Deconfining SU(2) Yang-Mills Thermodynamics and the Temperature Dependence of the Axion Mass

Dmitry Antonov, Ralf Hofmann, and Janning Meinert

Overview I

- **motivation: ultralight axions (fuzzy dark matter)**
  - small-scale structure in cosmology, Schrödinger-Poisson
  - CMB T-z relation $\rightarrow$ depercolation phenomena in the dark sector

- **interpretation of Veneziano-Witten (VW) at $T>0$**
  - deconfining thermal ground state
  - phase structure of SU(2) Yang-Mills thermodynamics
  - active vs passive YM factors

- **previous results on $\chi(T)$**
  - modelling approaches: dilute instanton gas, instanton liquid
  - lattice simulations
  - Guisti & Lüscher: why not $\chi(T) \propto T^4$ at high T?
  - Seiler: VW at $T=0 \rightarrow$ it is all in the contact term

Overview II

- **deconfining thermal ground state and axion mass $m_a(T)$**
  - relevant Harrington-Shepard (anti)calorons and their size
  - results on contact contribution to $\chi(T)$ in effective theory
  - contributions of effective quasiparticles to $\chi(T)$ in effective theory

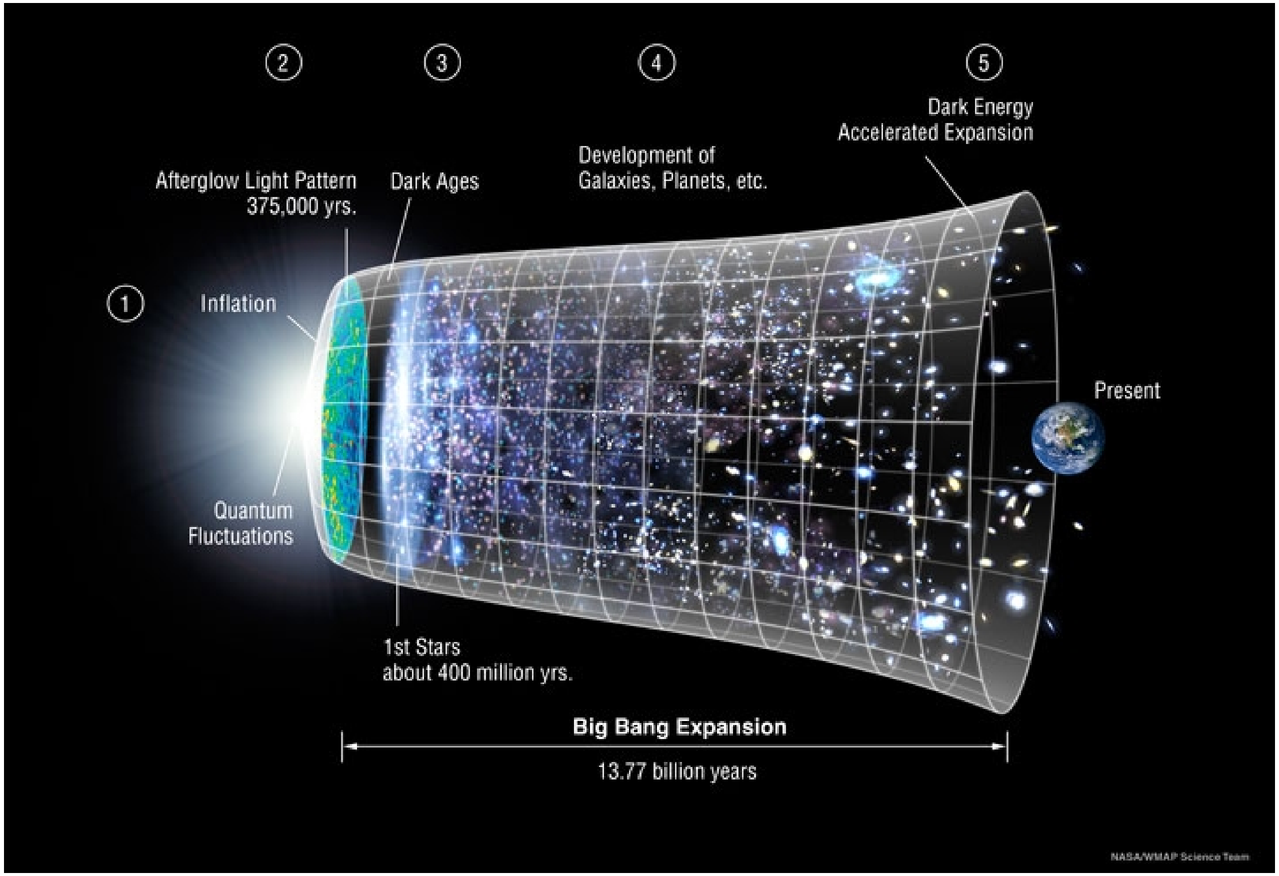
- **cosmological implication of $m_a(T)$**
  - CMB, lepton families, and associated axion masses
  - Bose condensation of ultralight axions

- **summary and outlook**

motivation: ultralight axions (fuzzy dark matter) I

- modulo some recent tensions in *ΛCDM*-model cosmological parameter values (H_0, S_8, ω_b) between high-z and low-z data extractions cold dark matter paradigm fares well on large scales $r \gg 1$Mpc

[A. G. Riess et al ApJ Lett 962 (2024), N. Aghanim A&A 641 (2020), D. J. Eisenstein & W. Hu ApJ, 496 (1998), J. M. Shull et al. ApJ 759 (2012), ...]



motivation: ultralight axions (fuzzy dark matter) II

- on small scales things are different:

Core-Cusp Problem in low surface brightness dwarf galaxies

[W. J. G de Blok Adv. Astron. (2010), …]

Dwarf galaxy problem

[A. Klypin et al. ApJ 522 (1999), …]

Galaxy morphology problem (missing buldges, many large disks)

[S. Sachdeva & K. Saha ApJ Lett 820 (2016), …]

High redshift galaxies (JWST)

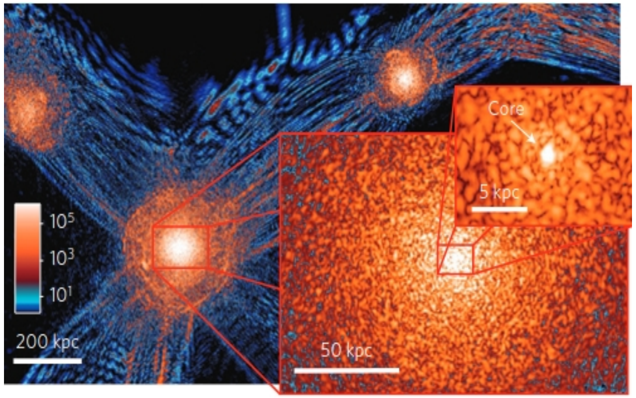
[E. Curtis-Lake et al. Nature Astronomy 7 (5) (2023), …]

motivation: ultralight axions (fuzzy dark matter) III

- ultralight axions yield selfgravitating quantum dynamics on galactic scales at axion mass $m_a \simeq 10^{-21} \cdots 10^{-24}$ eV, nonrelativitistic speeds and gravitational fields
  → Schrödinger-Poisson ,
  [S.-J. Sin PRD 50 (1994), …]

$$\nabla^2 V = 4\pi G(\rho_{\mathrm{dark}} + \rho_{\mathrm{visible}}) \qquad i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi(r)$$

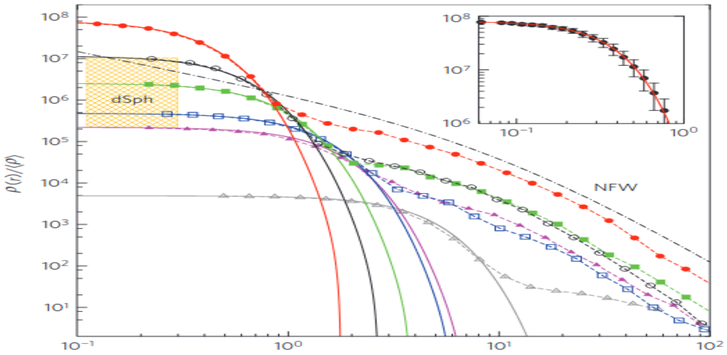
- simulating this numerically
  → fit of galaxy core-mass density as
  [H.-Y. Schive et al. Nature Physics 10 (2014), …]



$$\rho_{\mathrm{sol}}(r) = \frac{\rho_c}{(1 + 0.091(r/r_c)^2)^8}$$

subject to

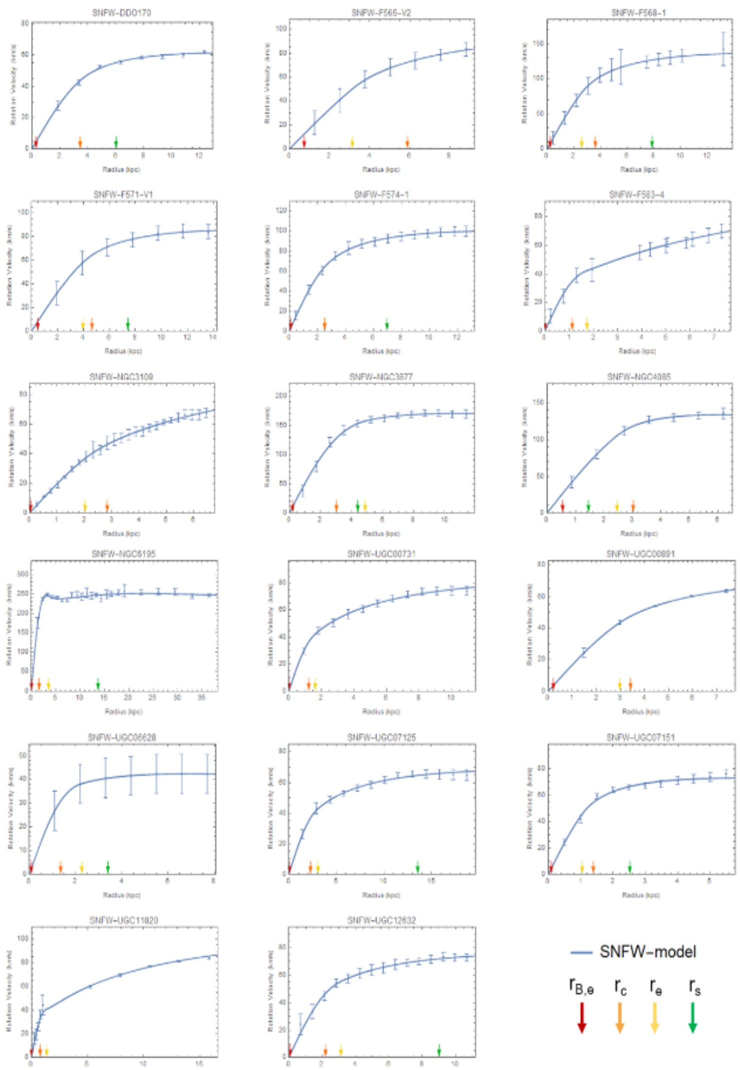
$$\rho_c \equiv 1.9 \times 10^9 (m_{a,e}/10^{-23}\mathrm{eV})^{-2} (r_c/\mathrm{kpc})^{-4} M_\odot \mathrm{kpc}^{-3}$$



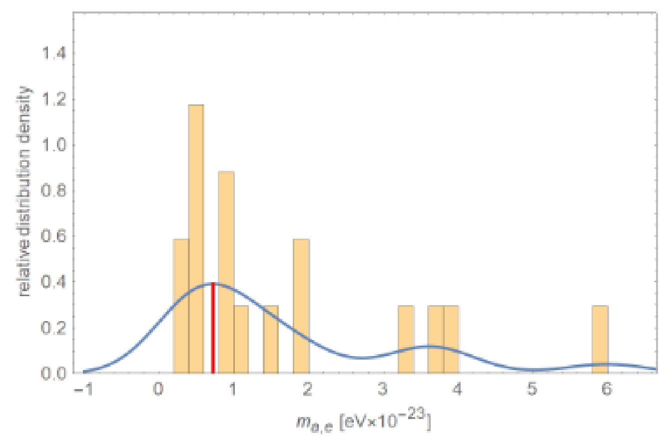
motivation: ultralight axions (fuzzy dark matter) IV

- fits of mass density to rotation curves of low surface brightness galaxies

  [J. Meinert & RH Universe 7 (2021), …]



Galaxy	Hubble Type	Luminosity [$L_\odot/pc^2$]	$\chi2$ /d.o.f.	$m_{a,e}$ [$eV\times10^{-23}$]	r_{200} [kpc]	M_{200} [$M_\odot\times10^{10}$]	$\rho_C\times10^7$ [$M_\odot/kpc^3$]	$\rho_s\times10^7$ [$M_\odot/kpc^3$]	r_s [kpc]	r_e [kpc]	r_c [kpc]	r_e/r_c
DDO170	10	73.93	0.73	$1.00_{\pm0.82}$	$36.9_{\pm8.2}$	$2.81_{\pm1.96}$	$1.34_{\pm0.38}$	$0.95_{\pm0.78}$	$6.02_{\pm1.61}$	$3.48_{\pm2.99}$	$3.45_{\pm1.39}$	1.01
F565–V2	10	40.26	0.02	$0.31_{\pm0.47}$	$60.7_{\pm11.5}$	$11.70_{\pm5.05}$	$1.60_{\pm0.21}$	$0.51_{\pm0.33}$	$12.50_{\pm4.33}$	$3.14_{\pm0.80}$	$5.89_{\pm4.38}$	0.53
F568–1	5	57.13	0.02	$0.41_{\pm0.24}$	$70.1_{\pm7.8}$	$22.60_{\pm6.67}$	$6.61_{\pm0.56}$	$2.67_{\pm1.04}$	$7.87_{\pm1.36}$	$2.59_{\pm0.60}$	$3.62_{\pm1.06}$	0.72
F571–V1	7	64.39	0.03	$0.50_{\pm0.30}$	$49.7_{\pm6.7}$	$7.12_{\pm2.83}$	$1.61_{\pm0.24}$	$1.22_{\pm0.68}$	$7.42_{\pm1.71}$	$3.96_{\pm1.21}$	$4.65_{\pm1.40}$	0.85
F574–1	7	128.48	0.02	$0.93_{\pm0.15}$	$54.0_{\pm3.1}$	$9.79_{\pm1.44}$	$5.41_{\pm0.26}$	$1.81_{\pm0.32}$	$6.98_{\pm0.46}$	$2.52_{\pm1.04}$	$2.52_{\pm0.21}$	1.00
F583–4	5	83.34	0.13	$3.87_{\pm1.37}$	$75.1_{\pm32.4}$	$17.90_{\pm13.80}$	$8.05_{\pm1.34}$	$0.12_{\pm0.13}$	$27.10_{\pm19.50}$	$1.73_{\pm0.41}$	$1.12_{\pm0.19}$	1.55
NGC3109	9	140.87	0.18	$1.14_{\pm0.39}$	$77.1_{\pm18.1}$	$19.90_{\pm5.45}$	$2.28_{\pm0.09}$	$0.14_{\pm0.08}$	$25.90_{\pm11.90}$	$2.03_{\pm0.56}$	$2.83_{\pm0.48}$	0.71
NGC3877	5	3410.59	0.17	$0.39_{\pm0.04}$	$69.4_{\pm12.5}$	$26.90_{\pm6.58}$	$15.00_{\pm0.91}$	$13.60_{\pm15.70}$	$4.40_{\pm2.43}$	$4.87_{\pm0.91}$	$3.03_{\pm0.15}$	1.61
NGC4085	5	5021.46	0.07	$0.41_{\pm0.18}$	$46.1_{\pm7.1}$	$9.26_{\pm2.78}$	$13.50_{\pm1.04}$	$103.00_{\pm135.00}$	$1.46_{\pm0.82}$	$2.46_{\pm0.41}$	$3.02_{\pm0.65}$	0.81
NGC6195	3	174.11	0.47	$0.48_{\pm0.03}$	$122.0_{\pm9.6}$	$122.00_{\pm24.60}$	$128.00_{\pm7.47}$	$2.67_{\pm0.60}$	$13.70_{\pm0.96}$	$3.55_{\pm0.12}$	$1.59_{\pm0.05}$	2.23
UGC00731	10	82.57	0.19	$3.39_{\pm0.87}$	$53.3_{\pm8.5}$	$7.62_{\pm2.90}$	$7.41_{\pm0.87}$	$0.39_{\pm0.16}$	$12.20_{\pm1.85}$	$1.62_{\pm0.34}$	$1.22_{\pm0.15}$	1.32
UGC00891	9	113.98	0.01	$0.93_{\pm0.10}$	$44.9_{\pm1.4}$	$4.78_{\pm0.36}$	$1.63_{\pm0.05}$	$0.57_{\pm0.06}$	$8.85_{\pm0.39}$	$2.97_{\pm0.19}$	$3.42_{\pm0.19}$	0.87
UGC06628	9	103.00	0.00	$3.61_{\pm0.10}$	$23.4_{\pm0.9}$	$0.77_{\pm0.05}$	$4.34_{\pm0.08}$	$1.30_{\pm0.21}$	$3.40_{\pm0.28}$	$2.27_{\pm0.07}$	$1.35_{\pm0.02}$	1.68
UGC07125	9	103.00	0.25	$1.81_{\pm0.65}$	$47.6_{\pm10.4}$	$4.97_{\pm2.68}$	$2.20_{\pm0.39}$	$0.22_{\pm0.12}$	$13.60_{\pm2.20}$	$3.06_{\pm0.94}$	$2.27_{\pm0.39}$	1.35
UGC07151	6	965.67	0.72	$1.94_{\pm2.32}$	$32.3_{\pm5.7}$	$2.53_{\pm1.58}$	$13.50_{\pm2.86}$	$7.49_{\pm5.62}$	$2.52_{\pm0.66}$	$1.05_{\pm0.46}$	$1.39_{\pm0.83}$	0.75
UGC11820	9	34.11	0.82	$5.99_{\pm4.76}$	$75.8_{\pm49.2}$	$18.10_{\pm34.70}$	$15.00_{\pm4.43}$	$0.11_{\pm0.20}$	$28.60_{\pm6.01}$	$1.35_{\pm0.66}$	$0.77_{\pm0.30}$	1.76
UGC12632	9	66.81	0.09	$1.47_{\pm0.23}$	$46.9_{\pm6.1}$	$5.56_{\pm1.48}$	$3.77_{\pm0.30}$	$0.61_{\pm0.24}$	$9.01_{\pm1.82}$	$3.12_{\pm0.49}$	$2.20_{\pm0.16}$	1.42



$\rightarrow$ $$m_{a,e} = 0.675 \times 10^{-23}\,\mathrm{eV}$$

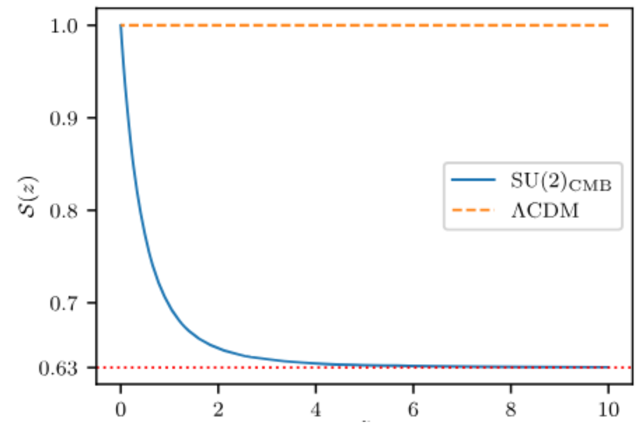
motivation: ultralight axions (fuzzy dark matter) V

- depercolation transition(s) in dark sector:
  [S. Hahn, RH, & D. Kramer MNRAS 482 (2019), J. Meinert & RH Universe 7 (2021)]

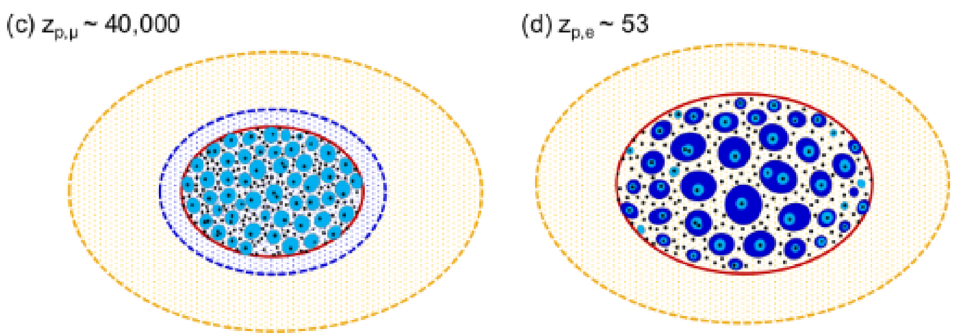
If CMB subject to SU(2) YMTD with $T_c = T_0 = 2.725$ K
[RH Ann. d. Phys. 18 (2009)]

then T–z relation $\boxed{T = S(z)\, T_0(z+1)}$ with
[S. Hahn and RH MPLA 33 (2018)]

implies that recombination occurs at
$z = 1730$ rather than $z = 1089$.



