

# The Polarization Tensor of the Massless Mode in Yang-Mills Thermodynamics

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21 September 2011



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## Review of Ralf's talk

- ▶ After coarse-graining, nonperturbative YMT ground state described by scalar field  $\phi(T)$   
[Herbst,Hofmann '04, Hofmann '05, Giacoia,Hofmann '05]
- ▶ Effective action for top. trivial sector

$$S_E[a_\mu] = \int_0^{\frac{1}{T}} dx_4 \int dx^3 \text{Tr} \left( \frac{1}{2} G_{\mu\nu} G_{\mu\nu} + D_\mu \phi D_\mu \phi + \Lambda^6 \phi^{-2} \right)$$

- ▶ In deconfining phase ( $T > \Lambda$ ):  $SU(2) \rightarrow U(1)$
- ▶ Two tree-level massive modes (TLH) with mass  $m^2(T) = 4e^2(T)|\phi(T)|^2 = 4e^2\Lambda^3/2\pi T$ ,  
one tree-level massless mode (TLM)
- ▶ Eff. coupling  $e(T)$  has plateau value  $e_{\text{plateau}} \sim \sqrt{8\pi} \approx 8.8$ ,  
no PT but loop expansion
- ▶  $|\phi|$  yields max. resolution

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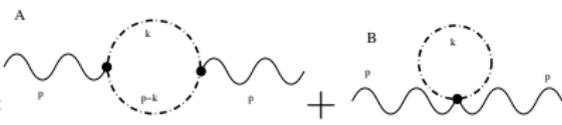
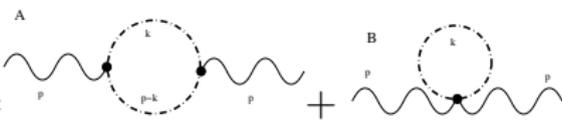
    Longitudinal Dispersion Relation

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## Polarization Tensor, Motivation

- ▶ Effect TLH modes on propagation of TLM modes described by polarization tensor  $\Sigma^{\mu\nu}$ .

- ▶ On one-loop level:  $\Sigma^{\mu\nu} =$   + 
- (one-loop sufficient, cmp. talk by Dariush)

- ▶ simplest radiative correction
- ▶ generic radiative correction
- ▶ interesting consequences for physics

## Polarization Tensor, Decomposition

$U(1)$  gauge symmetry unbroken,  $\Rightarrow \Sigma^{\mu\nu}$  4D transverse:  $p_\mu \Sigma^{\mu\nu} = 0$   
Decomposition into spatially transverse and longitudinal part:

$$\Sigma^{\mu\nu} = G(p_4, \mathbf{p}) P_T^{\mu\nu} + F(p_4, \mathbf{p}) P_L^{\mu\nu}$$

with

$$P_L^{\mu\nu} \equiv \delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} - P_T^{\mu\nu},$$

projecting also onto  $\mathbf{p}$ .

## Free Propagators

Free propagator for TLH and TLM modes in

- ▶ unitary gauge (particle content manifest)
- ▶ Coulomb gauge ( $\nabla \mathbf{A} = 0$ )

$$D_{\mu\nu,ab}^{TLH,0}(k) = -\delta_{ab} \tilde{D}_{\mu\nu} \frac{1}{k^2 + m^2}, \quad \{a, b\} \in \{1, 2\}$$

$$D_{\mu\nu,ab}^{TLM,0}(p) = -\delta_{a3} \delta_{b3} \left( P_{\mu\nu}^T \frac{1}{p^2} + \frac{u_\mu u_\nu}{\mathbf{p}^2} \right)$$

$u_\mu = \delta_{4\mu}$  four-velocity of head bath.

$\tilde{D}_{\mu\nu}$  projects out the component transverse to  $k$

$P_T^{\mu\nu}$  projects out the component transverse to  $\mathbf{p}$ .

Gauge fixed completely  $\Rightarrow$  no ghost fields needed.

