Thermal ground state in Yang-Mills thermodynamics

Ralf Hofmann

ITP Universität Heidelberg

1st Symposium on **Analysis of Quantum Field Theory**
9th International Conference of Numerical Analysis and Applied Mathematics
Haldiki, Greece, 19-25 September 2011

21 September 2011

Table of Contents I

Table of Contents II

Distinction of the electromagnetic field?

Consider invariance of action

$$\frac{1}{2g^2}\mathrm{tr}\int F_{\mu\nu}F_{\mu\nu}$$

under local transformation

$$A_\mu \xrightarrow{\Omega} \Omega A_\mu \Omega^\dagger + i\Omega\partial_\mu\Omega^\dagger\,,$$

where $\Omega = \Omega(x) \in$ semisimple Lie group G, $A_\mu = A_\mu(x) \in g$ gauge field, and $F_{\mu\nu} = F_{\mu\nu}(x)$ its curvature.

Before 1954, Pauli, Barker, and Gulmanelli considered Kaluza-Klein zero-mode reduction over "internal", compact manifold G with $\dim G > 1 \Rightarrow$ generally leads to inconsistency of 4D gravity

In 1954, Yang speaks on work with Mills (4D theory with SU(2) invariance), **Pauli in audience:**
Pauli asked, ,What is the mass of this field A_μ*?'. I said we did not know. Then I resumed my presentation, but soon Pauli asked the same question again. I said something to the effect that that was a very complicated problem, we had worked on it and come to no definite conclusions. I still remember his repartee: ,That is not sufficient excuse!'.*

Note to Yang by Pauli:
But I was and still am disgusted and discouraged of the vectorfield corresponding to **particles with zero rest-mass** *(I do not take your excuse for it with "complications" seriously), and the difficulty with the group due to the* **distinction of the electromagnetic field** *remains.'*

This talk and talks by **Markus** and **Dariush**:

- Pauli's conclusion that SU(2) irrelevant on basis of nonobservation of propagating DOEs **premature** (topological field configuration collectively break SU(2) dynamically $\leftrightarrow$ external Higgs mechanism for ew symmetry breaking in SM)
- Yang's feeling of mass generation ("complications") in a pure SU(2) theory confirmed:
  mass scale Λ enters the game **on the BPS level**, no conceptual problem as in PT
- collective nature of associated thermal ground state: both **infrared** and **ultraviolet** divergences do not appear in effective theory
- stable **topological defects (magnetic monopoles)** are generated collectively (and unresolvably) by **well-controlled radiative corrections** in effective theory: no need to rely on semiclassical approximation

Euclidean finite-temperature field theory

- representation of partition function Z (field theory of real scalar ϕ for simplicity)

$$
\begin{aligned}
Z &= \mathrm{Tr}\, e^{-\beta H} = \mathcal{N} \int_{\phi(\mathbf{x},0)=\phi_\alpha(\mathbf{x})}^{\phi(\mathbf{x},\beta)=\phi_\alpha(\mathbf{x})} \prod_{\mathbf{x},\tau'} d\phi(\mathbf{x},\tau') \times \\
&\quad \exp\left[-\int_0^\beta d\tau'' \int d^3y \left(\frac{1}{2}\,\partial_{\tau''}\phi\partial_{\tau''}\phi + \frac{1}{2}\,\nabla\phi\cdot\nabla\phi + V(\phi)\right)\right] \\
&\equiv \mathcal{N} \int_{\phi(\mathbf{x},0)=\phi_\alpha(\mathbf{x})}^{\phi(\mathbf{x},\beta)=\phi_\alpha(\mathbf{x})} \prod_{\mathbf{x},\tau'} d\phi(\mathbf{x},\tau') \, \exp\left[-\int_0^\beta d\tau'' \int d^3y \, \mathcal{L}_E\right] ,
\end{aligned}
$$

  where $\beta \equiv 1/T$.

- in gauge theory: admissible changes of gauge respect periodicity of A_μ
- in gauge-theory PT: additional gauge fixing required (Faddeev-Popov or better)

Euclidean finite-temperature field theory

- loop expansion of N-point functions in momentum space, propagator $\bar{D}$

$$\bar{D}(\mathbf{p}, \omega_n) = \frac{1}{\omega_n^2 + \mathbf{p}^2 + m^2}\,,$$

  where $\omega_n \equiv 2\pi\, nT$ $(n \in \mathbf{Z})$ nth Matsubara frequency.