→ $\Omega_{m,0}$ at recombination 4 times smaller than in ΛCDM
→ at some intermediate redshift z_p a depercolation transition from dark energy to dark matter must occur, from CMB fits it is suggested that $z_p \simeq 50$

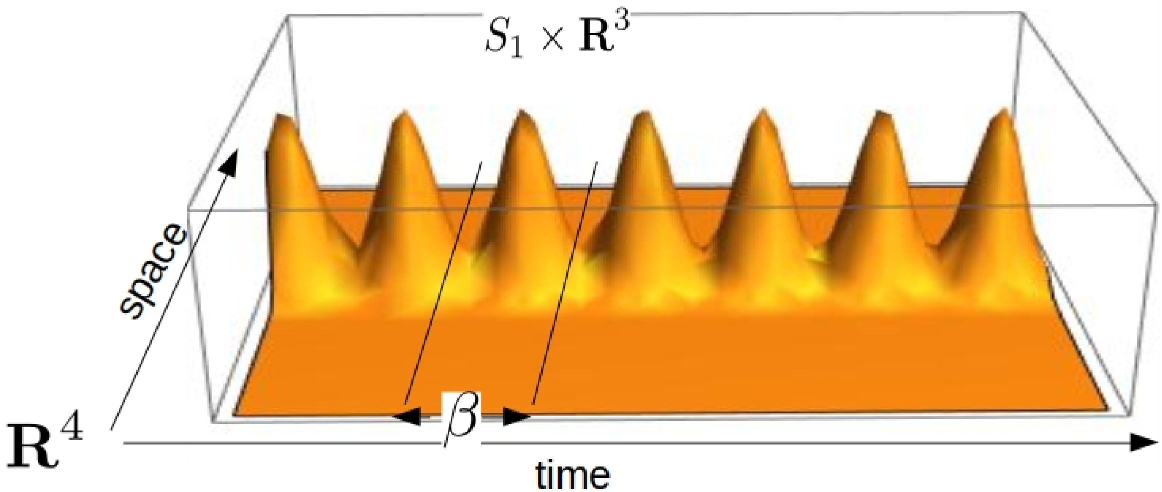


→ fuzzy dark matter explains such a transition (Hubble pull vs quantum correlations) and triggers active structure formation

interpretation of Veneziano-Witten (VW) at T>0 I

- the thermal ground state in deconfining SU(2) YMTD
  [RH, Book (2016), …]

  - Estimate (trivial holonomy): Composed of (densely packed) Harrington-Shepard (HS) (anti)calorons



superposition of (anti) instanton centers along temporal coordinate enforces **periodicity**

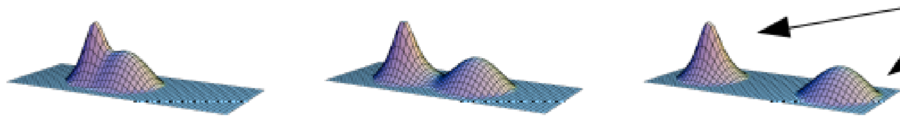
[Harrington & Shepard (1977)]

This **(anti) caloron** still has topological charge $k = \pm 1$ on $S_1 \times \mathbf{R}^3$.

  - Nontrivial holonomy:

    behavior of adjoint „Higgs field“ A_4 at spatial infinity determines transient magnetic substructure: non-trivial holonomy

    [Atiyah,Drinfeld,Hitchin, & Manin (1978); Nahm (1983); Lee & Lu (1998); Kraan & Van Baal (1998)]



pair of magnetic monopole and its antimonopole
['t Hooft (1974), Polyakov (1974), Prasad & Sommerfield (1975)]

static config.: attraction A_i cancelled by repulsion A_4

action density of $k = 1$ caloron with non-trivial holonomy on spatial plane

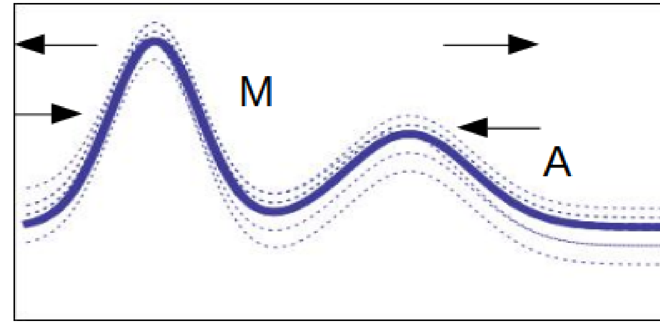
interpretation of Veneziano-Witten (VW) at $T>0$ II

- semiclassical one-loop approximation (nontrivial holonomy):

„integrating out“ Gaussian quantum fluctuations
about non-trivial holonomy caloron:

[Diakonov et al. (2004) along lines of 't Hooft (1976) for instanton]

$\rightarrow$



- small holonomy (likely): fall-back to **trivial holonomy** which therefore is **stable**
- large holonmy (unlikely): dissociation of caloron into its **monopole-antimonopole constituents**

- definition of family of phases of an inert, adjoint, & effective Higgs field

[U. Herbst, RH ISRN High Energy Phys. (2012), RH book (2016)]

$$\{\hat{\phi}^a\} \equiv \sum_{C,A} \mathrm{tr} \int d^3x \int d\rho\, t^a\, F_{\mu\nu}(\tau,\vec{0})\, \{(\tau,\vec{0}),(\tau,\vec{x})\} \times F_{\mu\nu}(\tau,\vec{x})\{(\tau,\vec{x}),(\tau,\vec{0})\},$$

where $\{(\tau,\vec{0}),(\tau,\vec{x})\} \equiv \mathcal{P}\exp\left[i\int_{(\tau,\vec{0})}^{(\tau,\vec{x})} dz_\mu\, A_\mu(z)\right], \{(\tau,\vec{x}),(\tau,\vec{0})\} \equiv \{(\tau,\vec{0}),(\tau,\vec{x})\}^\dagger\,.$

<u>unique</u>**:**
- no higher n points
- no higher k
- no curvature of lines
- no shiftability of base point

interpretation of Veneziano-Witten (VW) at $T>0$ III

- leads to:

$$\mathcal{L}_{\rm eff}[a_\mu] = {\rm tr}\,\left(\frac{1}{2}\,G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)\,.$$

where $|\phi| = \sqrt{\dfrac{\Lambda^3\beta}{2\pi}}$

[RH, Book (2016), …]

(anatomy of HS caloron, staticity of field strength for $r\gg\beta$:)
[D.J. Gross, R.D.Pisarski, & L.G. Yaffe Rev. Mod. Phys. 53 (1981)]

$$A_\mu = \bar\eta^a_{\mu\nu} t_a \partial_\nu \log \Pi(\tau, r)$$

$$\Pi = \begin{cases} \left(1+\frac{1}{3}\frac{s}{\beta}\right) + \frac{\rho^2}{x^2} & (|x| \ll \beta) \\ 1+\frac{s}{r} & (r\gg\beta) \end{cases}$$

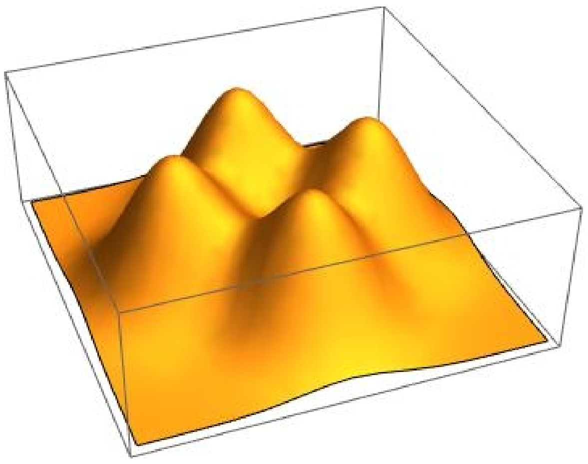
$$\left(s\equiv\frac{\pi\rho^2}{\beta}\right)$$

$$E^a_i = B^a_i \sim -\frac{\hat{x}^a\hat{x}_i}{r^2} \qquad (\,\beta \ll r \ll s\,)\,,$$

(static selfdual **monopole-field**)

$$E^a_i = B^a_i = s\,\frac{\delta^a_i - 3\,\hat{x}^a\hat{x}^i}{r^3} \qquad (r \gg s)\,.$$

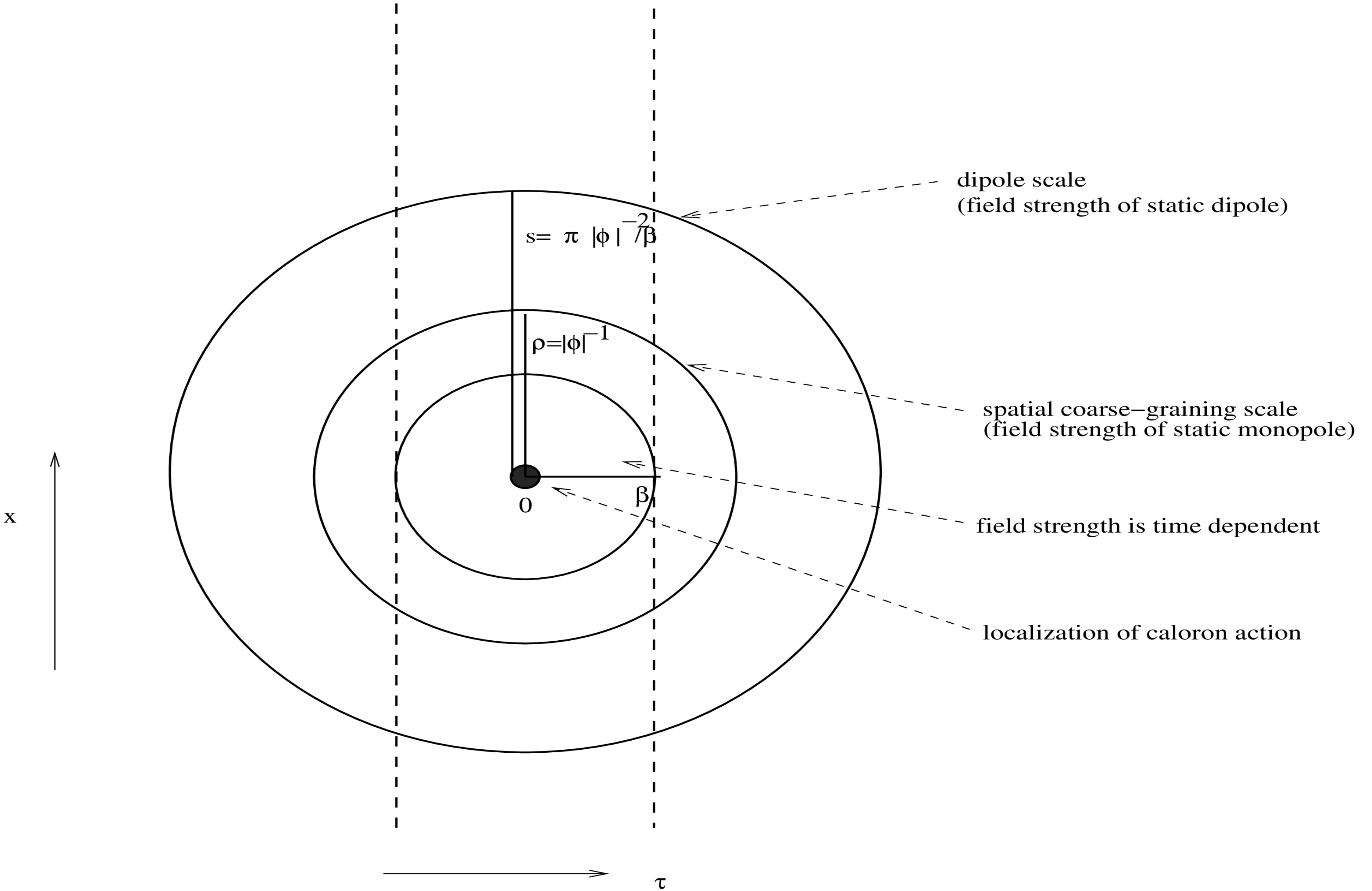
(static selfdual **dipole-field** with dipole moment: $p^a_i = s\,\delta^a_i$)



spatially densely packed (anti)caloron centers with overlapping peripheries and centers with $\rho \simeq |\phi|$

interpretation of Veneziano-Witten (VW) at $T>0$ IV

- anatomy of a relevant HS caloron
  [RH, Book (2016), …]



interpretation of Veneziano-Witten (VW) at $T>0$ V

- estimate of energy density and pressure of thermal ground state:
  [RH, Book (2016), …]

Euler-Lagrange for a_μ:

$$\boxed{D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]}$$

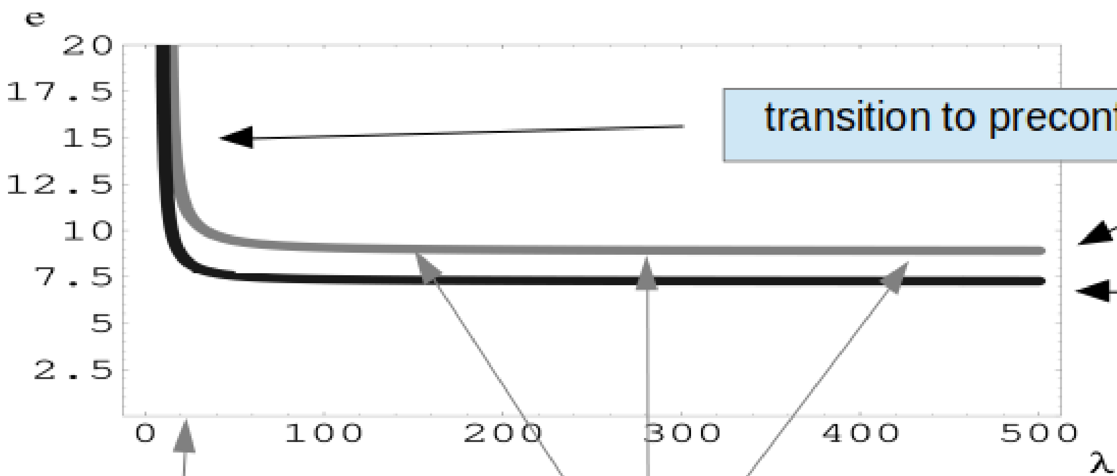
pure-gauge solution:

$$\boxed{a_\mu^{\rm gs} = \mp\delta_{\mu 4}\frac{2\pi}{e\beta}\,t_3} \quad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0) \quad \boxed{P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T\,.}$$

Overlapping, small and transient-holonomy (anti)calorons implying collapsing monopole-antimonopole pairs.

interpretation of Veneziano-Witten (VW) at $T>0$ VI

- evolution of coupling and interpretation of caloron action:
  [RH, Book (2016), …]



transition to preconfining phase

SU(2)

SU(3)

small-mass expansion, pressure
[L. Dolan & R. Jackiw, PRD 9 (1974)]

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8}\pi$$

coarse-graining dominated by $\rho \sim |\phi|^{-1}$

restoration of $\hbar$

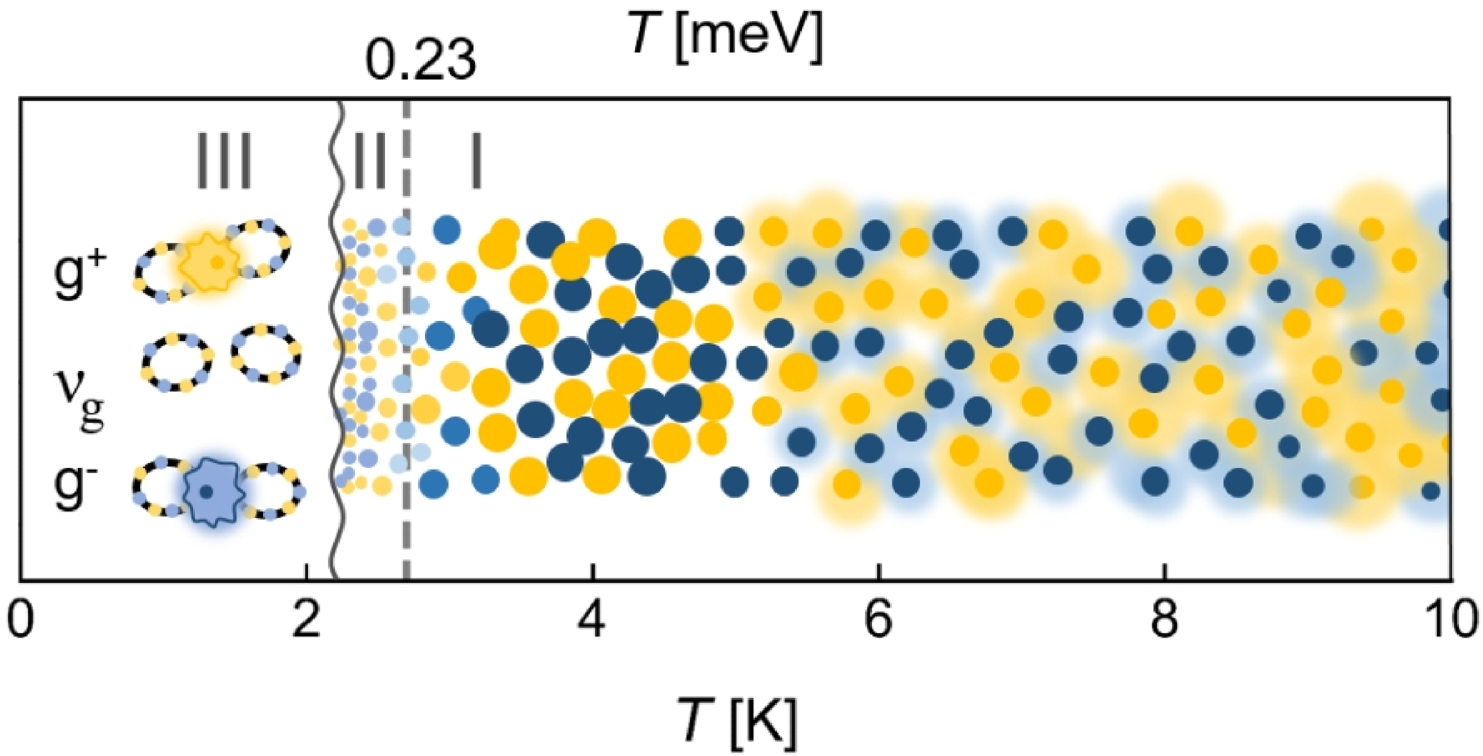
$$e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}}\,.$$

$$S_{C/A} = \hbar\,.$$

[S. B. Brodsky & P. Hoyer PRD 83 (2011); D. Kaviani & RH Quant. Matt (2012)]

interpretation of Veneziano-Witten (VW) at $T>0$ VII

- phase structure SU(2) YMTD:
  [RH, Book (2016), RH Ann. d. Phys. (2009), D.J. Fixsen ApJ 734 (2011)]



$$\mathrm{SU}(2)_{\mathrm{CMB}}$$

interpretation of Veneziano-Witten (VW) at $T>0$ VIII

- active vs passive SU(2) Yang-Mills factors:
  [D. Antonov, RH, J. Meinert Astronomy (2024)]

independent SU(2) factors s.t. their YM-scales act differently in deconfining and confining phases due to different nature of (relevant) topological charge carriers and their different densities

$$\boxed{\lim_{N_c\to\infty}\frac{m_A^2 f^2}{2N_f}=\lim_{N_c\to\infty}\int d^4x\langle Q(x)Q(0)\rangle_T\equiv\chi}$$

pure SU(N_c)

→ independent SU(2) factors generate independent axions s.t. different masses for one and the same Peccei-Quinn scale f/$(2\,N_f)^{1/2}$.

previous results on $\chi(T)$ I

- dilute instanton gas approximation

[D.J. Gross, R.D. Pisarski, & L.G. Yaffe Rev. Mod. Phys. 53 (1981), R.D. Pisarski, & L.G. Yaffe PLB 97B (1980), ...]

$$\boxed{\chi(T) \simeq \frac{Z_I(T) + Z_{\bar{I}}(T)}{\mathcal{V}} = 2\int_0^\infty d\rho\, D(\rho)\, G(\pi\rho T),}$$

with

$$D(\rho) \equiv \frac{d_{\overline{\mathrm{MS}}}}{\rho^5}\left(\frac{2\pi}{\alpha_{\overline{\mathrm{MS}}}(\mu_r)}\right)^6 \exp\left(-\frac{2\pi}{\alpha_{\overline{\mathrm{MS}}}(\mu_r)}\right) \times (\rho\,\mu_r)^{\beta_0}\left[1 + \mathcal{O}(\alpha_{\overline{\mathrm{MS}}}(\mu_r))\right],$$

$$G(x) \equiv \exp\left\{-2x^2 - 18A(x)\right\}$$

$$A(x) \simeq -\frac{1}{12}\ln\left[1 + (\pi\rho T)^2/3\right] + \alpha\left[1 + \gamma(\pi\rho T)^{-3/2}\right]^{-8}$$

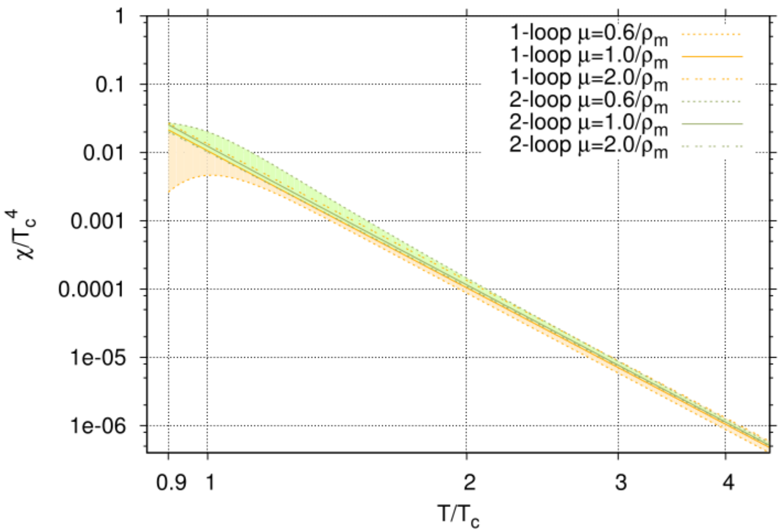
<u>differences compared to thermal ground state:</u>

- diluteness
- Debye screening prevents large instanton radii from contributing: $\rho < 1/(\sqrt{2}\pi T)$
  but in thermal ground state $\rho = \sqrt{2\pi T/\Lambda^3}$

previous results on $\chi(T)$ II

- dilute instanton gas approximation (SU(3))

[S. Borsanyi Phys. Lett B 752 (2016), ….]