## Dressed Propagator

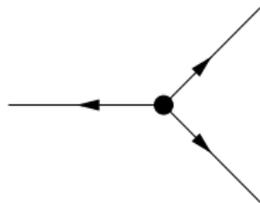
Propagator for interacting TLM mode (imaginary time)

$$D_{\mu\nu,ab}^{TLM}(p) = -\delta_{a3}\delta_{b3} \left( P_{\mu\nu}^T \frac{1}{p^2 + G} + \frac{p^2}{\mathbf{p}^2} \frac{u_\mu u_\nu}{p^2 + F} \right)$$

$F(p_4, \mathbf{p}) = \left(1 - \frac{p_4^2}{p^2}\right)^{-1} \Sigma^{44}$  describes propagation of longitudinal mode  $A_4$

For  $\mathbf{p} \parallel \mathbf{e}_3$ ,  $G(p_4, \mathbf{p}) = \Sigma^{11} = \Sigma^{22}$  describes propagation of transverse mode  $A_i$

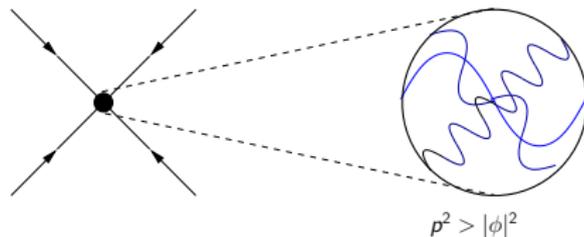
## Vertices and Momentum constraints



$$\Gamma_{[3]abc}^{\mu\nu\rho}$$

▶ Momentum conservation at each vertex.

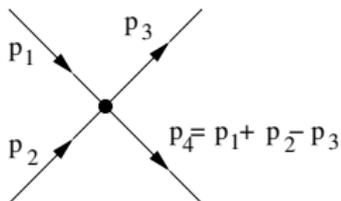
▶ Effective vertex contains modes with  $|p^2| > |\phi|^2$ :



▶ Exclude these modes in effective theory to avoid "double counting" (already included in  $a_{\mu}^{g.s.}$ ; cmp. talk by Ralf)

## Momentum constraints

$|\phi|$  yields maximum resolution in effective theory  $\Rightarrow$  constraints on momentum transfer in vertex



$$\text{s-channel: } |(p_1 + p_2)^2| \leq |\phi|^2$$

$$\text{t-channel: } |(p_1 - p_3)^2| \leq |\phi|^2$$

$$\text{u-channel: } |(p_2 - p_3)^2| \leq |\phi|^2$$

### Recall

Finite temperature QFT defined in imaginary time  $x_4$  with fields being periodic in  $x_4$ . Only discrete  $p_4$  momenta allowed (Matsubara sums).

### Problem

Momentum constraints formulated in terms of physical, continuous four momenta.

# Real time propagators

## Solution

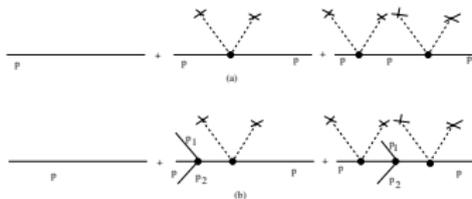
Express Matsubara sums as integrals over continuous real time  $t$ .  
[Kapusta, LeBellac]

Free propagator for TLH and TLM modes in unitary Coulomb gauge and real-time formalism

$$D_{\mu\nu,ab}^{TLM,0}(p) = \delta_{a3}\delta_{b3} \left\{ P_{\mu\nu}^T \left[ \frac{-i}{p^2 + i\epsilon} - 2\pi\delta(p^2) n_B(|p_0|/T) \right] + i \frac{u_\mu u_\nu}{p^2} \right\}$$

$$D_{\mu\nu,ab}^{TLH,0}(k) = -2\pi\delta_{ab}\tilde{D}_{\mu\nu}\delta(k^2 - m^2) n_B(|k_0|/T), \quad a, b \in \{1, 2\}$$

No vacuum propagator for TLH modes (cmp. talk by Ralf):



## Modified dispersion relation of TLM

Dressed propagator of transverse and longitudinal TLM mode (real time)

$$D_{\mu\nu,ab}^{TLM}(\mathbf{p}_t) = -\delta_{a3}\delta_{b3}P_{\mu\nu}^T \left[ \frac{i}{p_t^2 - G} + 2\pi\delta(p_t^2 - G) n_B(|\mathbf{p}_{0,t}|/T) \right]$$
$$D_{\mu\nu,ab}^{TLM}(\mathbf{p}_l) = \delta_{a3}\delta_{b3}u_\mu u_\nu \left[ \frac{p_l^2}{\mathbf{p}_l^2} \frac{i}{p_l^2 - F} - 2\pi\delta(p_l^2 - F) n_B(|\mathbf{p}_{0,l}|/T) \right]$$