- re-expressing (but not changing the contour for τ'' integration in Euclid. action) summation over n and integration over $\mathbf{p}$, $\sum_n \int d^3p$, by Cauchy's integral theorem $\Rightarrow$

$$-\frac{1}{\omega_n^2 + \mathbf{p}^2 + m^2} \longrightarrow \frac{i}{p^2 - m^2} + \delta(p^2 - m^2)\frac{2\pi}{\mathrm{e}^{\beta|p_0|} - 1}\,,$$

  where $\sum_n \int d^3p \longrightarrow \int d^4p$.

Real-time interpretation of loop integrals

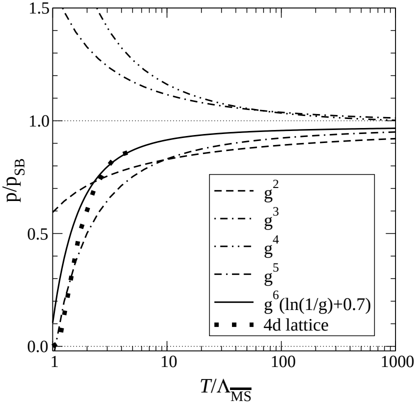
Remarks:

- A more elaborate τ'' integration contour in the action was considered in [Umezawa, Matsumoto, and Tachiki (1982), Niemi and Semenoff (1984)]. This doubles real-time DOEs to avoid **pinch singularities** in PT

- In Yang-Mills, where topological field configurations constructed for $0 \leq \tau'' \leq \beta$ (ground state!), such a change of contour for physics of propagating excitations is **inconsistent**.

Perturbative approach to pressure in Euclidean formulation

- in [Linde 1980] uselessness of PT after order g^6 pointed out (scale-separation argument for $g \ll 1$: momenta of order T (hard), gT (soft), and $g^2 T$ (ultrasoft); hard and soft OK; ultrasoft: weak screening of magnetic modes destroys perturbativity starting at g^6)
- SU(3) pressure in pure-YM PT



[Shuryak 1978, Kapusta 1979, Toimula 1983, Arnold and Zhai 1994, Zhai and Kastening 1994, Braaten and Nieto 1996, Kajantie 2003]

Trivial-holonomy calorons

- in singular gauge (winding number $|k| = 1$ is localized in a point) there is a **superposition principle** of instanton centers in **prepotential** Π ['t Hooft (1976), Jackiw and Rebbi (1976)]:

$$
\begin{aligned}
\bar{A}_\mu^{+,a}(x) &= -\bar{\eta}^a_{\mu\nu}\,\partial_\nu \log \Pi\,, \\
\bar{A}_\mu^{-,a}(x) &= -\eta^a_{\mu\nu}\,\partial_\nu \log \Pi\,.
\end{aligned}
$$

- can be used to satisfy at $|k| = 1$ periodic b.c. in strip $(0 \leq \tau \leq \beta) \times \mathbf{R}^3$ [Harrington and Shepard (1978)]:

$$
\begin{aligned}
\Pi(\tau, \mathbf{x}; \rho, \beta, x_0) &= 1 + \sum_{l=-\infty}^{l=\infty} \frac{\rho^2}{(x - x_l)^2} \\
&= 1 + \frac{\pi \rho^2}{\beta r} \frac{\sinh\left(\frac{2\pi r}{\beta}\right)}{\cosh\left(\frac{2\pi r}{\beta}\right) - \cos\left(\frac{2\pi \tau}{\beta}\right)}\,,
\end{aligned}
$$

where $r \equiv |\mathbf{x}|$.

Trivial-holonomy calorons, cntd.

- holonomy of $\bar{A}_\mu^{\pm,a}(x)$ at $r \to \infty$ trivial:

$$\Pi \stackrel{r\to\infty}{=} 1 + \frac{\pi\rho^2}{\beta r} \Rightarrow \lim_{r\to\infty} \bar{A}_4^{\pm} \propto \lim_{r\to\infty} \frac{1}{r^2} = 0 \Rightarrow$$

$$\mathcal{P}\,\exp\left[i\int_0^\beta d\tau\,\bar{A}_4^{\pm}\right] = \mathbf{1}_2\,.$$

- Gaussian quantum weight [Gross, Pisarski, and Yaffe (1981)]:

$$S_{\rm eff} = \frac{8\pi^2}{\bar{g}^2} + \frac{4}{3}\sigma^2 + 16\,A(\sigma) \quad (\sigma \equiv \pi\frac{\rho}{\beta})\,,$$

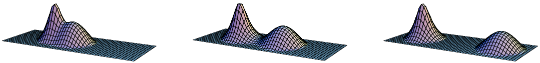
$$A(\sigma) \to -\frac{1}{6}\log\sigma \quad (\sigma\to\infty) \qquad A(\sigma) \to -\frac{\sigma^2}{36} \quad (\sigma \to 0)\,.$$

Conclusion of **semiclassical approx.**:
Trivial-holonomy-caloron weight exponentially suppressed at high T.