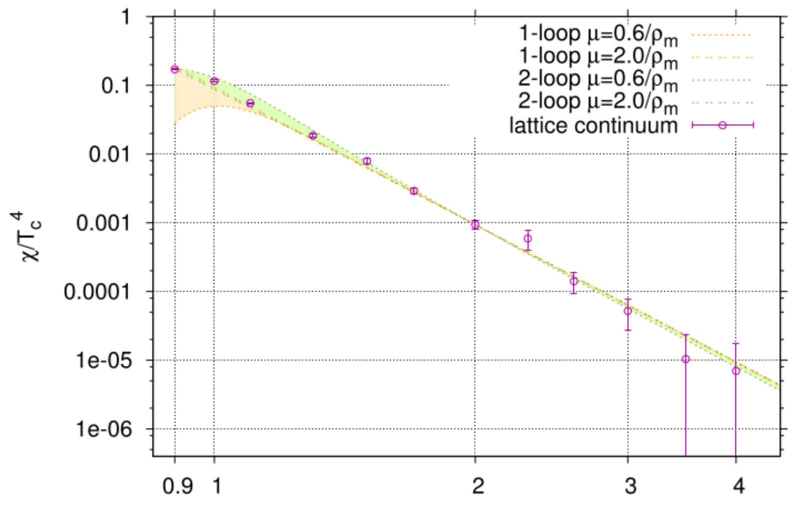
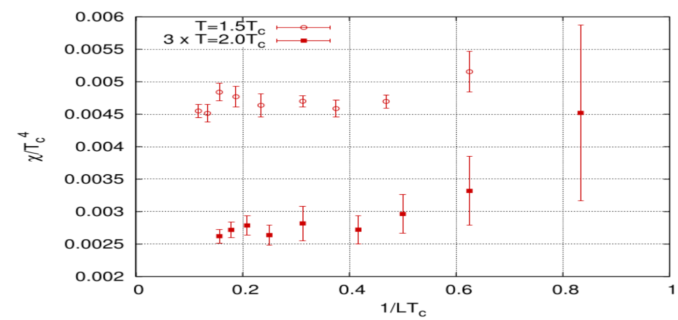


Table 2
Temperature slopes of the topological susceptibility predicted in the two-loop RGI DIGA, for a range of renormalization scales according to Eq. (24).

T/T_c	1.5	2	3	4	5
b ($\kappa = 0.6$)	−6.04	−6.26	−6.43	−6.50	−6.55
b ($\kappa = 1$)	−6.37	−6.46	−6.55	−6.59	−6.62
b ($\kappa = 2$)	−6.55	−6.59	−6.64	−6.67	−6.69



(exponent of power-law decay)

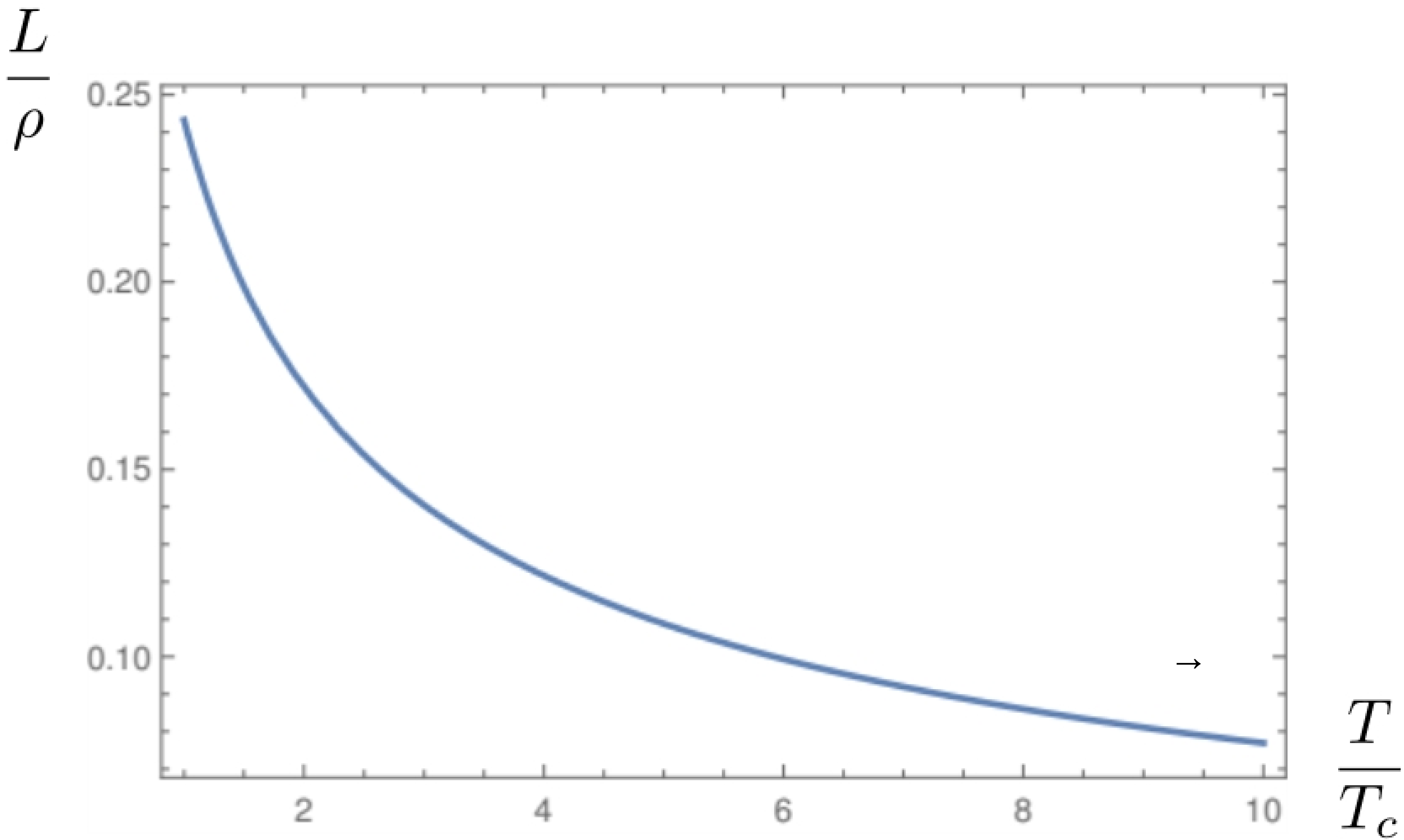
(claimed control of $V \to \infty$)

DIGA

rescaled DIGA (with factor of 10!), lattice

previous results on $\chi(T)$ III

- however: lattice size $L=2/T_c$ vs size ρ of a relevant caloron (thermal ground state) *[S. Borsanyi Phys. Lett B 752 (2016), RH book (2016)]*



→ lattice allows for calorons only that are irrelevant for emergence of thermal ground state

previous results on $\chi(T)$ IV

- SU(3) lattice fits of DIGA and Interacting Instanton Liquid Model (IILM)

  [E. Berkowitz et al. PRD 92 (2015), F. Burger et al. Nucl. Phys. A 967 (2017)]

$$\frac{\chi_{\mathrm{DIGM}}}{T_c^4} = \frac{C}{(T/T_c)^n},$$

$$\frac{\chi_{\mathrm{IILM}}}{T_c^4} = \frac{e^{d_0}}{(T/T_c)^{-d_1}} \exp\left[d_2\left(\ln\frac{T}{T_c}\right)^2 + d_3\left(\ln\frac{T}{T_c}\right)^3\right]$$

TABLE II. Fit parameters for the DIGM and IILM fit to all of our best data points.

DIGM	
$\chi^2/\mathrm{d.o.f.} = 1.2$	
C	0.0869 ± 0.0015
n	5.64 ± 0.04
IILM	
$\chi^2/\mathrm{d.o.f.} = 1.7$	
e^{d_0}	0.079 ± 0.006
d_1	-4.9 ± 0.5
d_2	-1.7 ± 1.0
d_3	1.2 ± 0.7

→ factor of ~10 in normalisation needed to match DIGA or IILM with lattice result

previous results on $\chi(T)$ V

- paper on approach via master field equations implying large lattice volumes

4 Conclusions

Dimensional analysis suggests that the topological susceptibility grows proportionally to T^4 at high temperatures T, but instead it decreases rapidly as a result of a nearly perfect cancellation of short- and long-distance contributions. This behaviour is commonly attributed to the topological nature of the charge density $q(x)$, i.e. to the fact that variations of $q(x)$ with respect to the gauge field are total derivatives. None of the non-perturbatively well-defined expressions for the susceptibility known to date however embodies this property of the charge density to the extent that the smallness of the susceptibility at high temperatures would be explained.

[L. Giusti & M. Lüscher Eur. Phys. J. C (2019)]

previous results on χ VI

- paper on $\langle Q(x)Q(0)\rangle$ $(T{=}0\ )$ which should be negative for $x \neq 0$ due to reflection positivity

$$\widehat{G}(p) = b_1 + b_2 p^2 + b_3 (p^2)^2 - \frac{R^2}{p^2 + m} \quad + (p^2)^3 \int_{m^2}^{\infty} \sigma(t) \frac{1}{(t + p^2)t^3} \, \mathrm{d}t$$

$$b_1 = \frac{R^2}{m_{\eta'}^2}. \tag{15}$$

By standard arguments one derives from this relation a WV-like formula, in which, however, the contact term b_1 takes the place of χ_t^{qu}. This was essentially the proposal made in [5] (where, however, an unsubtracted dispersion relation was used, which is only justified *after* approximating the spectral density ρ by a δ-function at the η' mass; see also [12]).

[E. Seiler Phys. Lett. B 525 (2002)]

$\rightarrow$ axion mass chiefly arises from contact term.

deconfining thermal ground state and axion mass $m_a(T)$ I

- relevant Harrington-Shepard (anti)calorons and their size

performing ρ integration in

$$\{\hat{\phi}^a\} \equiv \sum_{C,A} \mathrm{tr} \int d^3x \int d\rho \, t^a \, F_{\mu\nu}(\tau,\vec{0}) \, \{(\tau,\vec{0}),(\tau,\vec{x})\} \times F_{\mu\nu}(\tau,\vec{x})\{(\tau,\vec{x}),(\tau,\vec{0})\},$$

where $\{(\tau,\vec{0}),(\tau,\vec{x})\} \equiv \mathcal{P}\exp\left[i\int_{(\tau,\vec{0})}^{(\tau,\vec{x})} dz_\mu \, A_\mu(z)\right], \{(\tau,\vec{x}),(\tau,\vec{0})\} \equiv \{(\tau,\vec{0}),(\tau,\vec{x})\}^\dagger .$

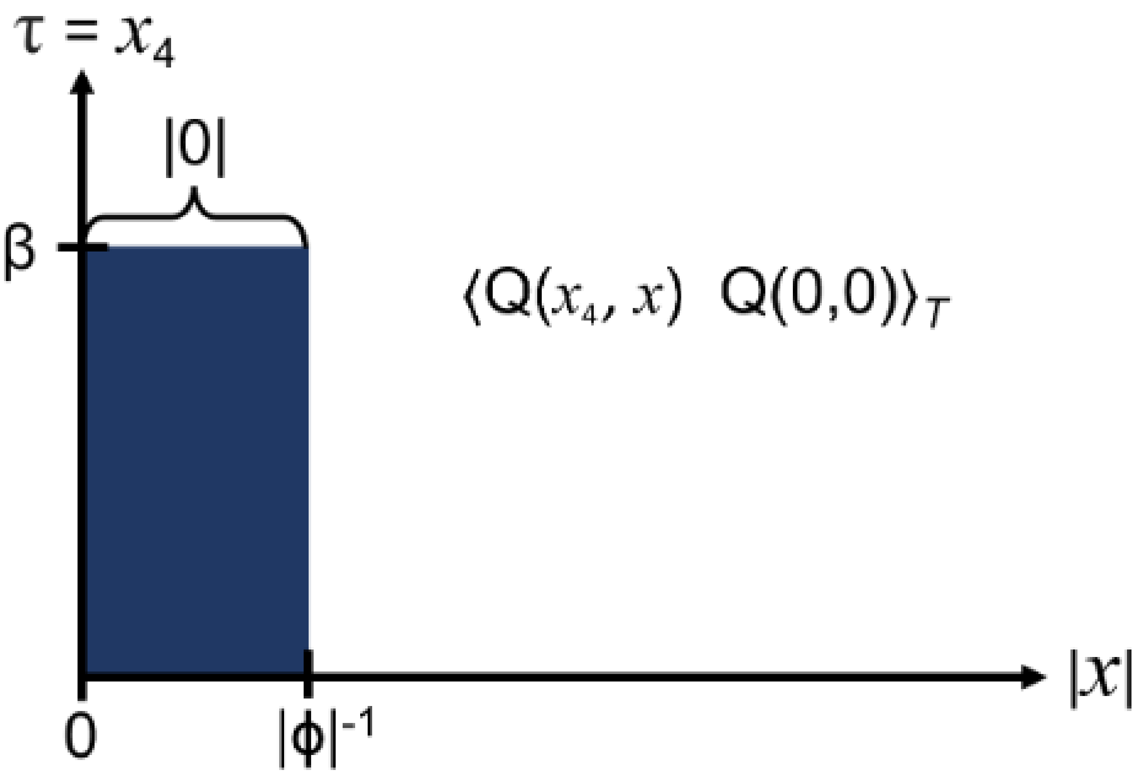
yields <u>cubic</u> dependence on cutoff, integral dominated by small region around $\rho = |\,\phi\,|^{-1}$ (or $\frac{\rho}{\beta} = \frac{|\,\phi\,|^{-1}}{\beta} = \frac{\lambda^{3/2}}{2\pi}$)

[U. Herbst & RH ISRN High Energy Phys. (2012), RH book (2016)]

→ emergence of thermal ground state through spatially densely packed caloron centers occurs with <u>increasing rather than decreasing caloron radii at increasing temperature</u>.

deconfining thermal ground state and axion mass $m_a(T)$ II

- results on contact contribution to $\chi(T)$ in effective theory, by (anti) caloron centers



→

[D. Antonov, RH & J. Meinert (2024)]

deconfining thermal ground state and axion mass $m_a(T)$ III

- technically:

[D. Antonov, RH & J. Meinert (2024)]

$$A_\mu^a = -\bar\eta^a_{\mu\kappa}\frac{\partial_\kappa \Pi}{\Pi} \quad \text{with} \quad \Pi(\tau,r) = 1+\frac{\rho^2}{\beta r}\frac{\sinh\frac{2\pi r}{\beta}}{\cosh\frac{2\pi r}{\beta}-\cos\frac{2\pi\tau}{\beta}} \quad \xrightarrow{\rho=|\phi|^{-1}=\sqrt{\frac{2\pi T}{\Lambda^3}}} \quad \Pi(\tau,r,T)=\bar\Pi(y,x,\lambda)=1+\frac{\lambda^3}{4\pi x}\frac{\sinh 2\pi x}{\cosh 2\pi x-\cos 2\pi y}$$

$$\rightarrow \quad F^a_{\mu\nu} = \bar\eta^a_{\mu\nu}P - \bar\eta^a_{\nu\kappa}P_{\mu\kappa} + \bar\eta^a_{\mu\kappa}P_{\nu\kappa} \quad \text{with} \quad P \equiv \frac{(\partial_\kappa\Pi)(\partial_\kappa\Pi)}{\Pi^2}, \quad P_{\mu\nu} \equiv \frac{\Pi(\partial_\mu\partial_\nu\Pi)-2(\partial_\mu\Pi)(\partial_\nu\Pi)}{\Pi^2}$$

$$\rightarrow \quad Q \equiv \frac{1}{32\pi^2}F^a_{\mu\nu}F^a_{\mu\nu} = \frac{1}{8\pi^2}\left(P^2_{\mu\nu}-P^2\right)$$

$$\rightarrow \quad \boxed{\begin{aligned} F^a_{\mu\nu}F^a_{\mu\nu} \;=\; & 4\Bigg(3\left(\frac{\partial_r\Pi}{\Pi}\right)^4+\left(\frac{\partial_r^2\Pi}{\Pi}\right)^2+2\left(\frac{\partial_r\Pi}{r\Pi}\right)^2+3\left(\frac{\partial_\tau\Pi}{\Pi}\right)^4+ \\ & \left(\frac{\partial_r\Pi}{\Pi}\right)^2\left(-4\frac{\partial_r^2\Pi}{\Pi}+6\left(\frac{\partial_\tau\Pi}{\Pi}\right)^2\right)-4\left(\frac{\partial_\tau\Pi}{\Pi}\right)^2\frac{\partial_\tau^2\Pi}{\Pi}+\left(\frac{\partial_\tau^2\Pi}{\Pi}\right)^2- \\ & 8\frac{\partial_r\Pi}{\Pi}\frac{\partial_\tau\Pi}{\Pi}\frac{\partial_r\partial_\tau\Pi}{\Pi}+2\left(\frac{\partial_r\partial_\tau\Pi}{\Pi}\right)^2\Bigg). \end{aligned}}$$

$$\rightarrow \quad \boxed{\lim_{r\to 0}Q(\tau,r,T) = T^4\lim_{x\to 0}\bar Q(y,x,\lambda) = 2\pi^2T^4\lambda^6\frac{(8+\lambda^3+4\cos 2\pi y)^2}{3(2+\lambda^3-2\cos 2\pi y)^4}}$$

deconfining thermal ground state and axion mass $m_a(T)$ III

- technically, cntd.:

[D. Antonov, RH & J. Meinert (2024)]

topological charge: $k = 4\pi \int_0^{\frac{\lambda^{3/2}}{2\pi}} dx\, x^2 \bar{Q}(\frac{1}{2}, x, \lambda)$ →

$k = 0.9768, 0.9927, 0.9938$ for $\lambda = \lambda_c, 2\,\lambda_c, 2.5\,\lambda_c$, respectively

→ caloron topological charge contained in center, peripheries do not contribute

→

$$
\begin{aligned}
\chi_{\text{cal. cen.}} &= \int_{\text{cal. cen.}} d\tau d\Omega dr\, r^2\, Q(\tau, r, T) Q(0,0,T) \\
&= 4\pi T^4\, \bar{Q}(0,0,\lambda) \int_0^1 dy \int_0^{\frac{\lambda^{3/2}}{2\pi}} dx\, x^2 \bar{Q}(y, x, \lambda) \\
&\sim 4\pi T^4\, \bar{Q}(0,0,\lambda) \int_0^{\frac{\lambda^{3/2}}{2\pi}} dx\, x^2 \bar{Q}(\frac{1}{2}, x, \lambda) \\
&\sim \frac{2}{3} \frac{(12+\lambda^3)^2 \pi^2}{\lambda^6}\, T^4 \\
&\xrightarrow{\lambda \gg \lambda_c} \frac{2}{3} \pi^2\, T^4 ,
\end{aligned}
$$

deconfining thermal ground state and axion mass $m_a(T)$ IV

- contributions of effective quasiparticles to $\chi(T)$ in effective theory
  [D. Antonov, RH & J. Meinert (2024)]

  - effective theory has thermal quasiparticle excitations which are either massless (photons) or $T-$ dependently massive (vector modes)

  - upon factorisation of $\chi(T)$ one obtains:

$$\langle Q(x)Q(0)\rangle_{\text{factorized}} = \frac{1}{128\pi^4}\left[\langle g^2 F^a_{4i}(x)F^b_{jk}(0)\rangle^2 + 2\langle g^2 F^a_{4i}(x)F^b_{ij}(0)\rangle\langle g^2 F^a_{4k}(x)F^b_{jk}(0)\rangle\right]$$
$$= -\frac{N_c^2-1}{32\pi^4}\mathbf{x}^2 f^2\,.$$

where $\langle g^2 E^a_i(x)B^b_k(0)\rangle = \delta^{ab}\varepsilon_{ikn}x_n f(x)\,.$

  - at sufficiently high T free massless modes (only $k=\pm 1$ Matsubara modes) propagate with:

$$f_{\text{free}}(x) \simeq \frac{2\pi e^2 T^3}{\mathbf{x}^2}\left(1+\frac{1}{2\pi T|\mathbf{x}|}\right)\mathrm{e}^{-2\pi T|\mathbf{x}|}\sin(2\pi T x_4)\,.$$

deconfining thermal ground state and axion mass $m_a(T)$ V

- therefore, correlator of topological charge:

$$\langle Q(x)Q(0)\rangle_{\text{factorized, free}} \simeq -\frac{N_c^2-1}{8\pi^2}\frac{(e^2T^3)^2}{\mathbf{x}^2}\left(1+\frac{1}{2\pi T|\mathbf{x}|}\right)^2 \mathrm{e}^{-4\pi T|\mathbf{x}|}\sin^2(2\pi T x_4)\,.$$

- finally, topological susceptibility:

$$\chi_{\text{factorized, free}} \sim \int \mathrm{d}^3x \int_0^\beta \mathrm{d}x_4 \langle Q(x)Q(0)\rangle_{\text{factorized, free}}$$

$$= -\frac{N_c^2-1}{8\pi^2}(e^2T^3)^2\frac{1}{2T}\cdot 4\pi\int_{|\phi|^{-1}}^\infty \mathrm{d}|\mathbf{x}|\left(1+\frac{1}{2\pi T|\mathbf{x}|}\right)^2 \mathrm{e}^{-4\pi T|\mathbf{x}|}$$

reflection positivity

$$\rightarrow \quad |\chi_{\text{factorized, free}}| \simeq \frac{N^2-1}{16\pi^2}(eT)^4\,\mathrm{e}^{-2\lambda^{3/2}} \leq 1.7\times 10^{-44}T^4\,,\quad (\lambda \geq 2\lambda_c, N_c=2)\,.$$

- for massive quasiparticles, additional exponental suppression through mass:

$$\rightarrow \quad \chi_{\text{factorized, free, massive}} \simeq -\frac{3}{2}\frac{N_c^2-1}{32\pi^4}(eT)^4\frac{1}{2T}\left[(2\pi T)^2+m^2\right]\cdot 4\pi\int_{|\phi|^{-1}}^\infty \mathrm{d}|\mathbf{x}|\,\mathrm{e}^{-2\sqrt{(2\pi T)^2+m^2}\,|\mathbf{x}|}$$

$$= -3\frac{N_c^2-1}{64\pi^3}e^4T^3\sqrt{(2\pi T)^2+m^2}\,\mathrm{e}^{-2\frac{\sqrt{(2\pi T)^2+m^2}}{2\pi T}\lambda^{3/2}}\,.$$

deconfining thermal ground state and axion mass $m_a(T)$ VI

summary:

- In effective theory $\chi(T)$ arises from a contact term due to (anti)caloron centers with

$$\chi_{\rm cal.cent.}(T) \to \frac{2}{3}\pi^2 T^4 \qquad (\lambda \gg \lambda_c)\,.$$

- Thermal quasiparticles practically do not contribute to $\chi(T)$.