Poles yield dispersion relations ( $p_0 = \omega + i\gamma$ , assume  $\gamma \ll \omega$ ):

$$\omega_t^2(\mathbf{p}_t) = \mathbf{p}_t^2 + \text{Re}G(\omega(\mathbf{p}_t), \mathbf{p}_t) \qquad \omega_l^2(\mathbf{p}_l) = \mathbf{p}_l^2 + \text{Re}F(\omega_L(\mathbf{p}_l), \mathbf{p}_l)$$
$$\gamma(\mathbf{p}_t) = -\text{Im}G(\omega(\mathbf{p}_t), \mathbf{p}_t)/2\omega \qquad \gamma_l(\mathbf{p}_l) = -\text{Im}F(\omega_l(\mathbf{p}_l), \mathbf{p}_l)/2\omega_l$$

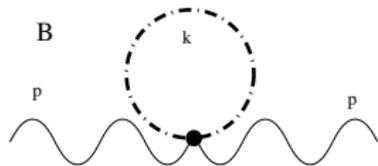
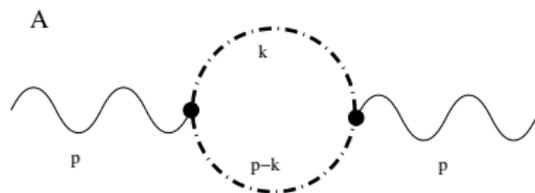
## Diagrams for $G$ and $F$

Choosing  $\mathbf{p} \parallel \mathbf{e}_3$ :

$$G(p_0, \mathbf{p}) = \Sigma^{11} = \Sigma^{22}$$

$$F(p_0, \mathbf{p}) = \left(1 - \frac{p_0^2}{p^2}\right)^{-1} \Sigma^{00}$$

$\Sigma^{\mu\nu}$  sum of two diagrams:



Purely imaginary:  $\Rightarrow$  yields  $\gamma$

One-loop level sufficient (see talk by Dariush)!

Purely real:  $\Rightarrow$  yields dispersion relation

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## Approximation $p^2 = 0$

Applying Feynman rules to  $\text{Re}G = \text{Re}\Sigma^{11} =$    
yields gap equation:

$$\text{Re}G(p_0, \mathbf{p}) = \Sigma_B^{11}(p) = 8\pi e^2 \int_{|(p+k)^2| \leq |\phi|^2} \left[ - \left( 3 - \frac{k^2}{m^2} \right) + \frac{k^1 k^1}{m^2} \right] \\ \times n_B(|k_0|/T) \delta(k^2 - m^2) \frac{d^4 k}{(2\pi)^4} \Big|_{p^2=G} \quad (1)$$

4 vertex imposes constraint  $|(p+k)^2| \leq |\phi|^2$

### Difficulty

Equation (1) is a transcendental equation for  $G$ .

### Approximation

Use  $p^2 = 0$  in constraint [Schwarz, Giacosa, Hofmann '06].

Valid if  $G \ll \mathbf{p}^2$ . Check later!

## Consequences of $p^2 = 0$

1. For finite  $\Sigma^{00}$ ,  $F(p_0, \mathbf{p}) = \left(1 - \frac{p_0^2}{p^2}\right)^{-1} \Sigma^{00}$  vanishes.  
Hence, no propagation of longitudinal modes.
2. Diagram A vanishes:



Momentum conservation at vertex forbids TLM mode with  $p^2 = 0$  to split into two on-shell particles with mass  $m$ .

3. Diagram A = 0, hence no imaginary part of  $G$ , hence  $\gamma = 0$  and assumption  $\gamma \ll \omega$  satisfied trivially.

## Calculation of diagram B, $p^2 = 0$

With  $p^2 = 0$ :

$$G(|\mathbf{p}|, \mathbf{p}) = 8\pi e^2 \int_{|2p\mathbf{k} + k^2| \leq |\phi|^2} \left[ g^{11} \left( 3 - \frac{k^2}{m^2} \right) + \frac{k^1 k^1}{m^2} \right] \\ \times n_B(|k_0|/T) \delta(k^2 - m^2) \frac{d^4 k}{(2\pi)^4}$$

Via  $\delta$ -function, integration over  $k_0$  yields  $k_0 \rightarrow \pm\sqrt{\mathbf{k}^2 + m^2}$ . Using  $p_0 \geq 0$ ,  $p^2 = 0$ ,  $k^2 = m^2 = 4e^2|\phi|^2$ ,  $\theta \equiv \angle(\mathbf{p}, \mathbf{k})$ , and  $k_0 = \pm\sqrt{\mathbf{k}^2 + m^2}$  constraint reads:

$$\left| 2|\mathbf{p}| \left( \pm\sqrt{\mathbf{k}^2 + 4e^2|\phi|^2} - |\mathbf{k}| \cos\theta \right) + 4e^2|\phi|^2 \right| \leq |\phi|^2$$

Integrand symmetric under  $k_0 \rightarrow -k_0$ , but constraint is not!