Nontrivial holonomy: Static magnetic dipoles

- construction based on [Ward 1977, Atiyah and Ward 1977, ADHM 1978, Drinfeld and Manin 1978, Manton 1978, Adler 1978, Rossi 1979, Nahm 1980-1983]

- explicitly carried out in [Lee and Lu 1998, Kraan and Van Baal 1998]: $A_4(\tau, r \to \infty) = -iut^3(0 \leq u \leq \frac{2\pi}{\beta})$.



action density of nontrivial-holonomy caloron with $k = 1$ plotted on 2D spatial slice

exact cancellation between A_4-mediated repulsion and A_i-mediated attraction; caloron radius ρ and thus monopole-core separation $D = \frac{\pi}{\beta}\,\rho^2$ increase from left to right (T and holonomy fixed)

Nontrivial holonomy, cntd.

computation of functional determinant about nontrivial holonomy carried out in [Gross, Pisarski, and Yaffe (1981), Diakonov et al. 2004] in (relevant) limit $\frac{D}{\beta} = \pi \left(\frac{\rho}{\beta}\right)^2 \gg 1$

conclusions:

- **total suppression** for nontrivial static holonomy in limit $V \to \infty$
- **attraction** of monop. and antimonop. for **small holonomy** $(0 \le u \le \frac{\pi}{\beta}(1 - \frac{1}{\sqrt{3}});\ \frac{\pi}{\beta}(1 + \frac{1}{\sqrt{3}}) \le u \le 2\,\frac{\pi}{\beta})$
- **repulsion** of monop. and antimonop. for **large holonomy** $(\frac{\pi}{\beta}(1 - \frac{1}{\sqrt{3}}) \le u \le \frac{\pi}{\beta}(1 + \frac{1}{\sqrt{3}}))$
- **unstability** of classical configuration under quantum noise $\Rightarrow$ **no entering of a priori estimate of thermal ground state**

Inert field ϕ

Observations and principles constraining construction of ϕ:

- $F_{\mu\nu} = \pm\tilde{F}_{\mu\nu} \Rightarrow$ vanishing energy-momentum:

$$
\begin{aligned}
\Theta_{\mu\nu} \quad = \quad & -2\,\mathrm{tr}\Big\{\delta_{\mu\nu}\left(\mp\mathbf{E}\cdot\mathbf{B} \pm \frac{1}{4}(2\mathbf{E}\cdot\mathbf{B} + 2\mathbf{B}\cdot\mathbf{E})\right) \\
& \mp(\delta_{\mu 4}\delta_{\nu i} + \delta_{\mu i}\delta_{\nu 4})\,(\mathbf{E}\times\mathbf{E})_i \\
& \pm\delta_{\mu i}\delta_{\nu(j\neq i)}\,(E_i B_j - E_i B_j) \pm \delta_{\mu(j\neq i)}\delta_{\nu i}\,(E_j B_i - E_j B_i)\Big\} \equiv 0\,.
\end{aligned}
$$

- spatial isotropy and homogeneity of *effective* local *not* associated with propagation of energy-momentum by *fundamental* gauge fields $\Rightarrow$ **inert scalar** ϕ
- modulo admissible gauge transformations ϕ does not depend on time
- relevance of ϕ (BPS) by gauge-invariant coupling to coarse-grained $k = 0$ sector (perturbative renormalizability) $\Rightarrow$ ϕ **adjoint scalar**

Inert field ϕ

Observations and principles constraining construction of ϕ, cntd:

- $F_{\mu\nu} \equiv \pm\tilde{F}_{\mu\nu} \Rightarrow$ any *local* "power" of $F_{\mu\nu}$ with an insertion of t^a **vanishes**
- **only trivial holonomy** in $F_{\mu\nu} \equiv \pm\tilde{F}_{\mu\nu}$ allowed
- $|\phi|$ is spacetime homogeneous $\Rightarrow$ information on ϕ's EOM is encoded in phase $\hat{\phi} \equiv \frac{\phi}{|\phi|}$
- definition of possible phases $\{\hat{\phi}\}$: due to BPS of $A_\mu^\pm$ **no explicit** T **dependence**, **flat measure** for admissible **integration over moduli** (excluding temporal shifts and global gauge rotations), Wilson lines between spatial points along **straight lines**

Inert field ϕ

Unique definition of $\{\hat{\phi}\}$ [Herbst and Hofmann 2004]:

$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \mathrm{tr} \int d^3x \int d\rho \, t^a \, F_{\mu\nu}(\tau, \mathbf{0}) \, \{(\tau, \mathbf{0}), (\tau, \mathbf{x})\}$$
$$\times F_{\mu\nu}(\tau, \mathbf{x}) \, \{(\tau, \mathbf{x}), (\tau, \mathbf{0})\} \, ,$$

where

$$\{(\tau, \mathbf{0}), (\tau, \mathbf{x})\} \equiv \mathcal{P} \exp \left[i \int_{(\tau,\mathbf{0})}^{(\tau,\mathbf{x})} dz_\mu \, A_\mu(z) \right] \, ,$$
$$\{(\tau, \mathbf{x}), (\tau, \mathbf{0})\} \equiv \{(\tau, \mathbf{0}), (\tau, \mathbf{x})\}^\dagger \, ,$$

and sum is over **Harrington-Shepard** (trivial-holonomy) caloron and anticaloron of scale ρ.

Higher n-point functions, higher topol. charge k? **No.**

(Would introduce mass dimension $d = 3 - n - m$ of object, $m > 1$ number of dimension-length caloron parameters at $k > 1$, but d needs to vanish.)

Inert field ϕ

Some observations, conventions:

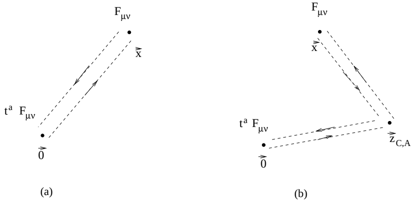
- $\hat{\phi}$ indeed transforms as an adjoint scalar:

$$\hat{\phi}^a(\tau) \to R^{ab}(\tau)\hat{\phi}^b(\tau)\,,$$

where R^{ab} is τ dependent matrix of adjoint rep.

$$R^{ab}(\tau)t^b = \Omega^\dagger(\tau,\mathbf{0})t^a\Omega(\tau,\mathbf{0})\,.$$

- What about shift of spatial center $\mathbf{0} \to \mathbf{z}_\pm$?



(a) graphical representation of **definition**

(b) only possible generalization to $\mathbf{z}_\pm \neq 0$

Shift of center amounts to spatially *global* gauge rotation induced by the group element $\Omega_z^\pm = \{(\tau,\mathbf{0}),(\tau,\mathbf{z}_\pm)\}$.

Inert field ϕ

Some observations, conventions, cntd:

- one has

$$\int_{(\tau,\mathbf{0})}^{(\tau,\mathbf{x})} dz_\mu A_\mu(z)|_\pm = \pm \int_0^1 ds\, x_i A_i(\tau, s\mathbf{x})$$

$$= \pm t_b x_b\, \partial_\tau \int_0^1 ds\, \log \Pi(\tau, sr, \rho) \;\Rightarrow$$

  integrand in the exponent of $\{(\tau,\mathbf{0}),(\tau,\mathbf{x})\}_\pm$ varies along a fixed direction in su(2) (a hedge hog); **Path-ordering can be ignored.**
- temporal shift freedom in $A_\mu^\pm$: set $\tau_\pm = 0$ and re-instate later
- parity: $F_{\mu\nu}(\tau,\mathbf{x})_+ = F_{\mu\nu}(\tau,-\mathbf{x})_-$ and

$$\begin{aligned}\{(\tau,\mathbf{0}),(\tau,\mathbf{x})\}_+ &= \left(\{(\tau,\mathbf{x}),(\tau,\mathbf{0})\}_+\right)^\dagger = \{(\tau,\mathbf{0}),(\tau,-\mathbf{x})\}_- \\ &= \left(\{(\tau,-\mathbf{x}),(\tau,\mathbf{0})\}_-\right)^\dagger \;\Rightarrow\end{aligned}$$

  $-$ contribution to the integrand in **definition** obtained by $\mathbf{x} \to -\mathbf{x}$ in $+$ contribution

Inert field ϕ

Some observations, conventions, cntd:

after tedious computation [Herbst and Hofmann 2004]
$+$ contribution to integrand in **definition** reads:

$$-\,i\,\beta^{-2}\frac{32\pi^4}{3}\,\frac{x^a}{r}\,\frac{\pi^2\hat{\rho}^4+\hat{\rho}^2(2+\cos(2\pi\hat{\tau}))}{\left(2\pi^2\hat{\rho}^2+1-\cos(2\pi\hat{\tau})\right)^2}\times F[\hat{g},\Pi]\,,$$

where $\hat{\rho}\equiv\frac{\rho}{\beta}$, $\hat{r}\equiv\frac{r}{\beta}$, $\hat{\tau}\equiv\frac{\tau}{\beta}$, and functional F is

$$\begin{aligned}F[\hat{g},\Pi]=2\cos(2\hat{g})&\left(2\frac{[\partial_\tau\Pi][\partial_r\Pi]}{\Pi^2}-\frac{\partial_\tau\partial_r\Pi}{\Pi}\right)\\&+\sin(2\hat{g})\left(2\frac{[\partial_r\Pi]^2}{\Pi^2}-2\frac{[\partial_\tau\Pi]^2}{\Pi^2}+\frac{\partial_\tau^2\Pi}{\Pi}-\frac{\partial_r^2\Pi}{\Pi}\right)\,,\end{aligned}$$

and

$$\{(\tau,\mathbf{0}),(\tau,\mathbf{x})\}_\pm\equiv\cos\hat{g}\pm 2it_b\frac{x^b}{r}\,\sin\hat{g}\,.$$