- This result is in stark contrast to dilute instanton gas (DIGA) or interacting instanton liquid (IILM) approaches or the lattice where

$$\chi(T) \propto T^{-5\cdots-7}$$

(see, however, M. Lüscher's remark)

Discrepancy likely to be rooted in insufficient lattice volumes and diluteness assumption in DIGA and IILM (implying small-size (anti)calorons to contribute only) when thermal ground state associates with densely packed (anti)calorons of size that is increasing with power ½ in T.

cosmological implications of $m_a(T)$ I

- CMB, lepton families and associated axion masses:

- in confining phases of SU(2) Yang-Mills theories and modulo mixing effects axion masses are given as

$$\boxed{m_a = \frac{\Lambda^2}{M_P}}$$

if a <u>common</u> Peccei-Quinn scale M_P is assumed.

[F. Giacosa, RH & M. Neubert JHEP02 (2008)]

- Λ can be exctracted from astrophysical data (rotation curves)

- If the lepton families of the SM are subject to (mixed) SU(2) Yang-Mills theories (J. Meinert's talk) then for first lepton family one can deduce a hierarchy →

$$\boxed{\frac{\Lambda_{e,\mathrm{dec}}}{\Lambda_{e,\mathrm{con}}} = \frac{3600\,\mathrm{eV}}{287\,\mathrm{eV}} = 12.54} \quad \text{with} \quad \frac{m_e}{\Lambda_{e,\mathrm{dec}}} = 141.82.$$

[J. Meinert & RH Universe 7 (2024)]

cosmological implications of $m_a(T)$ II

- for other lepton families' YM-scales $\Lambda_{\mu,\mathrm{con}}$ and $\Lambda_{\tau,\mathrm{con}}$ use:

$$\boxed{\Lambda_\mu = \Lambda_e\, m_\mu / m_e \ \text{ and } \Lambda_\tau = \Lambda_e\, m_\tau / m_e}$$

- this yields axion masses:

$$\boxed{\begin{aligned} m_{a,e} &\sim 0.675 \times 10^{-23}\,\mathrm{eV}\,, \\ m_{a,\mu} &\sim 2.89 \times 10^{-19}\,\mathrm{eV}\,, \\ m_{a,\tau} &\sim 8.17 \times 10^{-17}\,\mathrm{eV}\,. \end{aligned}}$$

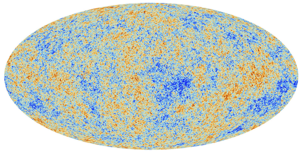
[J. Meinert & RH Universe 7 (2021)]

$\rightarrow$

- assuming gravitational virialisation, these axions can be shown to move nonrelativistically slowly: $v_{A,i}(z) = \frac{M(z) m_{A,i}(z)}{M_P^2}$ $\rightarrow$ $v \sim \sqrt{2}/314 \ll 1$

[S.-J. Sin PRD 50 (1994), J. Meinert & RH, Universe 7 (2021)]

cosmological implications of $m_a(T)$ III



- What about axion associated with $\mathrm{SU}(2)_{\mathrm{CMB}}$ for which $\Lambda_{\mathrm{CMB,dec}} \sim 10^{-4}\,\mathrm{eV}$?
  [RH Ann. d. Phys. 18 (2009)]

  - such axions with $m_{A,\mathrm{CMB},0} = 10^{-35}\,\mathrm{eV}$ form a super-horizon sized condensate: $\lambda_C > H_0^{-1}$

  - their speed is estimated to be: $\boxed{v_{A,\mathrm{CMB}}(z=0) \sim 5.04\times 10^{-4} \ll 1}$

  - in a matter dominated Universe, we have: $v_{A,i} = v_{A,i,0}(1/4)^{2/3}\,(z+1)^{1/2}$
  (in a radiation dominated Universe, $v_{A,i}$ does not evolve)

→ - with matter-radiation equality at $z \sim 3000$ we thus have:

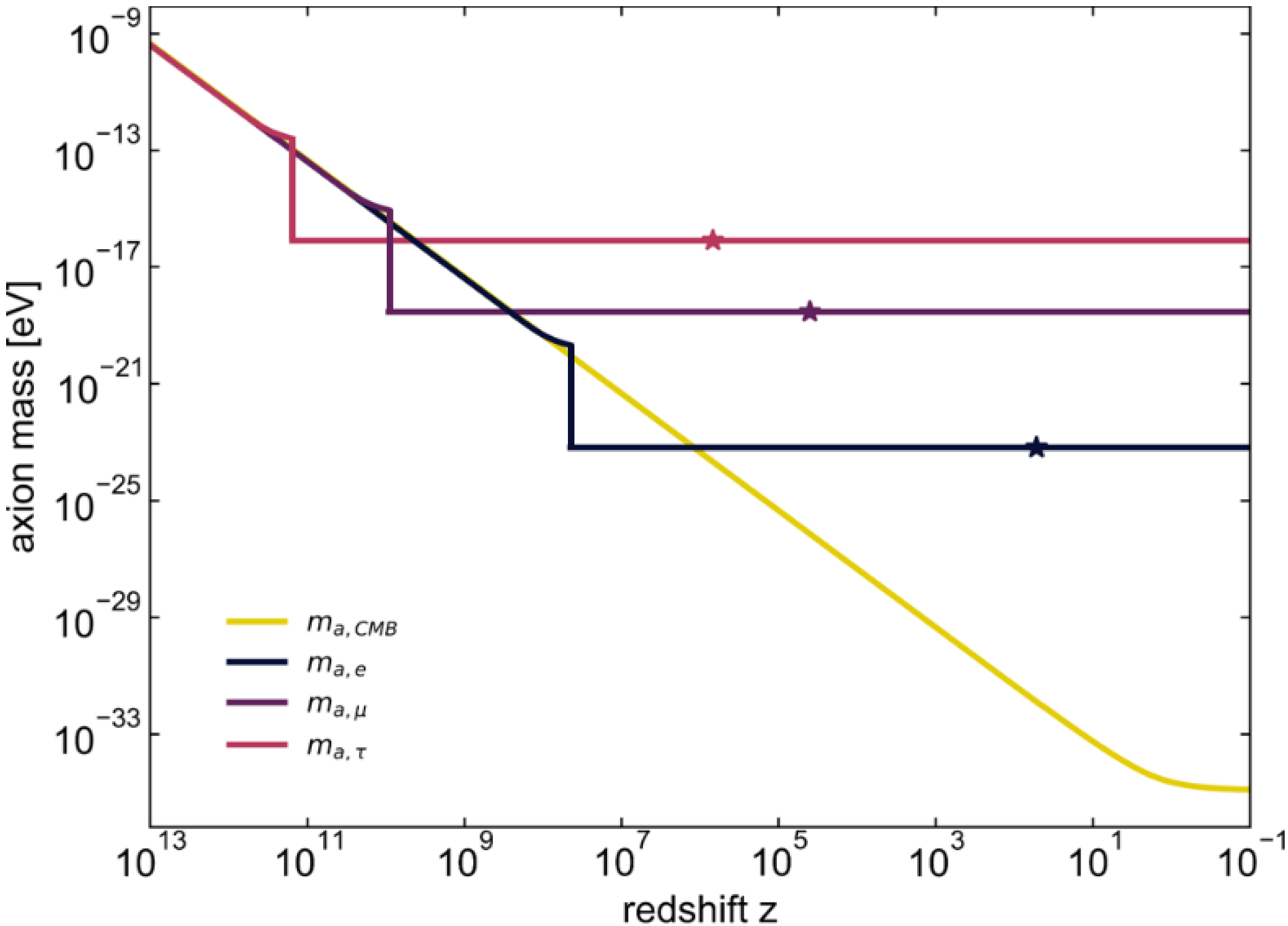
$$\boxed{(1/4)^{2/3}(3000+1)^{1/2} = 21.7}$$

therefore CMB axions remain nonrelativistic through entire expansion history

[D. Antonov, RH & J. Meinert (2024)]

cosmological implications of $m_a(T)$ IV

- Evolution of axion masses:



cosmological implications of $m_a(T)$ V

- Bose condensation of axions in deconfining phases: (nonrelativistic particles)

-

$$\boxed{T_{c,B} = \frac{2\pi}{m_A}\left(\frac{n_A}{\zeta\left(\frac{3}{2}\right)}\right)^{\frac{2}{3}}}$$

- assuming z−independent energy densities $\rho_{\Lambda_i} = m_{A,i} n_{A,i}$ which is justifiable in deconfining phases,

→ one derives from $m_{A,i} = \sqrt{\frac{2}{3}}\pi\frac{T^2}{\Lambda_i}$:

$\ll 1$

$$\boxed{T_{c,B,i} = \left(\frac{486}{\pi^4}\right)^{1/26}\left(\frac{\rho_{\Lambda_i}}{\zeta\left(\frac{3}{2}\right)}\right)^{2/13} M_P^{5/13}\,.}$$

- with $\rho_{\Lambda_{\rm CMB}} = (10^{-3}\,{\rm eV})^4$ and $M_P = 1.22\times 10^{28}\,{\rm eV}$ this yields:

$$\boxed{T_{c,B,{\rm CMB}} = 8.3\times 10^8\,{\rm eV}} \quad \text{or} \quad \boxed{z_{\rm c,B,CMB} > 10^{12}}\,.$$

summary

- motivation for ultralight axions:
  <u>observationally:</u> small-scale cosmology, depercolation
  <u>theoretically:</u> common PQ scale M_P for SU(2) Yang-Mills theories associating with the lepton families and with the CMB

- how to interpret VW at $T>0$: dec. thermal ground state, SU(2) phase structure, active YM factors on the right-hand side of VW

- previous approaches:
    - DIGA
    - ILLM
    - Lattice gauge theory

- deconfining thermal ground state and axion mass $m_a(T)$:
    - contact term
    - effective quasiparticle excitations

- cosmological implications of $m_a(T)$:
    - axions are slow
    - Bose condensation of various species is lifted at similarily high $z \simeq 10^{12}$

outlook

- Schrödinger-Poisson for multi-lump merger physics in galactic center subject to $m_{a,\mu} = 3 \times 10^{17}\mathrm{eV}$, onset of collapse using relativistic description

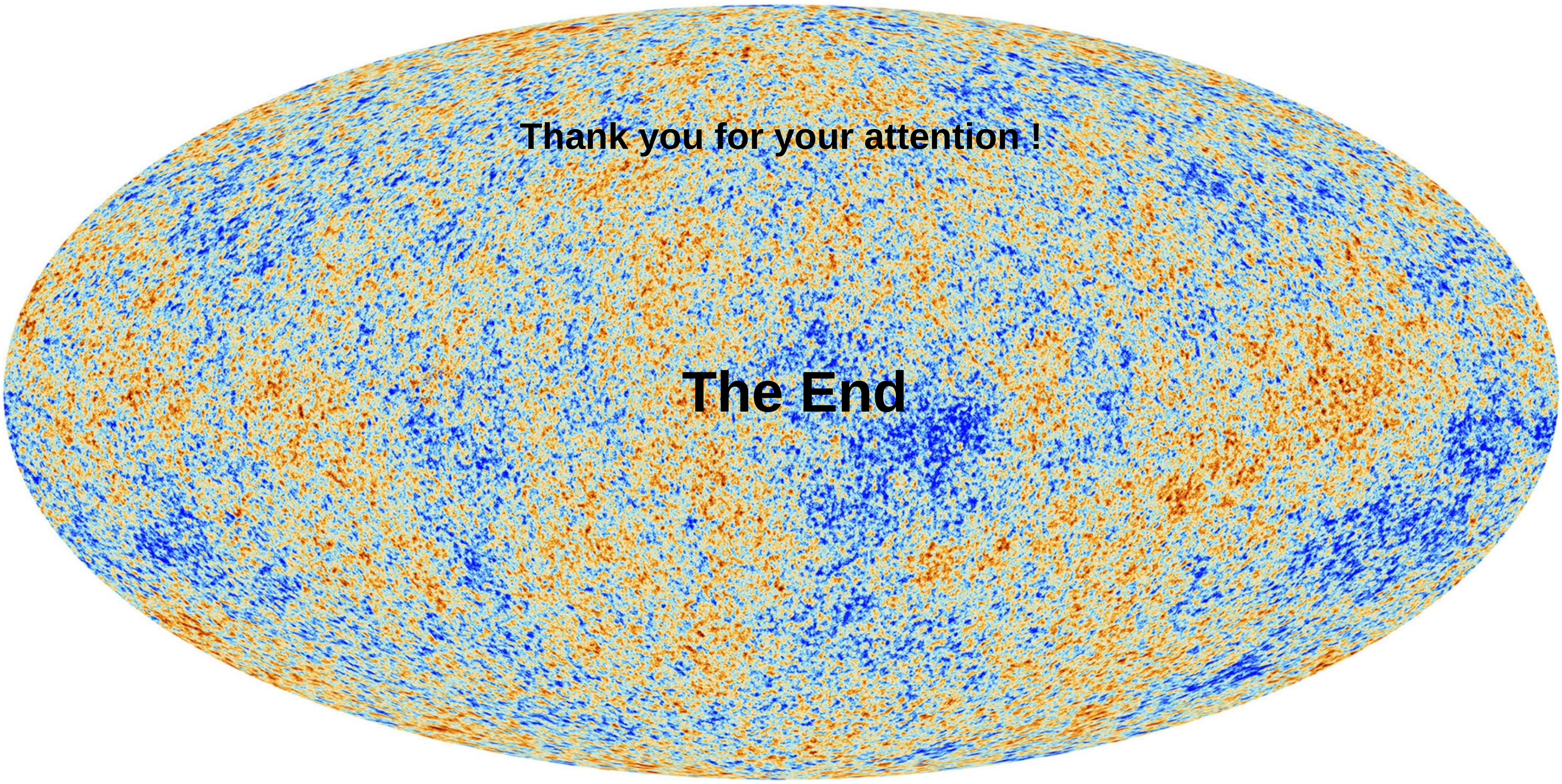
- Schrödinger-Poisson for globular clusters

- Schrödinger-Poisson for depercolation

- cosmological model for very early universe, including revised BBN (missing baryons)

- experimental search with terrestial blackbody for radiative anomalies at low T and Rayleigh-Jeans (ongoing: INRIM Torino)

- high electron density plasmas at $T \sim 10^4\,\mathrm{eV}$


Thank you for your attention !
The End

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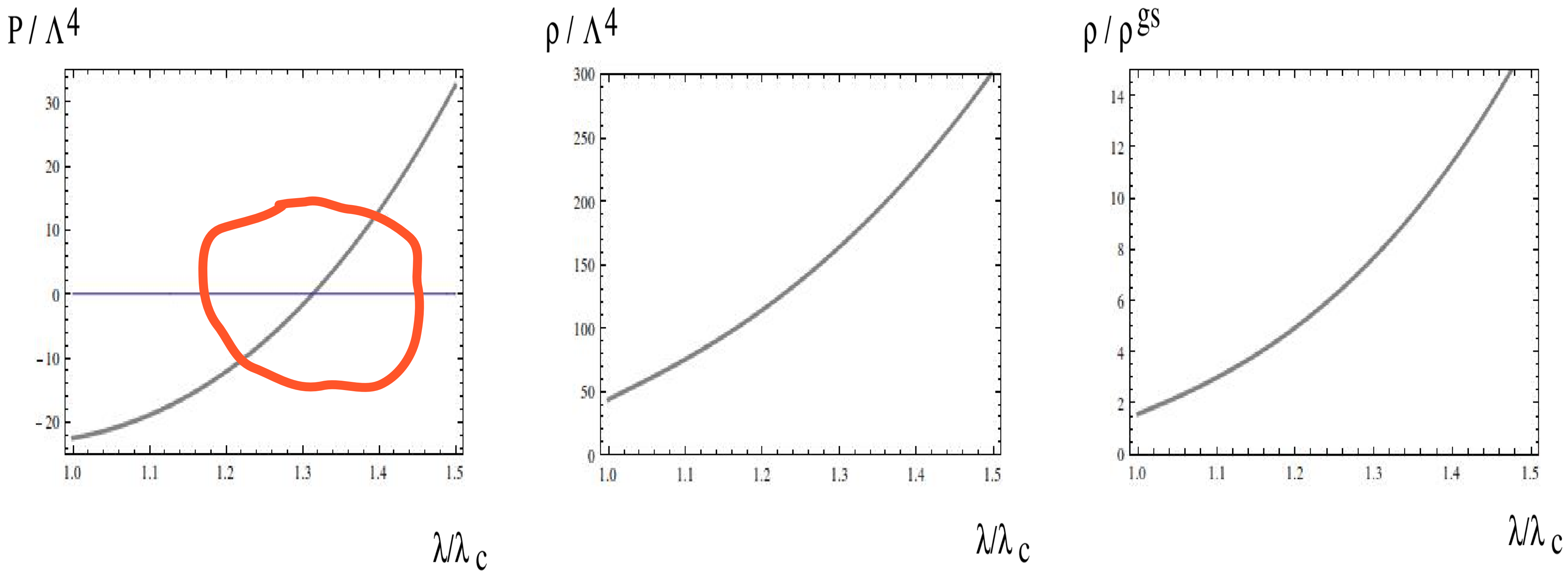
etc, etc ...

literature:

Yang-Mills thermodynamics:

R.H. (2016), The thermodynamics of quantum Yang-Mills theory. Theory and Applications, Second Edn. *World Scientific.*

Deconfining thermodynamics: pressure and energy density



(a) (b) (c)

total pressure

total energy density

energy densities:
total vs. ground state

Electric-magnetically dual interpretations of $\mathrm{U}(1) \subset \mathrm{SU}(2)$

– if SU(2) something to do with photons [RH (2005), Grandou & RH (2015), etc]
then **electric-magnetically dual** interpretation of $\mathrm{U}(1) \subset \mathrm{SU}(2)$ required:
in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\boxed{\alpha = \frac{Q^2}{4\pi\hbar}\,,}$$

– for α to be unitless:

$$\boxed{Q \propto \frac{1}{e}\,.} \qquad \left(e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}} \right)$$

But: magnetic coupling in SU(2)

$$\boxed{g = \frac{4\pi}{e}\,.}$$

SU(2) to be interpreted in an **electric-magnetically dual way**.
(e.g., magnetic monopole $\longleftrightarrow$ electric monopole,
electric dipole moment $\longleftrightarrow$ magnetic dipole moment, etc.)

Deconfining thermodynamics: radiative corrections

A (anti)caloron center **localises Planck's quantum of action** $\hbar$. [Kaviani, RH 2012]

Such centers therefore must be interpreted as effective **vertex induces** (scattering of a_μ -fields) or as **originators** of **all massive** fluctuations or **high-frequency massless** fluctuations

→ real-time **Feynman rules in unitary-Coulomb gauge.**

To not resolve a center in 2-2 scattering, momentum transfer in all Mandelstam variables s, t, u is bounded by $|\phi|^2$.

To not resolve a center in 2-1 scattering, off-shellness of massless mode is bounded by $|\phi|^2$.

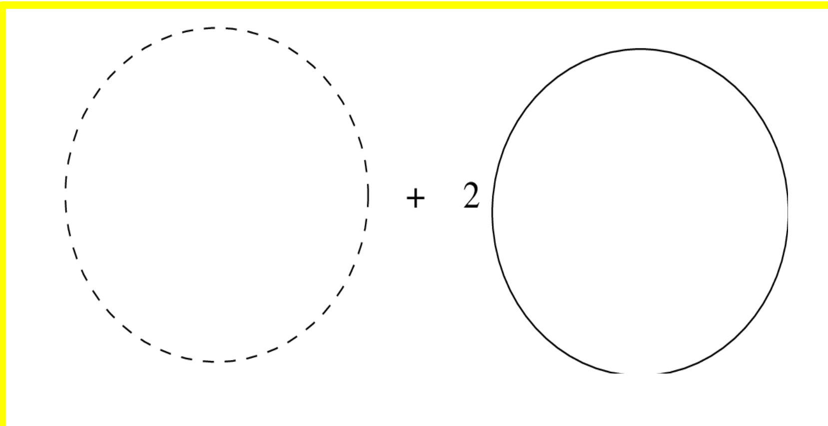
→ Radiative corrections are **infrared** (masses) and **ultraviolet** (bounds on momentum transfer) **finite**.