## Dealing with constraints, + sign

Consider + sign and observe:

$$\left| 2|\mathbf{p}| \left( +\sqrt{\mathbf{k}^2 + 4e^2|\phi|^2} - |\mathbf{k}| \cos \theta \right) + 4e^2|\phi|^2 \right| \leq |\phi|^2$$

1. Term in parentheses always positive.
2.  $e \gtrsim \sqrt{8}\pi \sim 8.8$

Hence, constraint never satisfied.

## Dealing with constraints, – sign

Using  $X \equiv |\mathbf{p}|/T$ ,  $\mathbf{y} \equiv \mathbf{k}/|\phi|$ ,  $|\phi|/T = 2\pi\lambda^{-3/2}$ , and  $\lambda \equiv 2\pi T/\Lambda$   
constraint reads

$$-1 \leq -\lambda^{3/2} \frac{X}{\pi} \left( \sqrt{\mathbf{y}^2 + 4e^2} + y_3 \right) + 4e^2 \leq 1$$

Convenient to use polar coordinates  $y_1 = \rho \cos \varphi$ ,  $y_2 = \rho \sin \varphi$ .  
Constraint then reads

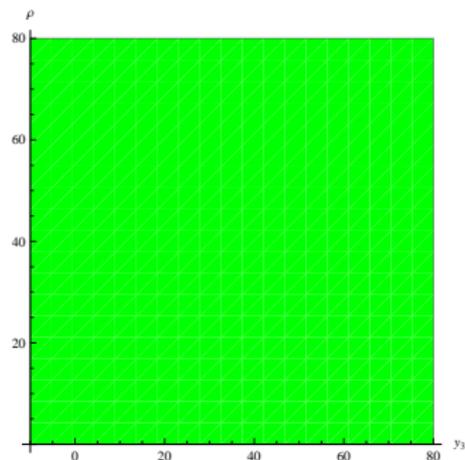
$$\frac{4e^2 - 1}{\lambda^{3/2}} \frac{\pi}{X} \leq \sqrt{\rho^2 + y_3^2 + 4e^2} + y_3 \leq \frac{4e^2 + 1}{\lambda^{3/2}} \frac{\pi}{X}$$

Integration over  $\varphi$  not constraint!

## Dealing with constraints, – sign

$$\frac{4e^2 - 1}{\lambda^{3/2}} \frac{\pi}{X} \leq \sqrt{\rho^2 + y_3^2} + 4e^2 + y_3 \leq \frac{4e^2 + 1}{\lambda^{3/2}} \frac{\pi}{X}$$

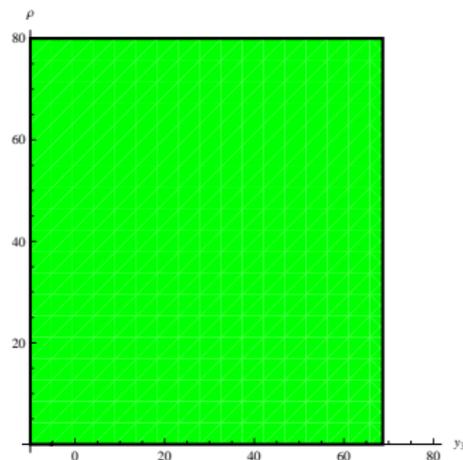
### 1. $y_3, \rho$ plane



## Dealing with constraints, – sign

$$\frac{4e^2 - 1}{\lambda^{3/2}} \frac{\pi}{X} \leq \sqrt{\rho^2 + y_3^2} + 4e^2 + y_3 \leq \frac{4e^2 + 1}{\lambda^{3/2}} \frac{\pi}{X}$$

1.  $y_3, \rho$  plane
2.  $y_3 < y_{\max} \equiv \frac{4e^2 + 1}{\lambda^{3/2}} \frac{\pi}{X}$