One shows that $\hat{g}$ saturates exponentially fast for $\hat{r}>1$.

Inert field ϕ

discussion:

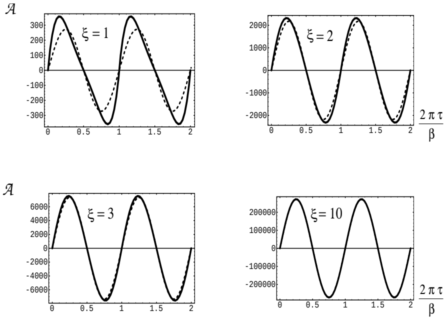
- angular integration would yield **zero** if radial integration was regular
- **but:** radial integration diverges logarithmically due to term $\frac{\partial_r^2 \Pi}{\Pi}$; this term arises from the **magnetic-magnetic** correlation (recall: no convergence in PT due to weakly screened magnetic sector!)
- zero$\times$infinity yields undetermined, multiplicative, and real constants $\Xi_\pm$
- without restriction of generality (global choice of gauge), angular integration regularized by defect azimuthal angle in 1-2 plane of su(2) for both $+$ and $-$ contributions $\Rightarrow$ **Members of $\{\hat{\phi}\}$ all move in hyperplane of su(2)!**
- re-instate $\tau \to \tau + \tau_\pm \Rightarrow$

Inert field ϕ

discussion, cntd:

result:

$$
\begin{aligned}
\{\hat{\phi}^a\} \;=\; & \{\Xi_+(\delta^{a1}\cos\alpha_+ + \delta^{a2}\sin\alpha_+)\,\mathcal{A}\,(2\pi(\hat{\tau}+\hat{\tau}_+)) \\
& +\Xi_-(\delta^{a1}\cos\alpha_- + \delta^{a2}\sin\alpha_-)\,\mathcal{A}\,(2\pi(\hat{\tau}+\hat{\tau}_-))\}\,, \quad \text{where}
\end{aligned}
$$



τ dependence of function $\mathcal{A}(\frac{2\pi\tau}{\beta})$;
saturation property (cutoff independence) for $\hat{\rho}$ integration.

Kernel of a differential operator D and potential for ϕ

- set $\{\hat{\phi}\}$ contains two real parameters for each "polarization": $\Xi_\pm$ and $\tau_\pm$; $\{\hat{\phi}\}$ is annihilated by **linear**, **second-order** differential operator $D = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2 \Rightarrow$

  $\{\hat{\phi}\}$ coincides with **kernel** of D and determines D uniquely
- linearity $\Rightarrow$ also $D\phi = 0$
- **but:** D depends on β explicitly, not allowed (BPS, caloron action given by topolog. charge)
- therefore seek potential $V(|\phi|^2)$ such that (Euclidean) action principle applied to

$$\mathcal{L}_\phi = \mathrm{tr}\left((\partial_\tau \phi)^2 + V(\phi^2)\right) .$$

  yields solutions annihilated by D, where $\mathcal{L}_\phi$ does not depend on β explicitly; demand that energy density $\Theta_{44} = 0$ on those solutions

Potential for and modulus of ϕ

- pick motion in 1-2 plane of su(2) (gauge invariance $\Rightarrow$ V **central** potential $\Rightarrow$ cons. angular momentum); ansatz:

$$\phi = 2\,|\phi|\,t_1\,\exp(\pm\frac{4\pi i}{\beta}t_3\tau)\,.$$

(circular motion in 1-2 plane, $|\phi|$ time independent!)

- apply E-L to $\mathcal{L}_\phi \Rightarrow$

$$\begin{aligned}\partial_\tau^2\phi^a &= \frac{\partial V(|\phi|^2)}{\partial|\phi|^2}\,\phi^a \text{ (in components) } \Leftrightarrow \\ \partial_\tau^2\phi &= \frac{\partial V(\phi^2)}{\partial\phi^2}\,\phi \text{ (in matrix form)}\,.\end{aligned}$$

- $\Theta_{44} = 0$ on ansatz $\phi \Rightarrow |\phi|^2\left(\frac{2\pi}{\beta}\right)^2 - V(|\phi|^2) = 0$

  but also: $\partial_\tau^2\phi + \left(\frac{2\pi}{\beta}\right)^2\phi = 0 \Rightarrow$

$$\frac{\partial V(|\phi|^2)}{\partial|\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2}\,.$$

Potential for and modulus of ϕ, cntd

- $\Rightarrow V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}$
  where Λ integration constant of mass dim. unity.