Deconfining thermodynamics: radiative corrections

– loop expansion of pressure:

[RH 2005]

$O(1)$



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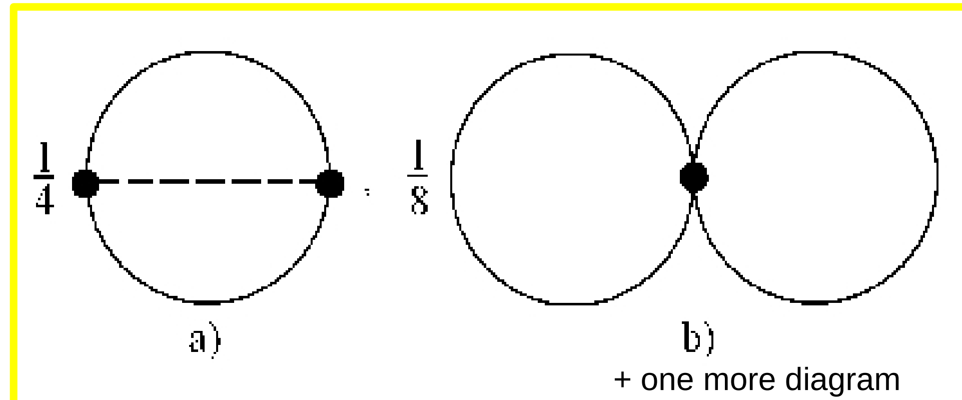
[Schwarz, Giacosa, RH 2007]

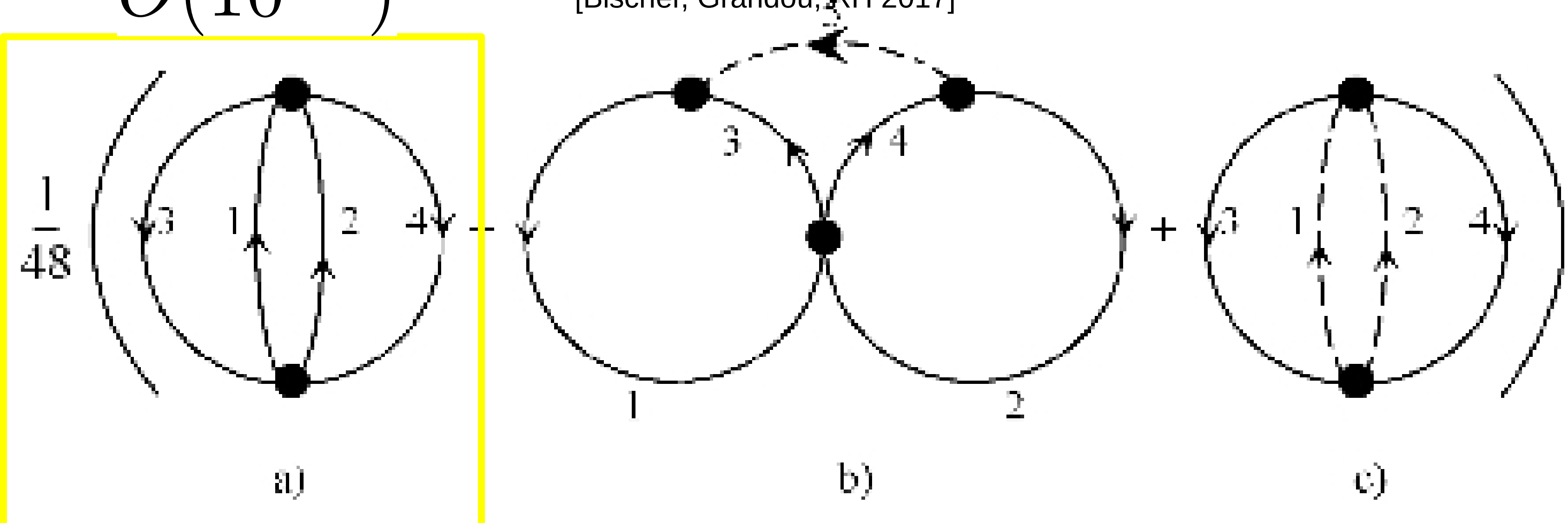
$O(10^{-2})$



$O(10^{-5})$

[Bischer, Grandou, RH 2017]

+

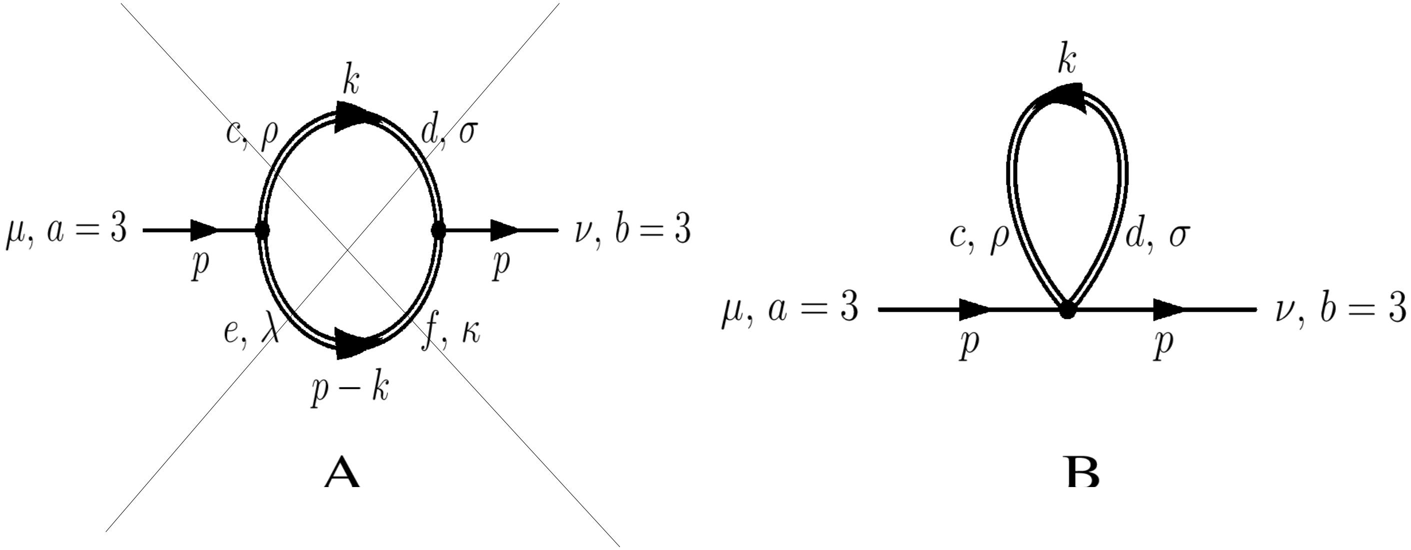


(subject to resummation)

Deconfining thermodynamics: radiative corrections

– polarisation tensor of massless mode

[Schwarz, Giacosa, RH 2007; Ludescher, RH 2008; Falquze, RH, Baumbach 2011]



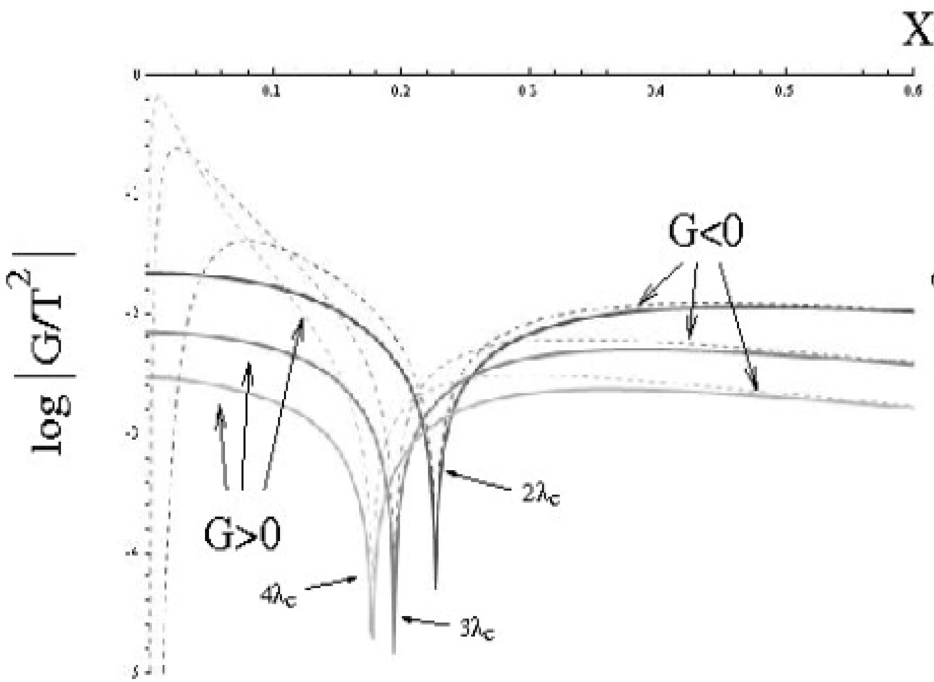
gap equations for **transverse** and **longitudinal** screening functions G and F

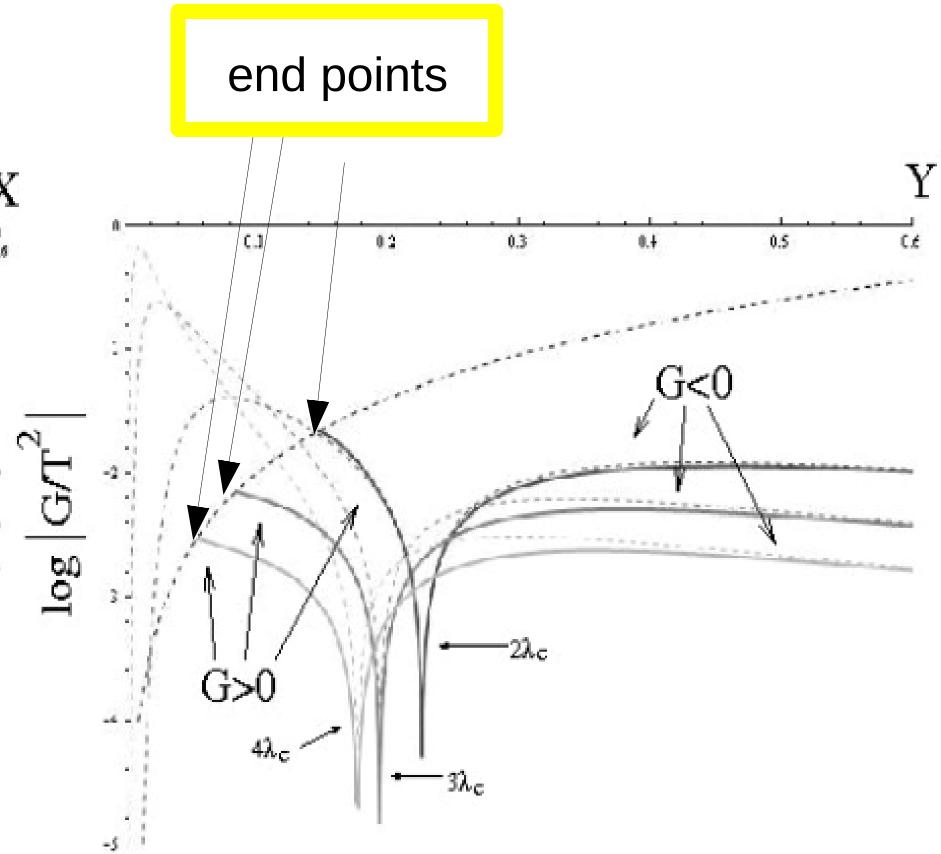
upon resummation: **radiatively corrected/invoked dispersion laws**

$$\boxed{\omega_T^2 = \vec{p}^2 + G} \qquad \boxed{\omega_L^2 = \vec{p}^2 + F}$$

Transverse modes:

[Ludescher, RH 2008]



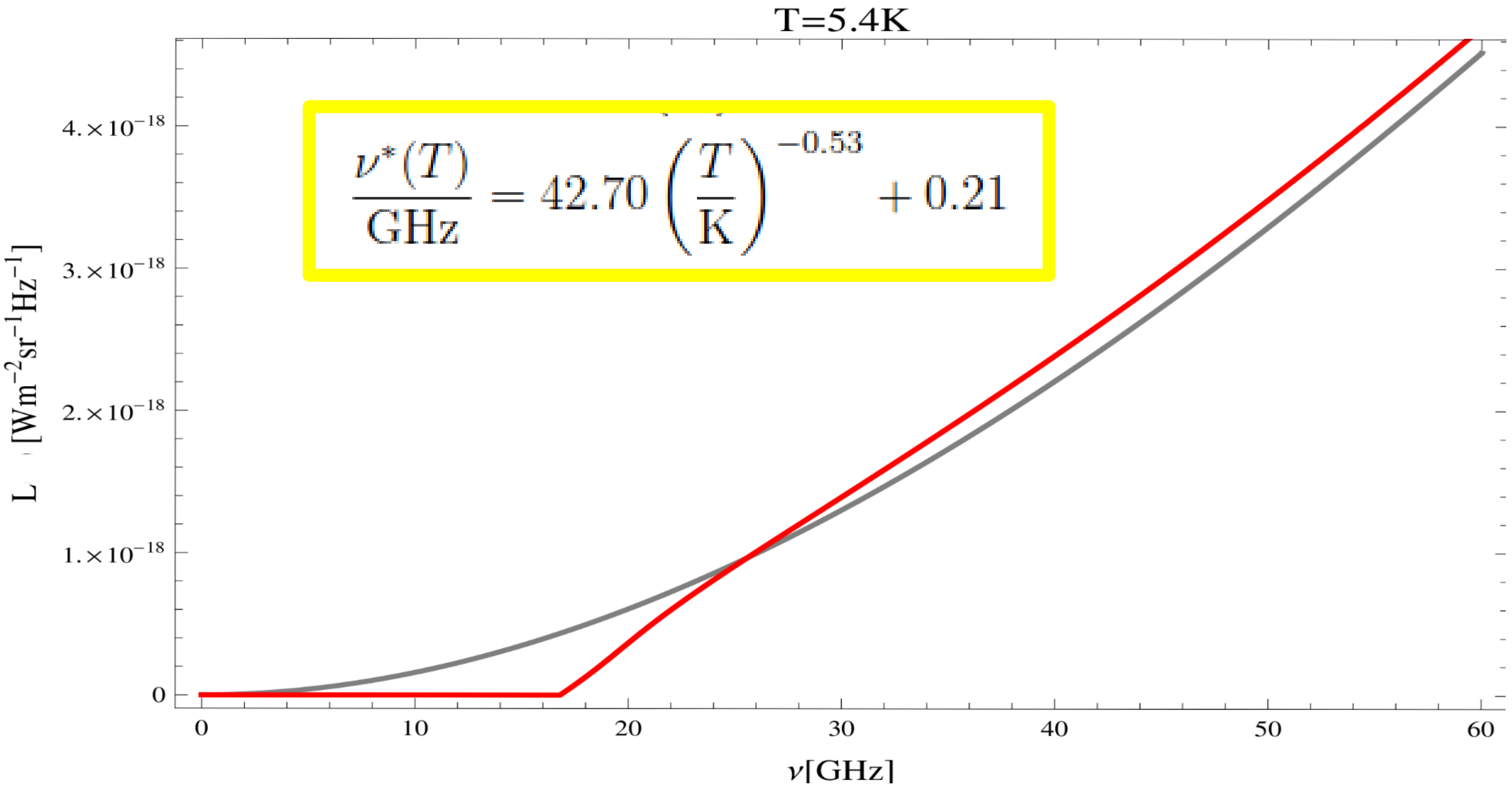


→ **modified spectral radiance in black-body (BB) radiation**

Deconfining thermodynamics: radiative corrections, BB anomaly

Transverse modes:

[Falquez, RH, Baumbach 2011]



$$\frac{\nu^*(T)}{\mathrm{GHz}} = 42.70\left(\frac{T}{\mathrm{K}}\right)^{-0.53} + 0.21$$

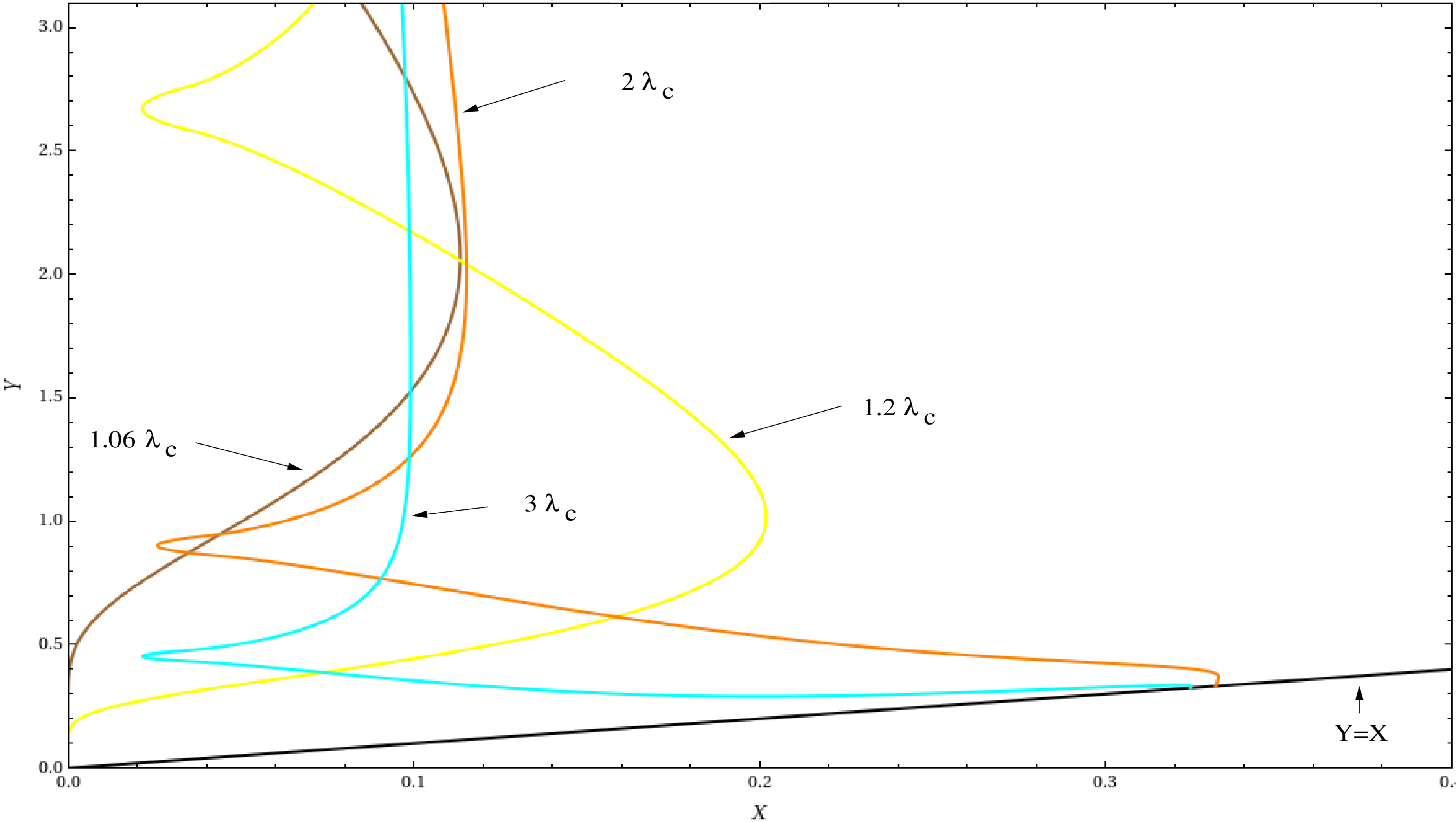
(Yang-Mills scale or T_c fixed by CMB observation at low frequencies, later)

Deconfining thermodynamics: radiative corrections

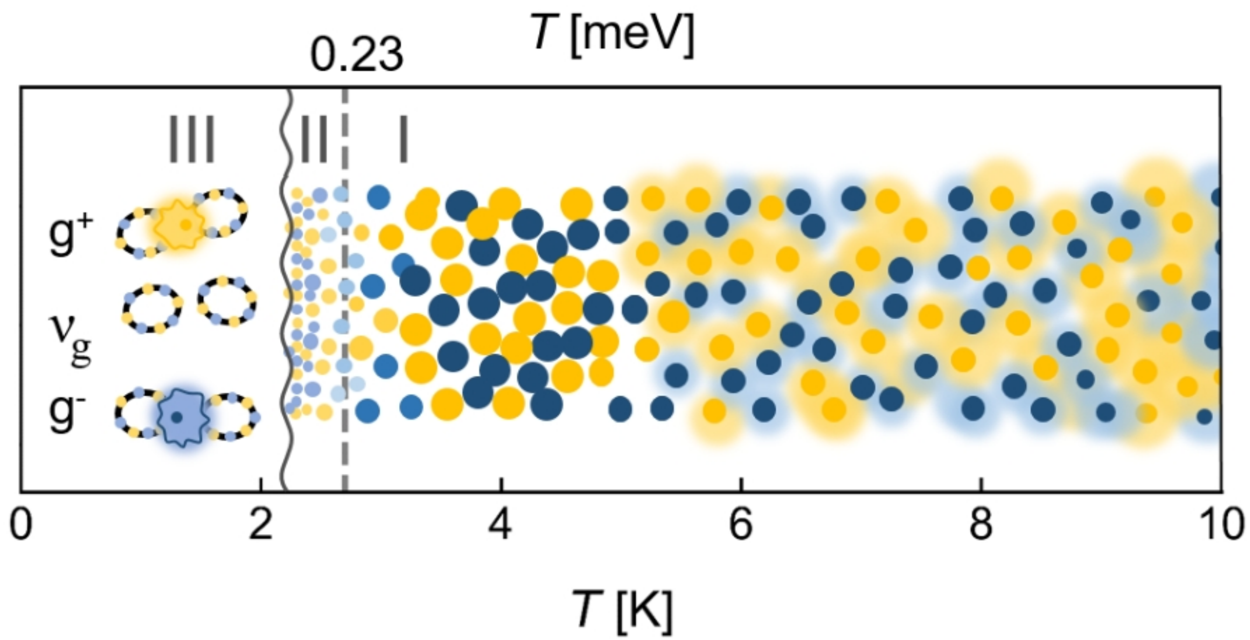
Longitudinal modes:

[Falquez, RH, Baumbach 2012]

