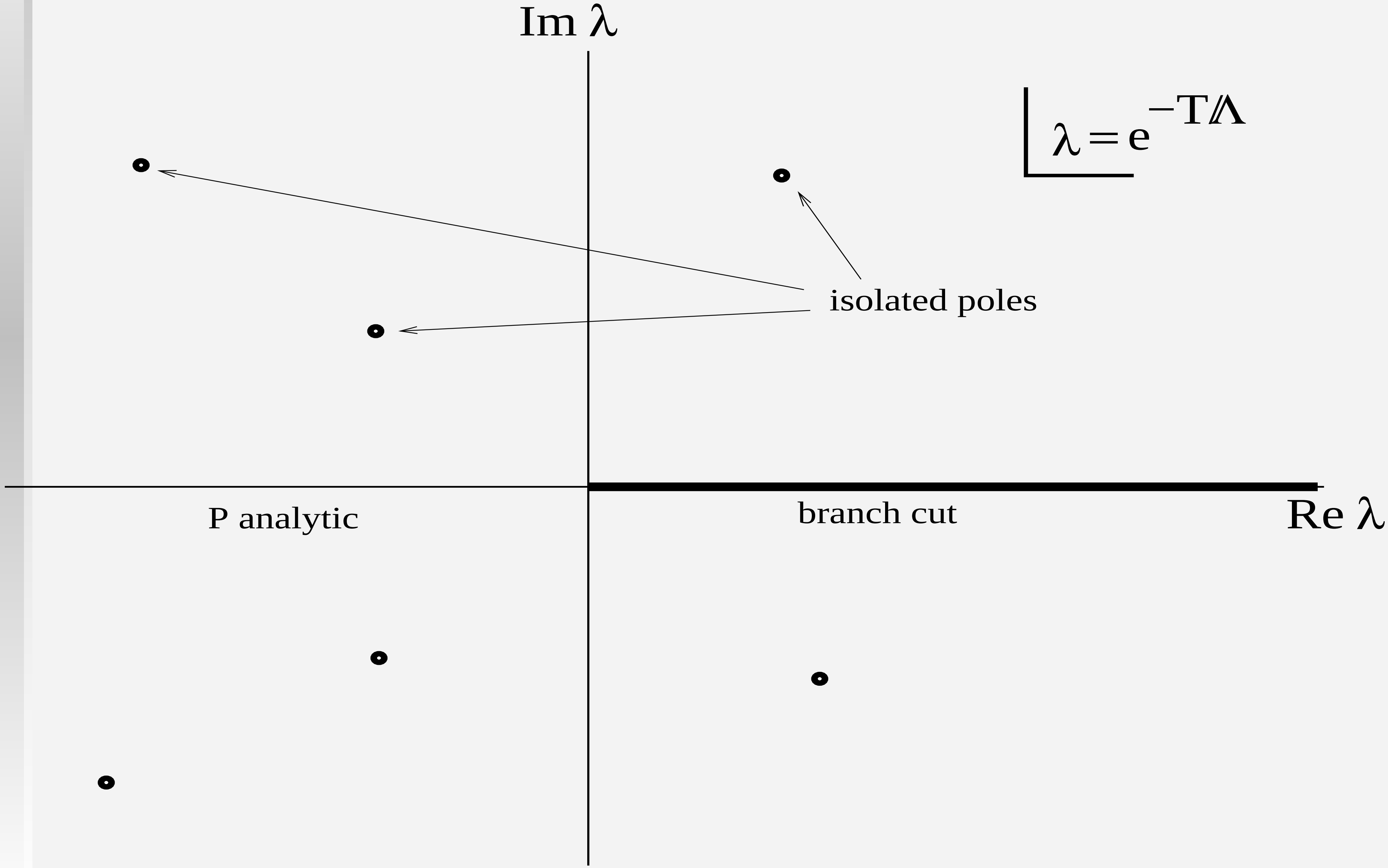
where $\hat{\beta} \equiv \Lambda/T$ and $\lambda \equiv \exp[-\hat{\beta}]$.
[uses Bender, Wu 1968–1976]

– Borel transformation and analytic continuation
$\Rightarrow$ **analytic** dependence on Borel parameter t (polylogs) for $\lambda < 0$

– inverse Borel trafo:
$\Rightarrow$ analytic dependence for $\lambda < 0$ and

meromorphic in entire λ-plane except for $\lambda \geq 0$

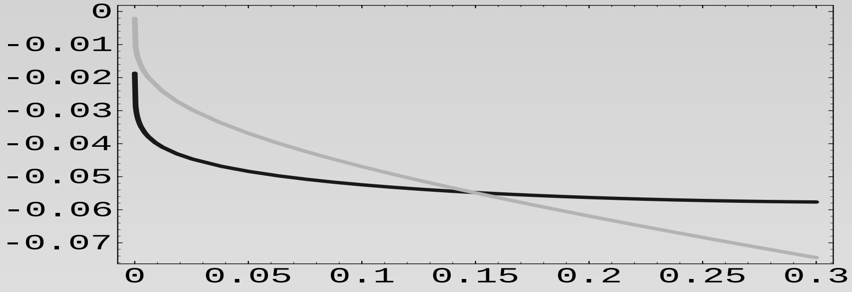
analyticity structure of physical pressure P:



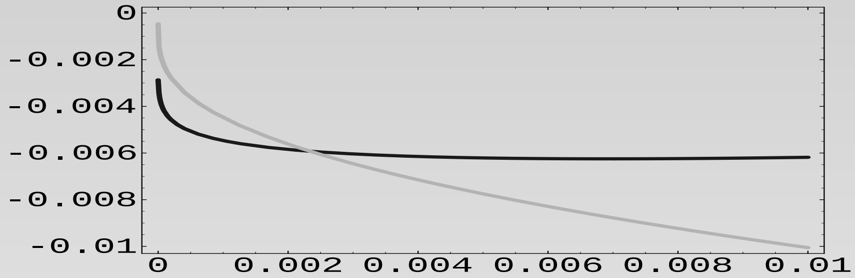
– Re P continuous across cut:

– sign-ambiguous Im P grows slower than Re P

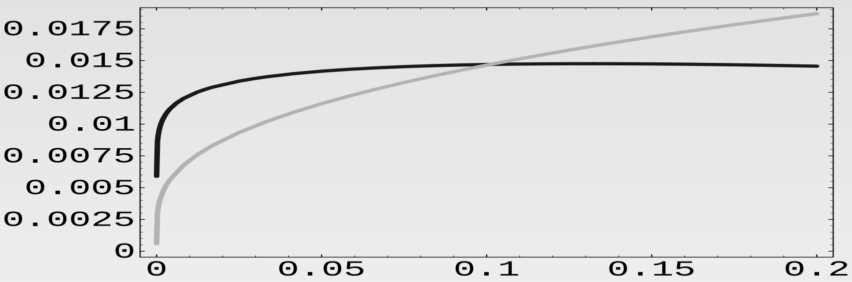
– turbulences become relevant for sufficiently high T only



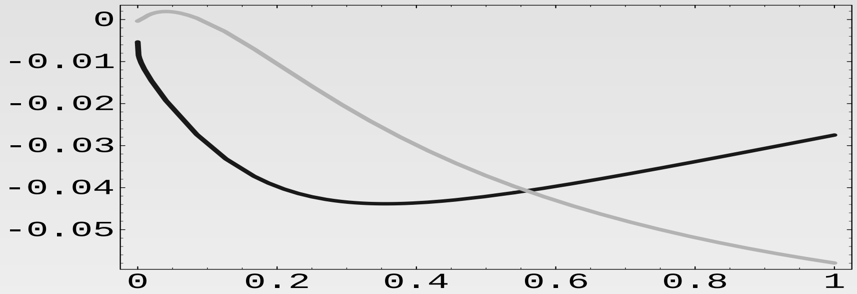
(a)



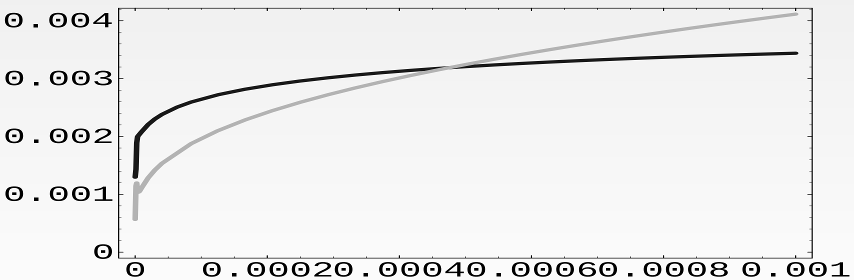
(b)



(c)



(d)



(e)

summary and conclusions

- **deconf. phase:**
  - magnetic-magnetic correlations in (anti)calorons generate adj. Higgs field
  - rapid saturation of average
  - negative ground-state pressure by microscopic holonomy shifts (annihilating M-A pairs)
  - thermal quasiparticles on tree level (adj. Higgs mech.)
  - very small radiative corrections, termination of expansion in terms of irreducible loops

- **preconf. phase:**
  - averaged magnetic flux of M-A pair through S_2^∞ generates phase of complex scalar
  - dual gauge field Meissner massive
  - loop expansion trivial

- **conf. phase:**
  - averaged center flux of CVL pair through min. surface spanned by S_1^∞ generates discrete values of phase of complex scalar; modulus=condensate
  - excitations are single or selfintersecting CVL's of factorially growing multiplicity
  - asymptotic-series representation of pressure
  - Borel summability for complex values of T
  - analytic continuation: rapidly (slowly) rising modulus of real (imaginary) part
  - interpretation: growing relevance of turbulences with increasing T

- physics applications:
  - CMB
  - late-time cosmology (axion + SU(2))
  - electroweak symmetry breaking

Thank you.