- $\Rightarrow |\phi| = \sqrt{\frac{\Lambda^3\beta}{2\pi}}$ (power-like decay of field ϕ with increasing T)

The field ϕ describes coarse-grained effect of **noninteracting** *trivial-holonomy calorons and anticalorons. It does not propagate, and its modulus $|\phi|$ sets the scale of off-shellness down to which quantum fluctuations, arising from the sector $k = 0$, must be considered "integrated out" in full effective theory (see also Markus' talk).*

- Indeed: cutting off ρ and r integrations at $|\phi|^{-1}$, τ dependence of $\mathcal{A}(\frac{2\pi\tau}{\beta})$ is perfect sine
  (Error at level smaller than 10^{-22} if knowledge about $T_c = \frac{\lambda_c\Lambda}{2\pi}$ with $\lambda_c = 13.87$ is used, later.)

BPS equation for ϕ

In addition to E-L equation ϕ satisfies **first-order**, BPS equation:

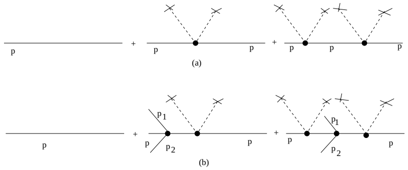
$$\partial_\tau \phi = \pm 2i\,\Lambda^3\, t_3\, \phi^{-1} = \pm i\, V^{1/2}(\phi)\,.$$

Because ϕ satisfies both, second-order E-L and first-order BPS equation, usual shift ambiguity in ground-state energy density, as allowed by E-L equation, **absent** *in SU(2) Yang-Mills thermodynamics.*

Effective action for deconfining phase

Coupling the coarse-grained $k = 0$ sector to ϕ, following constraints:

- perturbative renormalizability
  ['t Hooft, Veltman, Lee, and Zinn-Justin 1971-1973]
  $\Rightarrow$ form invariance of action for effective $k = 0$ gauge field a_μ from integrating fundamental $k = 0$ fluctuations only, no higher dim. ops. for a_μ only
- no energy-momentum transfer to $\phi \Rightarrow$ absence of higher dim. ops. involving a_μ **and** ϕ
- gauge invariance $\Rightarrow \partial_\mu\phi \rightarrow D_\mu\phi \equiv \partial_\mu\phi - ie[a_\mu, \phi]$ (e **effective** coupling); no momentum transfer to ϕ if (unitary gauge $\phi = 2|\phi|\, t_3$) massive 1,2 modes propagate on-shell only



Effective action and ground-state estimate

unique effective action density:

$$\mathcal{L}_{\text{eff}}[a_\mu] = \text{tr}\left(\frac{1}{2}\,G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)\,,$$

$$\text{where } G_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - ie[a_\mu, a_\nu] \equiv G^a_{\mu\nu}\,t_a$$

ground-state estimate:

- E-L EOM from $\mathcal{L}_{\text{eff}}[a_\mu]$

$$D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]\,.$$

- solved by zero-curvature (pure-gauge) config. a^{gs}_μ:

$$\begin{aligned} a^{\text{gs}}_\mu &= \mp\delta_{\mu 4}\frac{2\pi}{e\beta}\,t_3 \quad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0) \quad \Rightarrow \\ \rho^{\text{gs}} &= -P^{\text{gs}} = 4\pi\Lambda^3\,T\,. \end{aligned}$$

Unresolvable interactions between $k = 0$ and $|k| = 1$ lifted ρ^{gs} from zero (BPS). EOS of a cosmological constant; pressure **negative**. *(Short-lived, attracting magnetic (anti)monopoles by temporary shifts of (anti)caloron holonomies from trivial to small through absorption of hard plane-wave fluctuations.)*