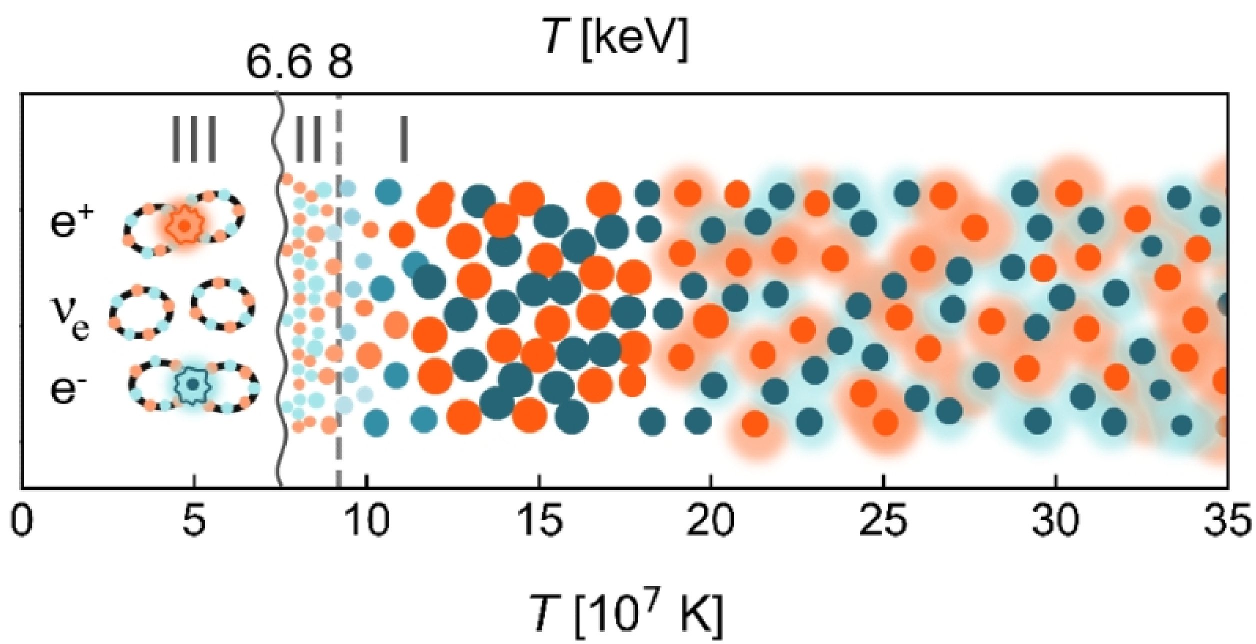
various low-momentum branches,
physics implications: later



Sketch of entire phase structure (each phase discussed in lecture):



$\mathrm{SU(2)_{CMB}}$



$\mathrm{SU(2)_{e}}$

[RH, Meinert, 2024]

R. Hofmann

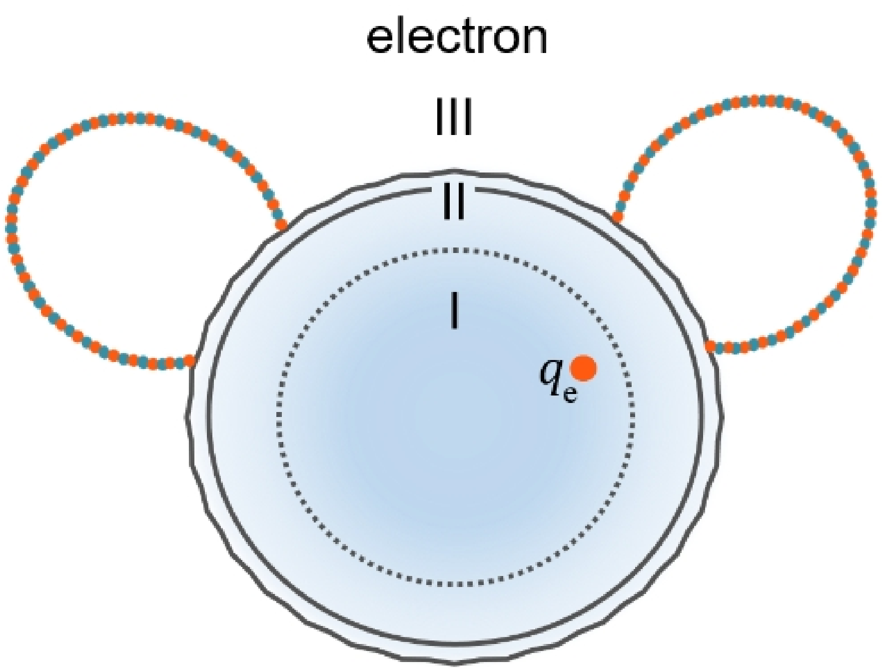
Sketch of entire phase structure: particles

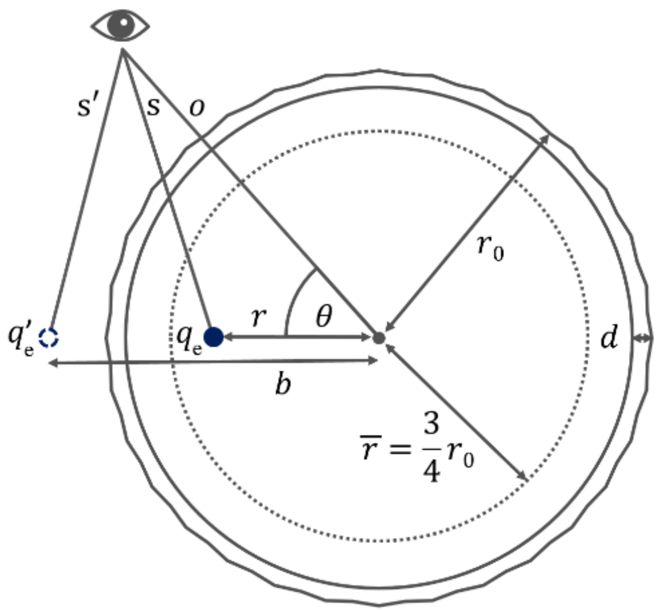
properties of deconfining phase that <u>suggest</u> particle interpretation of fine-radius blobs:

(i) isolated electric charges (electric-magnetically dual interpretation of, e.g., monopoles)

(ii) zero of pressure

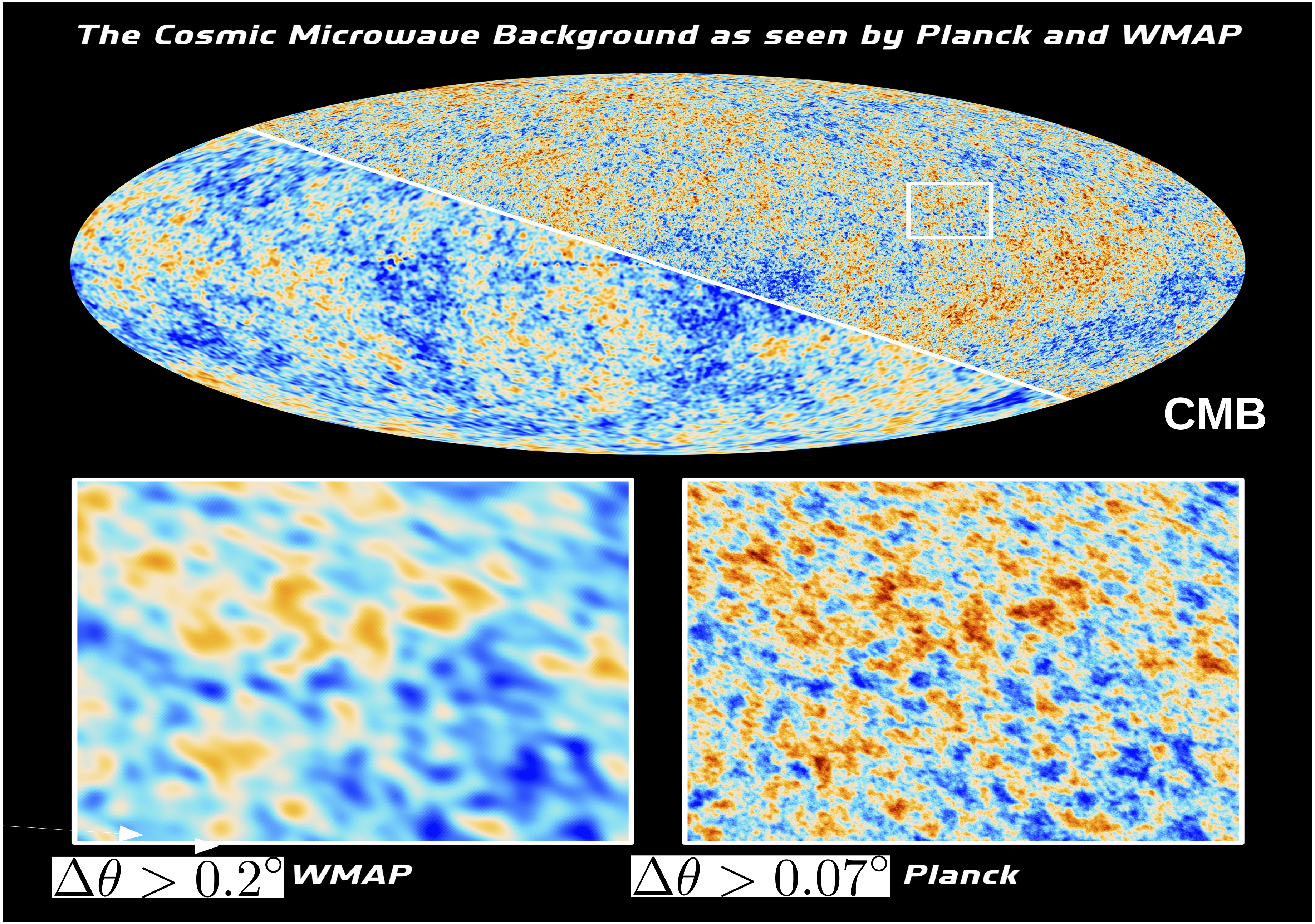
(iii) quantum of action $\hbar$ localised within (anti)caloron center $S_{\rho\sim|\phi|^{-1}} = \frac{8\pi^2}{e^2} = \hbar.$ [RH & Kaviani, 2012]



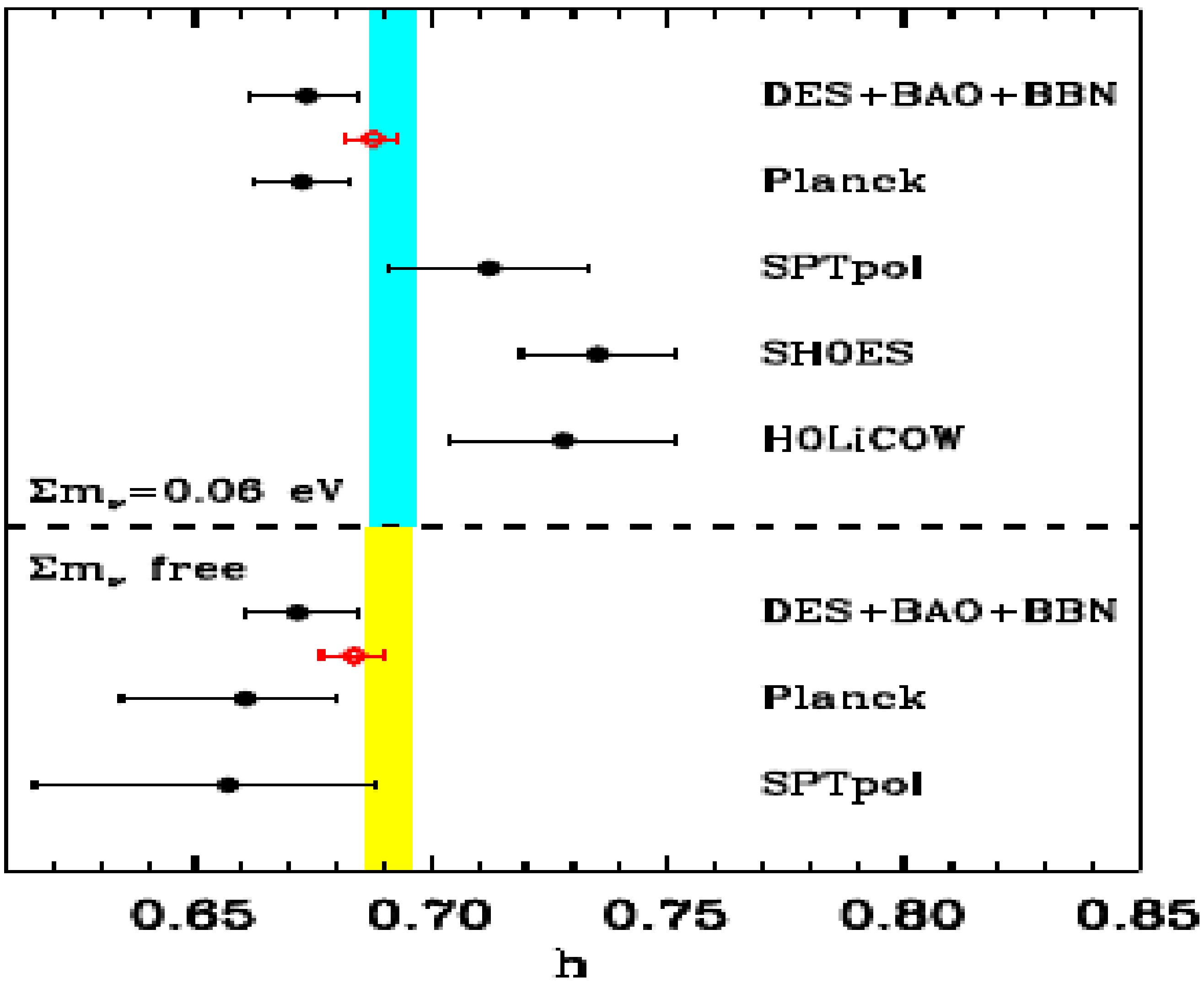


[RH & Meinert, 2024]

CMB: Observational situation after WMAP and PLANCK

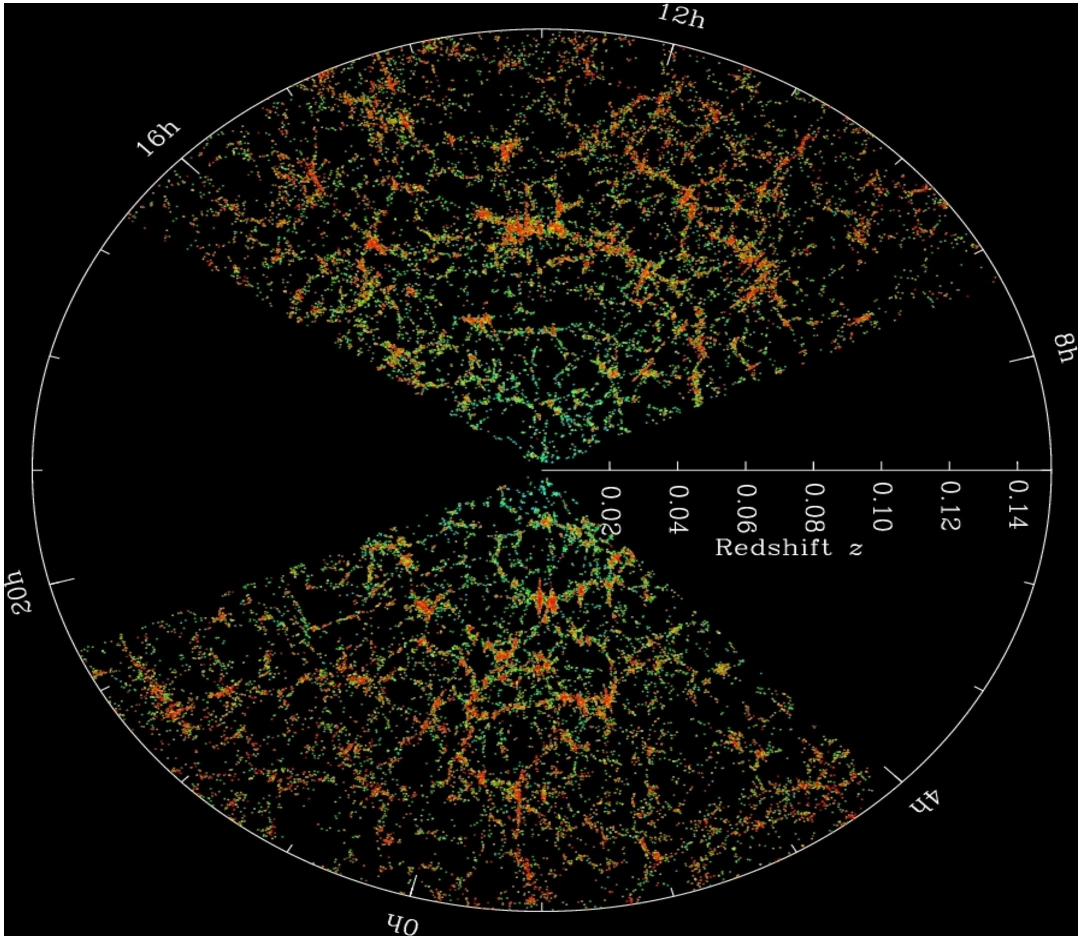


CMB: Observational situation after WMAP and PLANCK



[Abbott et al., 2018]

matter correlation function, BAO, weak lensing, CMB PS, BBN
vs.
distance-redshift calibrations to (standardised candles):
SN Ia, TRGB, Cepheids and time delays (strong lensing)



[Sloan Digital Sky Survey]

local-global parameter tensions:

$$H_0$$

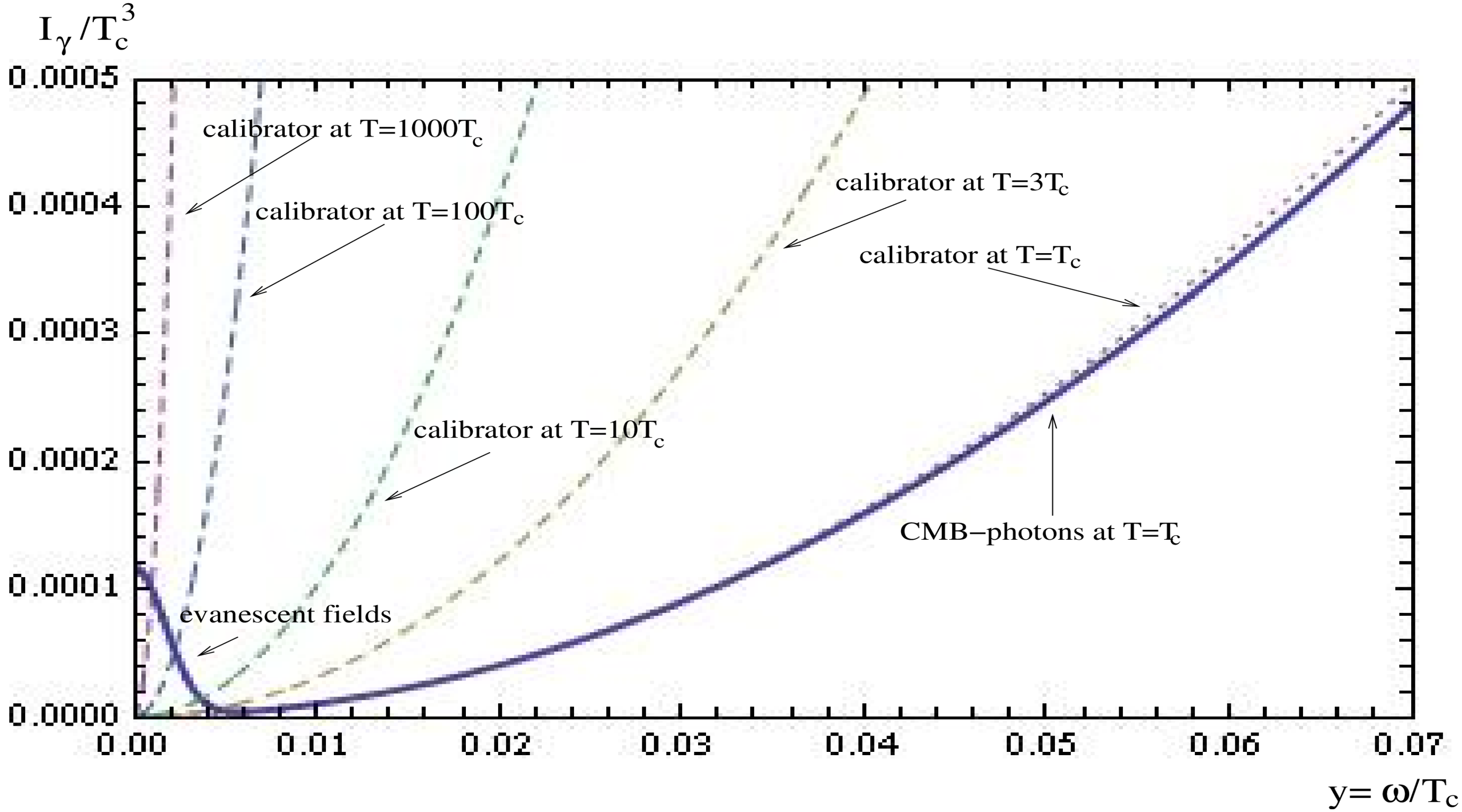
$$S_8 = \sigma_8 \left(\Omega_{\mathrm{m}}/0.3\right)^{0.5}$$

$$\omega_b$$

Low-temperature photon gases: Fixation of Yang-Mills scale

– low-frequency CMB radiance spectrum

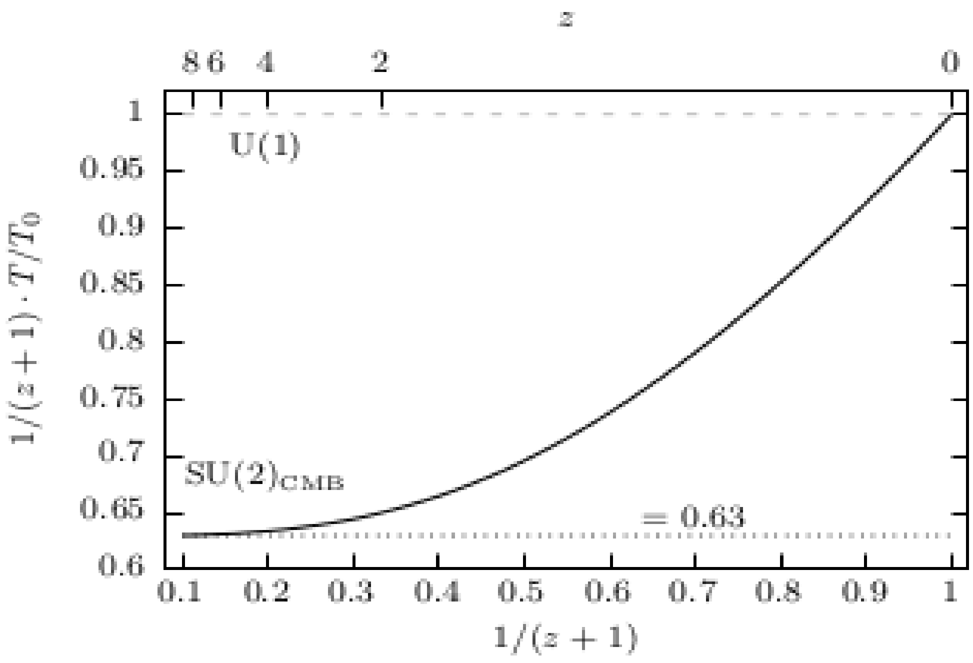
[terrest. observations 1981-1999; Arcade2 2009]



– interpretation as onset of deconfining-preconfining phase transition through **Meissner mass** ⟶ **evanescence of low-frequency waves (spectral redistribution); sharply** fixes [RH 2009]

$$\boxed{T_c = T_0 = 2.725\,\mathrm{K} \quad \Rightarrow \Lambda = \frac{2\pi T_c}{13.87} \sim 10^{-4}\,\mathrm{eV}} \longrightarrow \boxed{\mathrm{SU}(2)_{\mathrm{CMB}}}$$

CMB: temperature (T)-redshift (z) relation



follows from energy conservation in FLRW universe upon deconfining-phase SU(2) equation of state $P = P(\rho)$: [RH 2015]

$$\boxed{\frac{d\rho}{da} = -\frac{3}{a}(P+\rho)}$$

immediate consequences:

- **discrepancy** resolved between **re-ionisation redshifts** as extracted from

(i) fit to **TT angular power spectrum** of CMB [Planck coll. 2013, 2015]

(ii) **Gunn-Peterson trough** in high-z quasar spectra [Becker et al 2001]

→ **modification of high-z cosmological model**, possible explanation of discrepancy in H_0 from $\Lambda\mathrm{CDM}$ fits to CMB power spectra and local observation

[Planck coll . 2013,2015; Riess et al 2016; HoliCow 2016]

CMB: temperature (T)-redshift (z) relation: 3D Ising exponent

[Hahn & RH (2017)]

$$\boxed{\frac{d\rho}{da} = -\frac{3}{a}(P+\rho)}$$

exact solution:

$$\boxed{a = \exp\left(-\frac{1}{3}\log\frac{s(T)}{s(T_0)}\right)}$$

where S denotes entropy density

asymptotic solution: $$\boxed{a = \left(\frac{1}{4}\right)^{\frac{1}{3}}\frac{T_0}{T} \qquad \left(\frac{T}{T_0} \gg 1\right)}$$ (*)

deduces from two points of scale invariance in phase diagram: $T = T_0\,;\; T = \infty$

Note that: $$\boxed{\left(\frac{1}{4}\right)^{\frac{1}{3}} = 0.629960(5)}$$

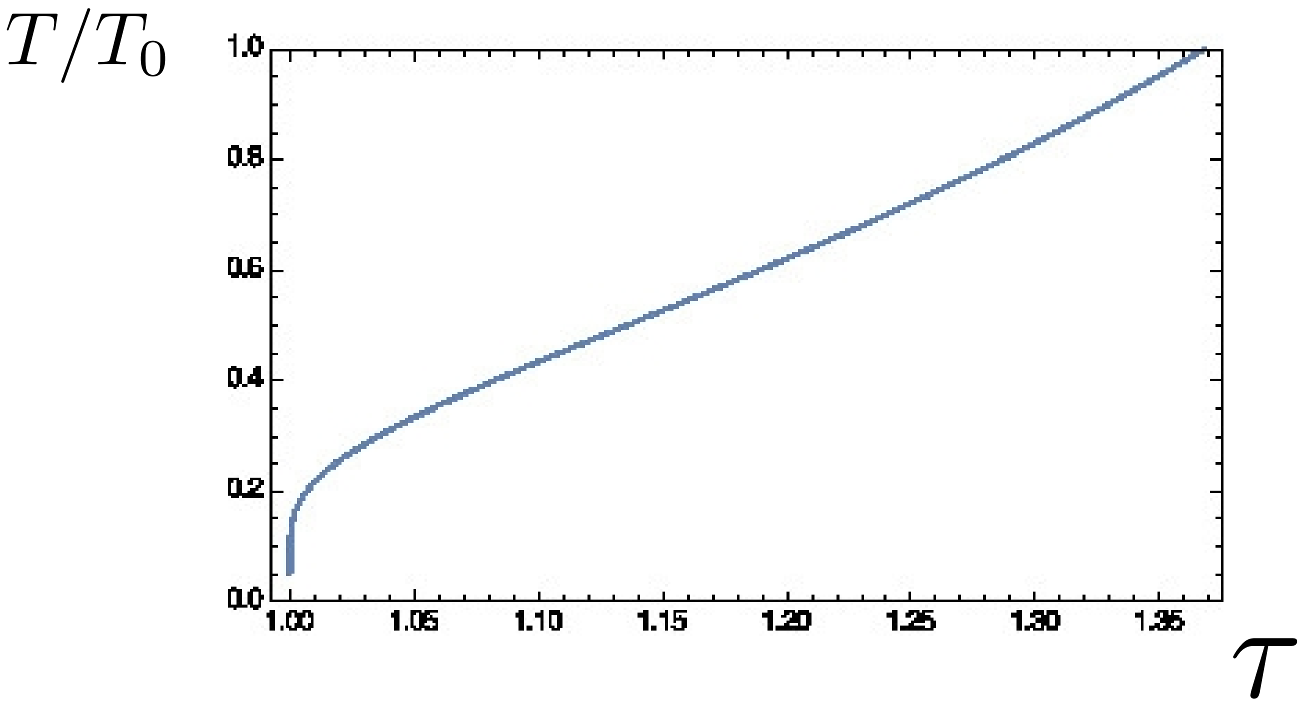
$\longrightarrow$ Deviates from numerically determined exponent of 3D Ising [Kos et al (2016)]
(same univ. class like SU(2) YMTD concerning dec.-prec. transition)
for correlation length l by only 0.001 % ! [Svetitsky & Yaffe (1983)]

Why? – continuing (*) down to $T = 0$ $\longrightarrow$
T fictitious temperature in SU(2) YMTD

– consider conformal/monotonic map from physical Ising temperature θ to T/T_0

$$\boxed{\frac{T}{T_0} = -\frac{1}{\log(\tau - 1)} \qquad \left(\tau \equiv \frac{\theta}{\theta_c}\right)}$$ (**)

CMB: temperature (T)-redshift (z) relation: 3D Ising exponent



– exponentiation of (*) under consideration of (**) yields

$$\boxed{\exp(a) = (\tau - 1)^{-\left(\frac{1}{4}\right)^{\frac{1}{3}}}} \longrightarrow$$

– interpretation of $\exp(a)$ as l/l_0 (system size a where l_0 a suitable reference length, cooling system down at fixed internal energy is accomplished by increasing system size, in critical regime, correlation length l diverges exponentially faster)

High-z cosmological model

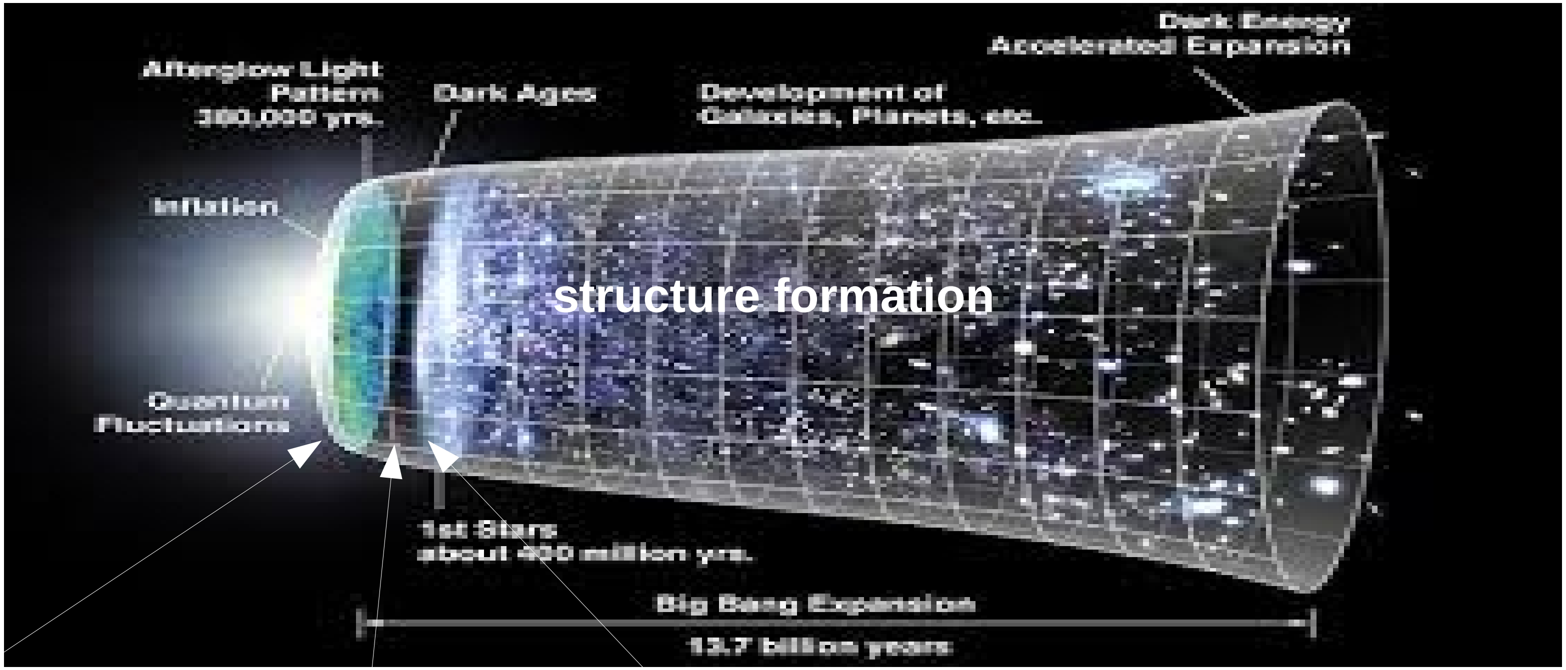
– re-combination z_* at a $\sim 1/0.63$ times higher redshift compared to $\Lambda\mathrm{CDM}$

→ minimally, **no dark matter** at z_* [Hahn, RH 2017]

→ predicts (comov) sound horizon (at baryon drag) such that H_0 **in agreement with local observation**

→ **But:** requires interpolation to low-z $\Lambda\mathrm{CDM}$ [Bernal et al 2016,Riess et al 2018; Bonvin et al 2016]

(de-percolation of Planck-scale axion vortices)



re-combination de-percolation ICN re-ionisation

$\mathrm{SU(2)_{CMB}}$ and a Planck-scale axion (PSA): dark sector

– **ultralight pseudo-scalar field** φ first proposed by **Frieman et al. 1995** and revived by **Wilczek et al 2004** to serve as quintessence

[Peccei, Sola ,Wetterich 1987; Wetterich 1988; Peebles, Ratra 1988]

– **conceptual underpinning:** radiative protection of a rather strongly determined potential arising from an explicit, quantum-anomaly induced breaking (topological charge!) of a dynamically broken global U(1) symmetry

[Adler,W.A. Bardeen,Bell,Jackiw1969; Fujikawa 1979;Peccei, Quinn 1977]

– **dynamical breaking (Peccei-Quinn scale):**
Planckian physics in a de Sitter spacetime $\longrightarrow$ PQ scale: $\boxed{M_P \sim 10^{19}\,\mathrm{GeV}}$

[Giacosa, Neubert, RH 2008]

explicit breaking:
ground states of Yang-Mills theories

– **on cosmological scales**, only presently **deconfining YM theory:** $\mathrm{SU(2)_{CMB}}$

$\longrightarrow$ **axion mass:** $\boxed{m_a = \frac{\Lambda^2_{\mathrm{CMB}}}{M_P} \sim 10^{-36}\,\mathrm{eV} \sim H_0}$

$\mathrm{SU}(2)_{\mathrm{CMB}}$ and a Planck-scale axion (PSA): dark sector

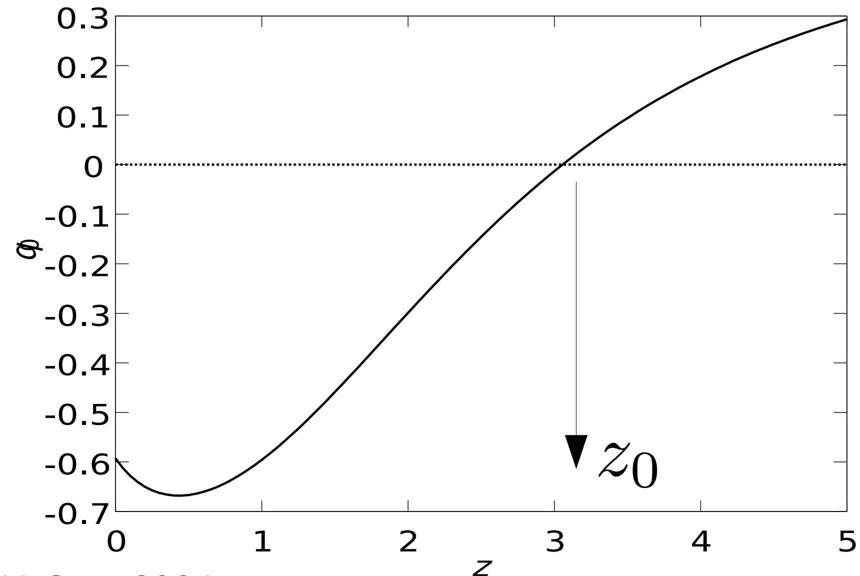
– **interesting coincidence:** $m_a \sim H_0$

(in view of $\boxed{\ddot{\varphi} + 3H\dot{\varphi} + \frac{\mathrm{d}}{\mathrm{d}\varphi}V(\varphi) \sim \ddot{\varphi} + 3H\dot{\varphi} + m_a^2\varphi = 0}$)

$\longrightarrow$ energy density in **damped oscillations** (DM eos) **comparable** to potential energy density (DE eos)

Notice: in $\Lambda\mathrm{CDM}$ $\boxed{\Omega_\Lambda = 0.7 \sim \Omega_{\mathrm{DM},0} = 0.25}$

However: deacceleration parameter $q_0(z)$



$\longrightarrow$ Universe accelerates **too early** for **viable structure formation** ($z_0 > 3$ as opposed to $z_0 \sim 0.7$)

Homogeneously oscillating PSA field falsified.

$SU(2)_{\rm CMB}$ and PSA: percolated and depercolated PSA vortices

–transitions from deconfining to confining phases in SU(2) YM are **highly nonthermal** (Hagedorn)

[RH 2007]

$\longrightarrow$ U(1) phase φ may wind around S_1 $\longrightarrow$ **PSA vortices**

– PSA percolate in **Berezinski-Kosterlitz-Thouless** transition subsequent to **Hagedorn** (not unreasonable to assume **DE e.o.s. for percolate**)

– de-percolation at some redshift z_p , DE e.o.s. transforms into DM e.o.s.

$$\boxed{\hat\rho_{\rm DS} = \hat\rho_\Lambda + \hat\rho_{\rm CDM,0}\cdot\begin{cases}(z\ +1)^3 & (z<z_p)\\ (z_p+1)^3 & (z\geq z_p)\end{cases}}$$

(interpolation of high-z model to ΛCDM)

– z_p fitted to angular scale of sound horizon at photon decoupling θ_*

$$\boxed{\theta_* = \frac{r_s(z_{{\rm lf},*})}{\int_0^{z_{{\rm lf},*}}\frac{dz}{H(z)}} = 0.597^\circ}$$

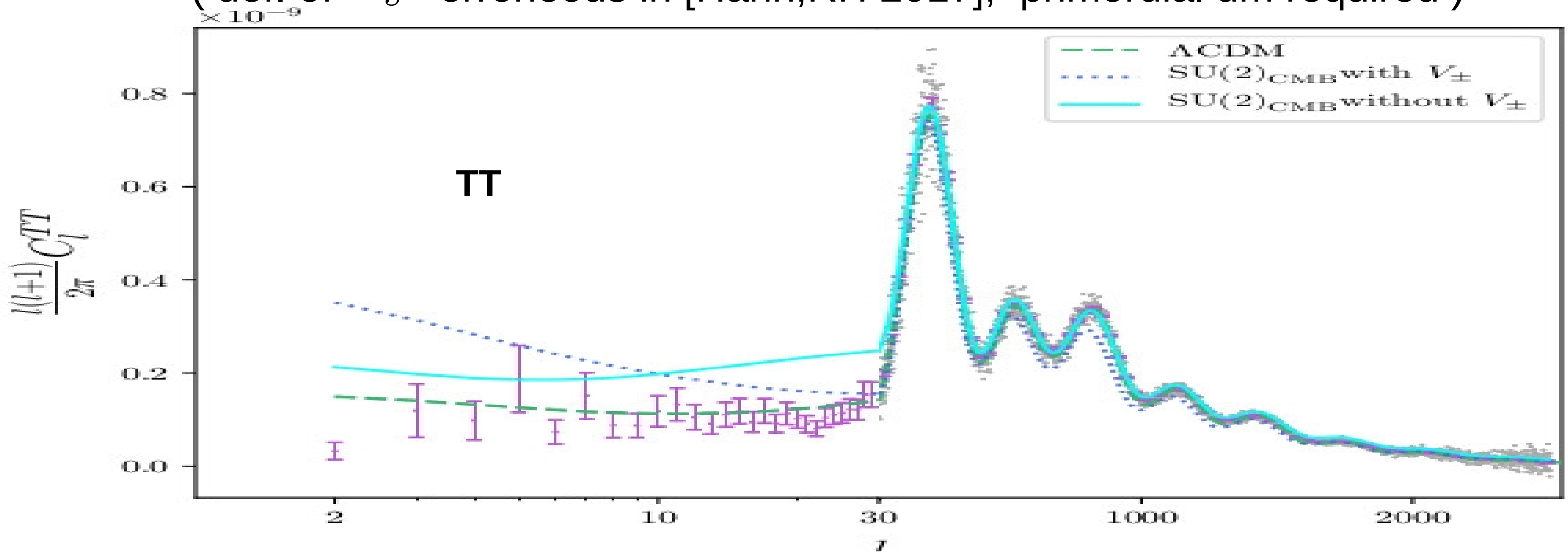
high-z model

[Planck coll. 2012, 2015]

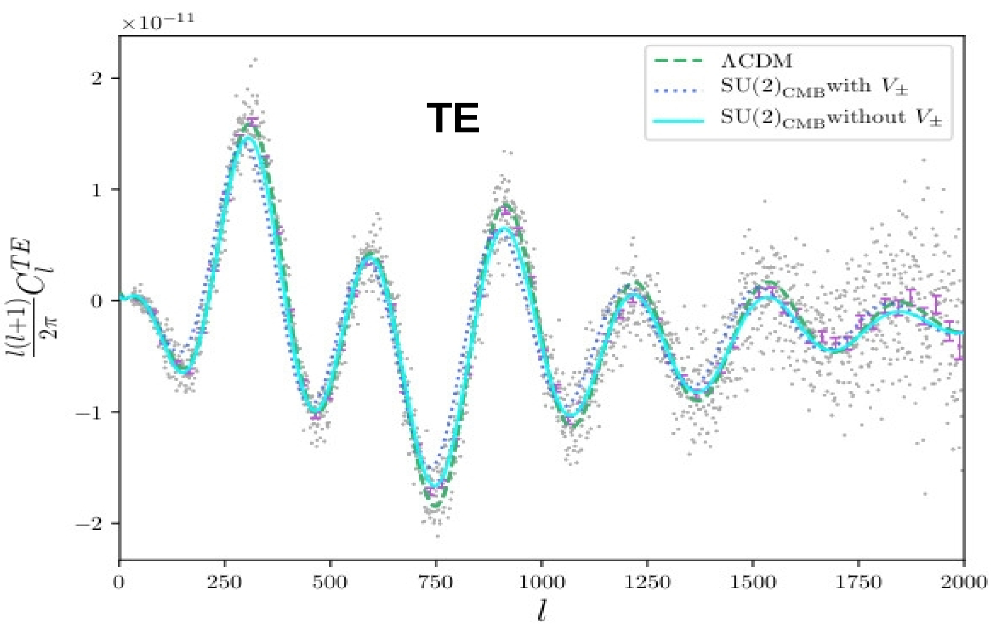
low-z model

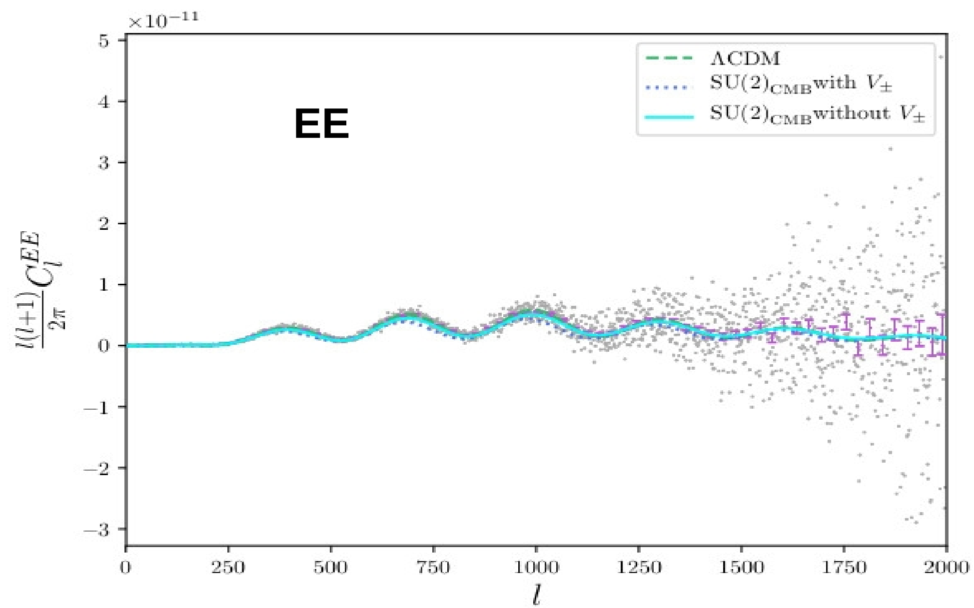
$\mathrm{SU(2)_{CMB}}$ and PSA: fits to angular power spectra