Winding to unitary gauge: $\mathbf{Z}_2$ degeneracy

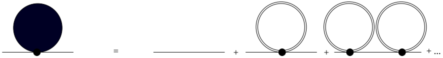
- consider gauge rotation $\tilde{\Omega}(\tau) = \Omega_{\rm gl}\, Z(\tau)\, \Omega(\tau)$ where $\Omega(\tau) \equiv \exp[\pm 2\pi i \frac{\tau}{\beta} t_3]$, $Z(\tau) = \Big(2\Theta(\tau - \frac{\beta}{2}) - 1\Big)\, \mathbf{1}_2$, and $\Omega_{\rm gl} = \exp[i\frac{\pi}{2} t_2]$
- $\tilde{\Omega}(\tau)$ transforms $a^{\rm gs}_\mu$ to $a^{\rm gs}_\mu \equiv 0$ and ϕ to $\phi = 2t^3|\phi|$
- $\tilde{\Omega}(\tau)$ is **admissible** because respects periodicity of δa_μ:

$$
\begin{aligned}
a_\mu &\to \tilde{\Omega}(a^{\rm gs}_\mu + \delta a_\mu)\tilde{\Omega}^\dagger + \frac{i}{e}\tilde{\Omega}\partial_\mu\tilde{\Omega}^\dagger \\
&= \Omega_{\rm gl}\left(\Omega(a^{\rm gs}_\mu + \delta a_\mu)\Omega^\dagger + \frac{i}{e}\Big(\Omega\partial_\mu\Omega^\dagger + Z\partial_\mu Z\Big)\right)\Omega^\dagger_{\rm gl} \\
&= \Omega_{\rm gl}\left(\Omega\delta a_\mu\Omega^\dagger + \frac{2i}{e}\delta(\tau - \frac{\beta}{2})Z\right)\Omega^\dagger_{\rm gl} = \Omega_{\rm gl}\Omega\,\delta a_\mu\,(\Omega_{\rm gl}\Omega)^\dagger\,.
\end{aligned}
$$

- $\tilde{\Omega}(\tau)$ transforms Polyakov loop from $-\mathbf{1}_2$ to $\mathbf{1}_2 \Rightarrow$ ground-state estimate is (electric) $\mathbf{Z}_2$ degenerate $\Rightarrow$ **deconfining phase**

Mass spectrum; outlook resummed radiative corrections

- computation in physical and completely fixed **unitary, Coulomb gauge** ($\phi = 2t^3|\phi|$, $\partial_i a_i^3 = 0$), see Markus' talk
- mass spectrum: $m^2 \equiv m_1^2 = m_2^2 = 4e^2 \frac{\Lambda^3}{2\pi T}$, $m_3 = 0$.
- resummation of **polarization tensor of massless mode** as



$\Rightarrow$ small linear-in-T correction to tree-level ground-state estimate [Falquez, Hofmann, Baumbach 2010]

$$\text{tree-level:} \qquad \frac{\rho^{\text{gs}}}{T^4} = 3117.09\,\lambda^{-3}\,,$$

$$\text{one-loop resummed:} \qquad \frac{\Delta\rho^{\text{gs}}}{T^4} = 3.95\,\lambda^{-3}\,.$$

- large hierarchy between loop orders (conjecture about **termination at finite irreducible order**, see Dariush' talk), so one-loop correction **practically exact**

T dependence of e: selfconsistent thermal quasiparticles

P and ρ at one loop:

$$P(\lambda) = -\Lambda^4 \left\{ \frac{2\lambda^4}{(2\pi)^6} \left[2\bar{P}(0) + 6\bar{P}(2a) \right] + 2\lambda \right\} ,$$

$$\rho(\lambda) = \Lambda^4 \left\{ \frac{2\lambda^4}{(2\pi)^6} \left[2\bar{\rho}(0) + 6\bar{\rho}(2a) \right] + 2\lambda \right\} ,$$

where

$$\bar{P}(y) \equiv \int_0^\infty dx\, x^2 \log\left[1 - \exp(-\sqrt{x^2+y^2}) \right] ,$$

$$\bar{\rho}(y) \equiv \int_0^\infty dx\, x^2 \frac{\sqrt{x^2+y^2}}{\exp(\sqrt{x^2+y^2}) - 1} ,$$

and $a \equiv \frac{m}{2T} = 2\pi e \lambda^{-3/2}$. For later use introduce function $D(2a)$ as

$$\partial_{y^2} \bar{P}\Big|_{y=2a} = -\frac{1}{4\pi^2} \int_0^\infty dx\, \frac{x^2}{\sqrt{x^2+(2a)^2}} \frac{1}{e^{\sqrt{x^2+(2a)^2}} - 1} \equiv -\frac{1}{4\pi^2} D(2a) .$$

Legendre transformation and evolution equation

- for $m(T)$ to respect Legendre trafo (fundamental partition function) between P and $\rho \Leftrightarrow \partial_m P = 0$
- $\Rightarrow$ first-order **evolution equation**

$$\partial_a \lambda = -\frac{24\lambda^4 a}{(2\pi)^6} \frac{D(2a)}{1 + \frac{24\lambda^3 a^2}{(2\pi)^6} D(2a)} \,.$$

or

$$1 = -\frac{24\lambda^3}{(2\pi)^6} \left(\lambda \frac{da}{d\lambda} + a \right) a \, D(2a) \,.$$