(def. of r_s erroneous in [Hahn,RH 2017], primordial dm required)

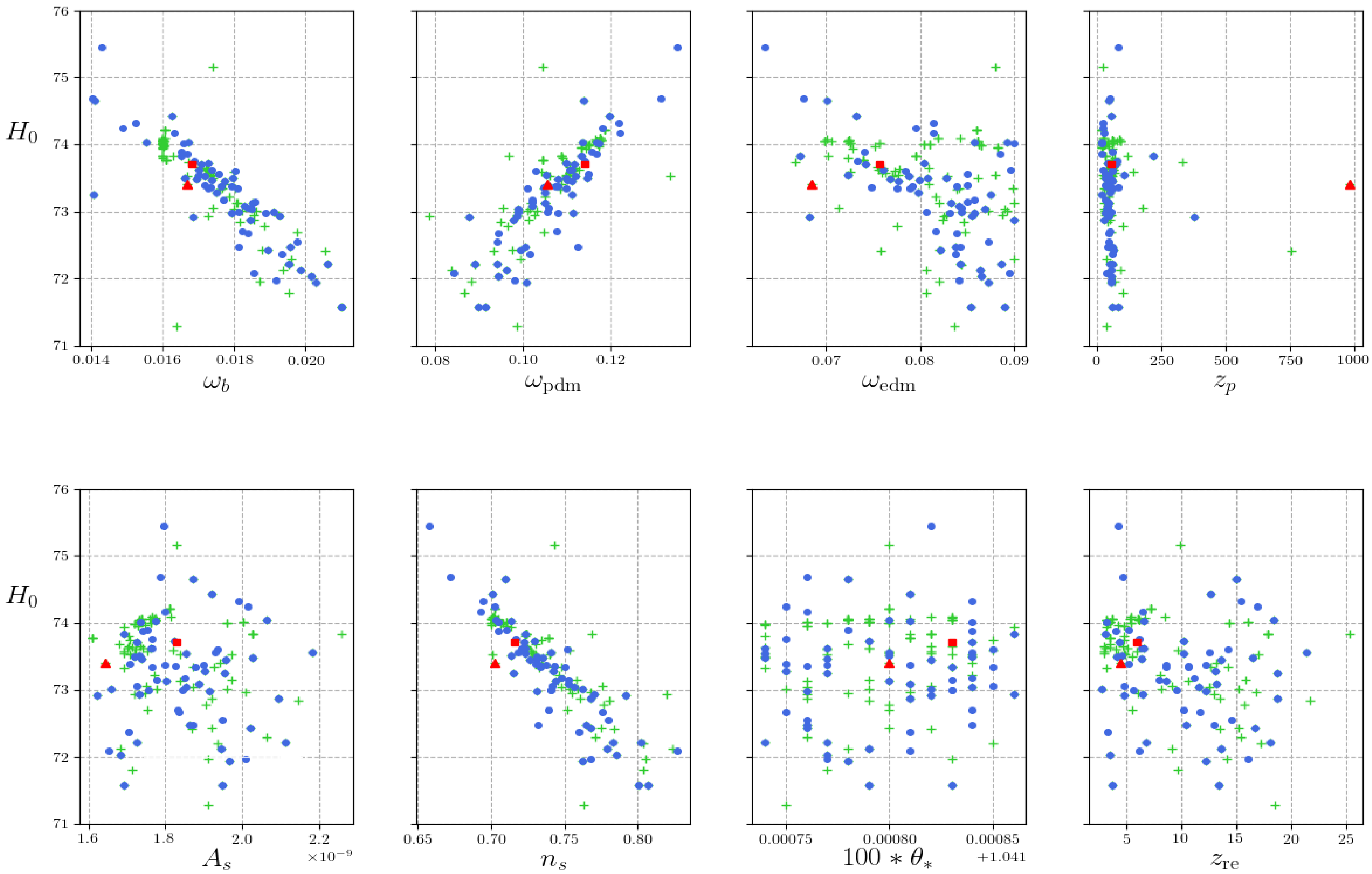


[Hahn,RH,Kramer 2018]



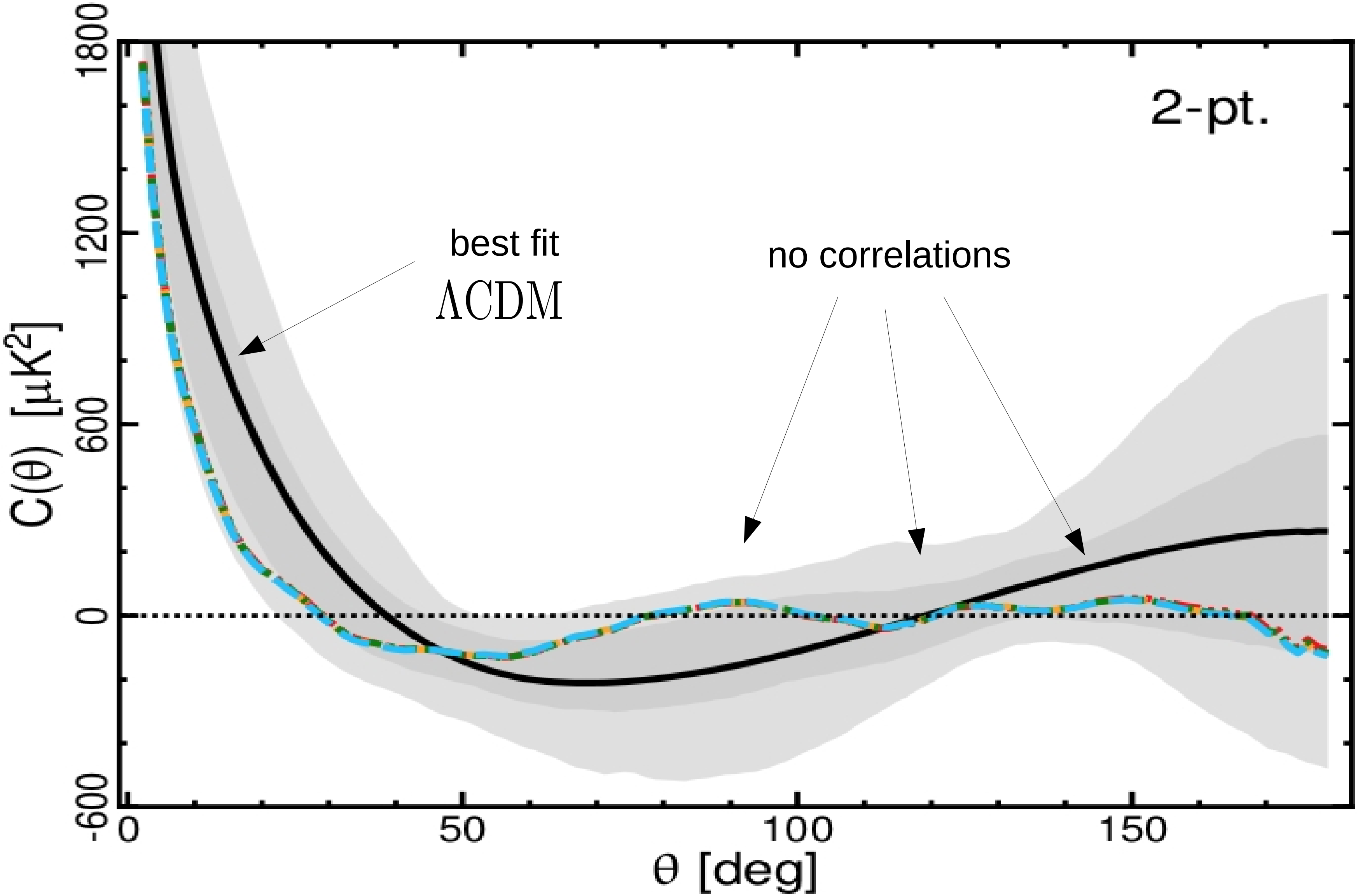


$\mathrm{SU(2)_{CMB}}$ and PSA: H_0 and r_{re} and n_s



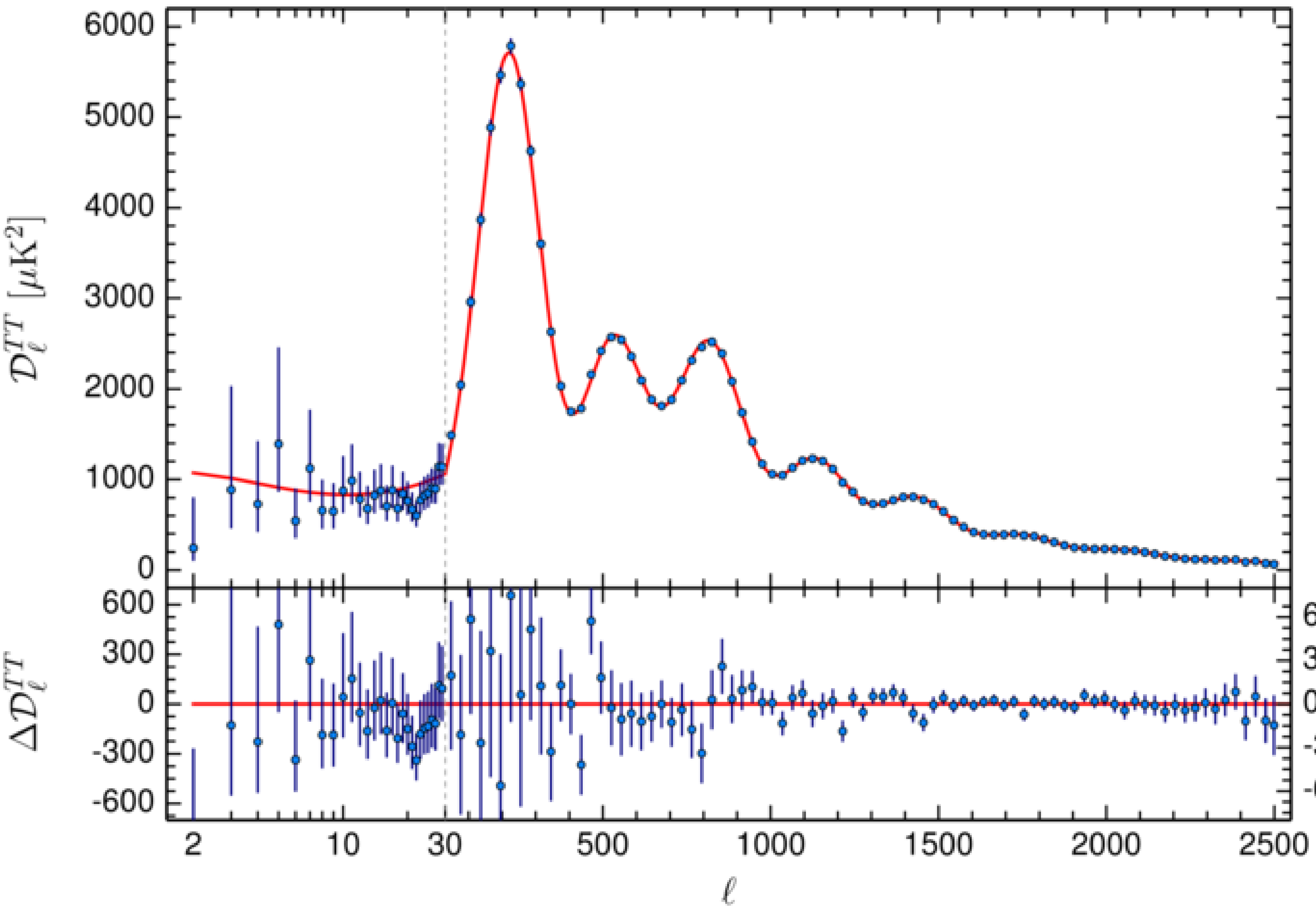
$\mathrm{SU}(2)_{\mathrm{CMB}}$ and thermal photon dispersion law: CMB at large angles

– TT correlation function $C(\theta)$ (PLANCK) [courtesy: Schwarz, Copi, Huterer, Starkman 2015]



$\mathrm{SU}(2)_{\mathrm{CMB}}$ and thermal photon dispersion law: CMB at large angles

– TT (binned) power spectrum (PLANCK): „Fourier transform" of $C(\theta)$ under the assumption of **statistical isotropy**



$\mathrm{SU(2)_{CMB}}$ and thermal photon dispersion law: CMB at large angles

– unexpected features of **CMB at large angles**
[Copi et al 2005-2015, based on WMAP and PLANCK data and others]

- lack of correlation in $C(\theta)$ for $\theta > 60^\circ$
(p-values well below 1% in $\Lambda\mathrm{CDM}$)

- alignments of low-l multipoles
(quadrupole with octopole and together with kinematic dipole direction)

- CMB cold spot
(non-Gaussianity of T-fluctuations in its vicinity)
[Vielva 2010]

- power and variance asymmetries between
northern and southern ecliptic hemispheres

$\mathrm{SU}(2)_{\mathrm{CMB}}$ and thermal photon dispersion law: CMB at large angles

– to address these in $\mathrm{SU}(2)_{\mathrm{CMB}}$ **CMB Boltzmann hierarchies** plus **evolution of curvature perturbations** subject to new cosmological model with modified T-z relation must be solved
[under investigation]

– **first-shot approach:**

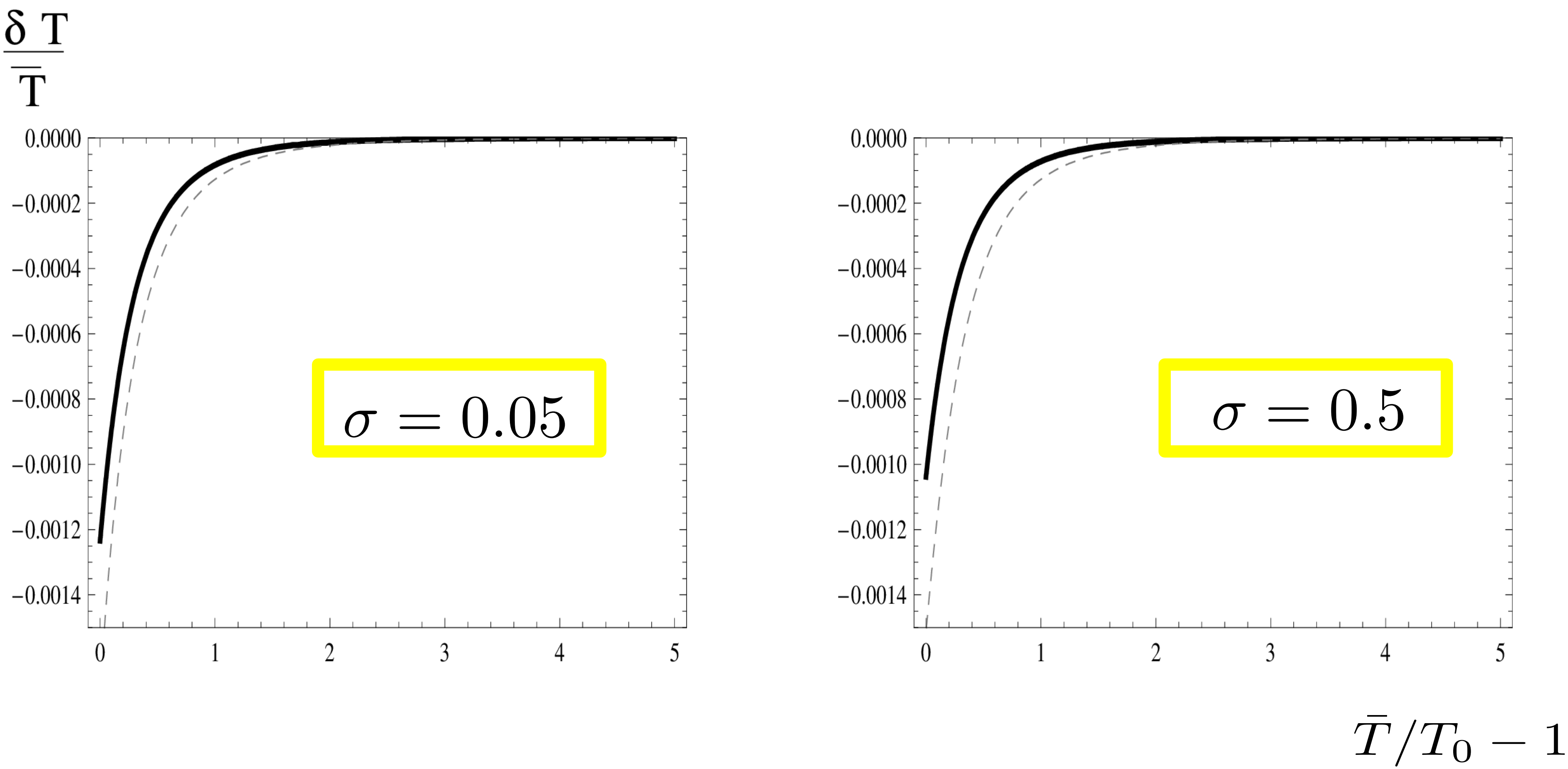
treat T as a scalar field, introduce kinetic term, and take potential from integrated BB anomaly $\longrightarrow$ e.o.m. (spherical symm. + linear fluct.)

$$0 = \partial_\tau \partial_\tau \delta T - \left(\frac{\mathrm{d}a}{a\,\mathrm{d}\tau}\right)^2 \left[\partial_\sigma \partial_\sigma \delta T + \frac{2}{\sigma}\partial_\sigma \delta T\right] - \frac{3}{\bar{T}}\partial_\tau \bar{T}\,\partial_\tau \delta T + \frac{\bar{T}_0^2}{kH_0^2}\left[\frac{1}{2}\left.\frac{\mathrm{d}^2\hat{\rho}}{\mathrm{d}T^2}\right|_{T=\bar{T}}\delta T + \frac{1}{2}\left.\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}T}\right|_{T=\bar{T}}\right]$$

to be determined from Doppler inferred discrepancy between measured and predicted dipole

[Szopa, RH 2007; Ludescher, RH 2009]

$\mathrm{SU}(2)_{\mathrm{CMB}}$ and thermal photon dispersion law: CMB at large angles



at around z~1: rapid formation of temperature depression

thesis problems

- improvements of cosmo model, CMB large-angle anomalies (q-grid)

- “microscopics“ of PSA-lumps percolation/depercolation by virtue of SP system $\rightarrow$ initial conditions for evolution of matter fluctuations after de-percolation [Meinert, RH, 2021], [Tudes, RH, 2023,2024]

$$m_{a,e} = \frac{\Lambda_e^2}{M_P} = 5\times 10^{-24}\,\mathrm{eV}\,,\quad m_{a,\mu} = \frac{\Lambda_\mu^2}{M_P} = 1.8\times 10^{-19}\,\mathrm{eV}\,,\quad m_{a,\tau} = \frac{\Lambda_\tau^2}{M_P} = 5\times 10^{-17}\,\mathrm{eV}\,,$$

- cores of spiral galaxies, globular clusters [Tudes, RH, 2024]

- axion mass in deconfining phase aus SU(2) Yang-Mills (Veneziano-Witten at $T>T_c$) [Antonov, Blumenberg, RH, 2024]

$$m_a^2 = \frac{2N_f}{F^2}\chi,$$

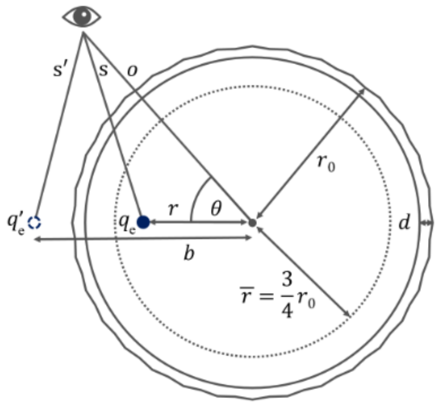
whereby

$$\chi = \int d^4x\,\langle \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\kappa\xi} Tr\left[\mathcal{F}_{\mu\nu}(0)\,\mathcal{F}_{\kappa\xi}(0)\right] \frac{g^2}{32\pi^2}\epsilon_{\alpha\beta\lambda\delta} Tr\left[\mathcal{F}_{\alpha\beta}(x)\,\mathcal{F}_{\lambda\delta}(x)\right]\rangle,$$

thesis problems

- model of electron and other charged leptons as thermal quantum solitons in SU(2) YMTD

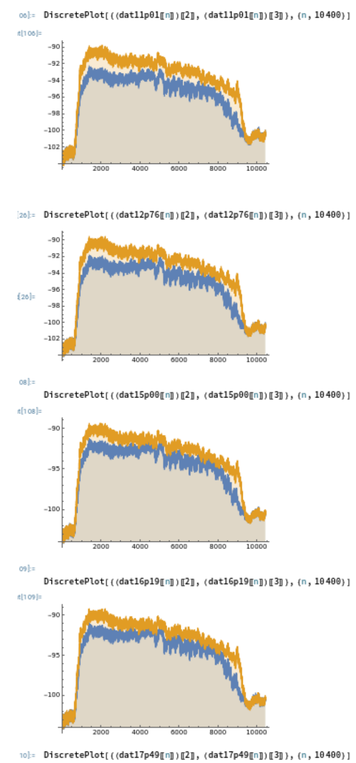
  [RH & Meinert, 2024; de Broglie, 1924]



  (determination of scale of $\mathrm{SU}(2)_\mathrm{e}$, electroweak symmetry breaking, dimensionless electroweak parameters, lepton decay, life times, embedding into axion lumps, magnetic correlations)

- BBN and $\mathrm{SU}(2)_\mathrm{e}$

- high-T, high-μ electron gases (magnetic confinement)



- data analysis: blackbody experiments at INRIM Torino and PTB Berlin

- convergence of loop expansion: dec. Phase of SU(2) YMTD

- Riemann's zeta function and SU(2) YMTD *