- $\Rightarrow$ dependence $a(\lambda)$ monotonic decreasing
  $\Rightarrow$ for $\lambda \gg 1$ a must fall below unity
- **fixed points of evolution equation**:

  repulsive at $a = 0$ $(\lambda \to \infty)$

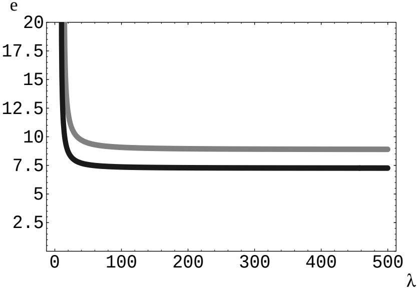
  attractive at $a = \infty$ $(\lambda = \lambda_c)$

Solution to evolution equation

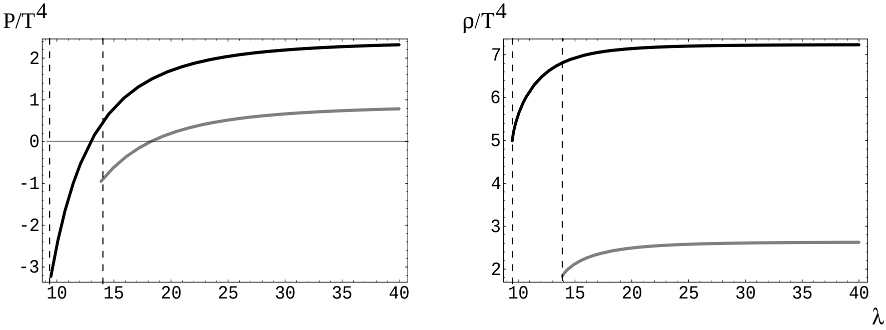
- $a \ll 1$ [Dolan, Jackiw 1974] $\Rightarrow 1 = -\frac{\lambda^3}{(2\pi)^4}\left(\lambda \frac{da}{d\lambda} + a\right) a$;
  solution $(a(\lambda_i) = a_i \ll 1)$:

$$a(\lambda) = 4\sqrt{2}\pi^2\lambda^{-3/2}\left(1 - \frac{\lambda}{\lambda_i}\left[1 - \frac{a_i^2\lambda_i^3}{32\pi^4}\right]\right)^{1/2} .$$

  $\Rightarrow$ attractor $a(\lambda) = 4\sqrt{2}\pi^2\lambda^{-3/2}$ as long as $a \ll 1$
  $\Rightarrow e = \sqrt{8}\pi$ as long as $a \ll 1$ (amusingly: $S = \frac{8\pi^2}{e^2} = 1$)
- full solution for $e(\lambda) \Rightarrow \lambda_c = 13.87$:



T dependence of P and ρ



- notice **negativity** of P shortly above λ_c
- relative correction to one-loop quasiparticle P and ρ by radiative effects: $< 1\%$, see talks by Markus and Dariush

Summary and outlook

Summary:

- brief motivation why nonperturbative approach to YMTD necessary: mass generation, poor convergence of pert. orders
- mini review on calorons: trivial vs. nontrivial holonomy for $|k| = 1$ plus semiclassical approx.
- construction of thermal ground-state estimate: inert field ϕ; BPS and E-L; potential
- discussion of constraints of effective action: pert. renormalizability plus inertness of $\phi \Rightarrow$ unique answer
- full ground-state estimate, deconfining nature, tree-level quasiparticles
- evolution of effective coupling
- T dependence pressure and energy density

Summary and outlook

Outlook:

- radiative corrections: polarization tensor of massless mode (Markus)
- radiative corrections: stable but unresolvable monopoles (Markus)
- radiative corrections: two-loop and three-loop cases (Dariush)
- radiative corrections: loop expansion of pressure, conjecture on termination at finite irreducible order (Dariush)
- two other phases:
  - **preconfining** (thermal ground state: condensate of massless monopoles and antimonopoles)
  - **confining** (ground state of zero energy density: condensate of single, round-point like center-vortex loops)

Summary and outlook

Some physics implications:

(i) mechanism for ew SB (LHC: not much of a Higgs signal so far)

(ii) postulate: SU(2) (10^{-4} eV) describes photon **propagation**

$\Rightarrow$ black-body spectral anomaly at $T \sim 3\,$K and low frequencies
(cold H1 clouds, large-angle anomalies in TT of CMB, UEGE)

$\Rightarrow$ Planck-scale axion plus such an SU(2) yield **Dark Energy**

Thank you.