## Dealing with constraints, – sign

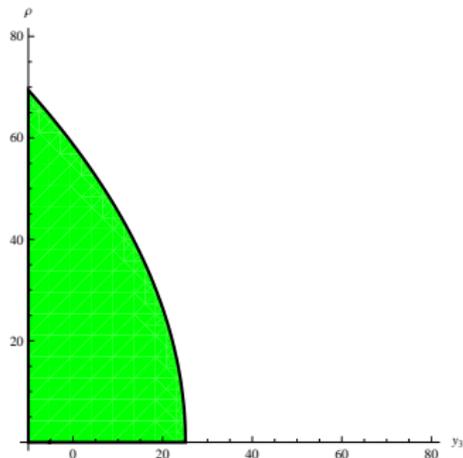
$$\frac{4e^2 - 1}{\lambda^{3/2}} \frac{\pi}{X} \leq \sqrt{\rho^2 + y_3^2} + 4e^2 + y_3 \leq \frac{4e^2 + 1}{\lambda^{3/2}} \frac{\pi}{X}$$

1.  $y_3, \rho$  plane

2.  $y_3 < y_{\max} \equiv \frac{4e^2 + 1}{\lambda^{3/2}} \frac{\pi}{X}$

3.  $\rho \leq \rho_{\max}(y_3) \equiv$   
$$\sqrt{\left(\frac{\pi}{X}\right)^2 \frac{(4e^2 + 1)^2}{\lambda^3} - \frac{2\pi}{X} \frac{4e^2 + 1}{\lambda^{3/2} y_3 - 4e^2}}$$
  
for

$$y_3 \leq y_3^M \equiv \frac{\pi}{2X} \frac{4e^2 + 1}{\lambda^{3/2}} - \frac{2\lambda^{3/2} X}{\pi} \frac{e^2}{4e^2 + 1}$$



## Dealing with constraints, – sign

$$\frac{4e^2 - 1}{\lambda^{3/2}} \frac{\pi}{X} \leq \sqrt{\rho^2 + y_3^2} + 4e^2 + y_3 \leq \frac{4e^2 + 1}{\lambda^{3/2}} \frac{\pi}{X}$$

1.  $y_3, \rho$  plane

2.  $y_3 < y_{\max} \equiv \frac{4e^2+1}{\lambda^{3/2}} \frac{\pi}{X}$

3.  $\rho \leq \rho_{\max}(y_3) \equiv$   

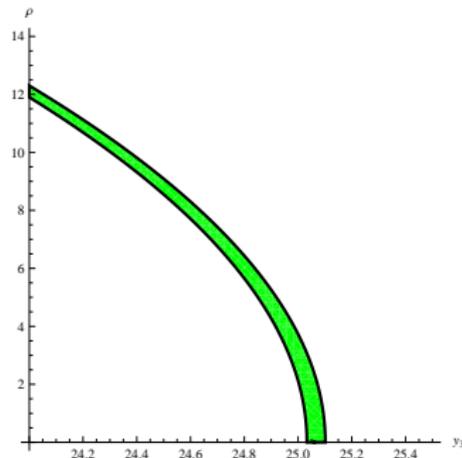
$$\sqrt{\left(\frac{\pi}{X}\right)^2 \frac{(4e^2+1)^2}{\lambda^3} - \frac{2\pi}{X} \frac{4e^2+1}{\lambda^{3/2}y_3 - 4e^2}}$$
 for

$$y_3 \leq y_3^M \equiv \frac{\pi}{2X} \frac{4e^2+1}{\lambda^{3/2}} - \frac{2\lambda^{3/2}X}{\pi} \frac{e^2}{4e^2+1}$$

4.  $\rho \geq \rho_{\min}(y_3) \equiv$   

$$\sqrt{\left(\frac{\pi}{X}\right)^2 \frac{(4e^2-1)^2}{\lambda^3} - \frac{2\pi}{X} \frac{4e^2-1}{\lambda^{3/2}y_3 - 4e^2}}$$
 for

$$y_3 \leq y_3^m \equiv \frac{\pi}{2X} \frac{4e^2-1}{\lambda^{3/2}} - \frac{2\lambda^{3/2}X}{\pi} \frac{e^2}{4e^2-1}$$



## Expression for $G$ , $p^2 = 0$

Constraint in terms of boundaries for  $(y_3, \rho)$  integration:

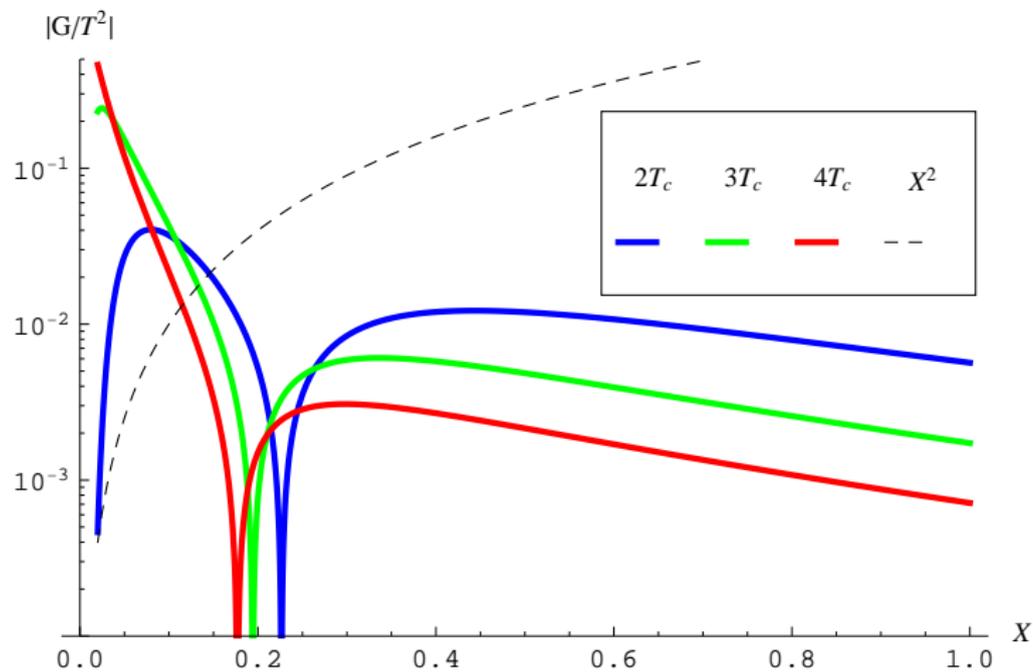
$$\frac{G(X, T)}{T^2} = \left[ \int_{-\infty}^{y_3^m} dy_3 \int_{\rho_{\min}}^{\rho_{\max}} d\rho + \int_{y_3^m}^{y_3^M} dy_3 \int_0^{\rho_{\max}} d\rho \right] \frac{e^2 \rho}{\lambda^3} \left( \frac{\rho^2}{4e^2} - 4 \right) \frac{n_B \left( 2\pi \lambda^{-3/2} \sqrt{\rho^2 + y_3^2 + 4e^2} \right)}{\sqrt{\rho^2 + y_3^2 + 4e^2}}$$

$X = |\mathbf{p}|/T$  momentum of external TLM mode in units of temperature,

$\lambda \equiv 2\pi T/\Lambda$  temperature in units of YM scale  $\Lambda$ .

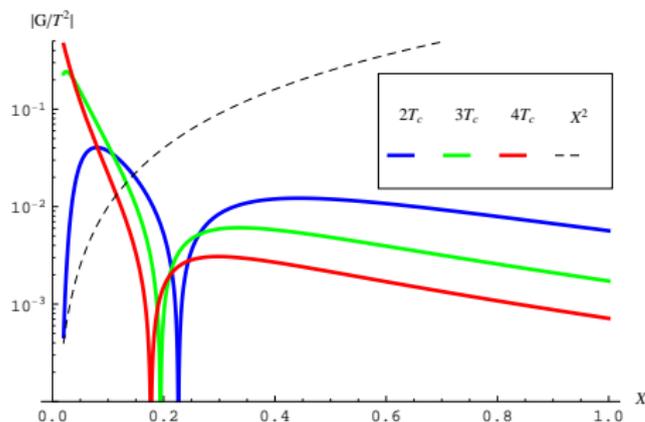
Integration performed numerically via Gaussian quadrature (unlike Monte-Carlo for 3-loop, comp. talk by Dariush).

Result for  $G, p^2 = 0$



## Result for $G, p^2 = 0$

All results based on  $|G/T^2| \ll X^2$ !



- ▶  $X \gtrsim 0.2$ :  $G < 0$   
(anti-screening)
- ▶ Dip:  $G = 0$
- ▶  $X \lesssim 0.2$ :  $G > 0$   
(screening)

- ▶  $G \ll p^2$  for  $X \gtrsim 0.2$
- ▶  $G = 0$  at  $X \sim 0.2$ , dispersion relation solved selfconsistently
- ▶  $G \geq p^2$  for  $X \lesssim 0.1$ , **approximation breaks down**

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## Full calculation of $G$

Gap equation:

$$\begin{aligned}\operatorname{Re}G(p_0, \mathbf{p}) &= 8\pi e^2 \int_{|(p+k)^2| \leq |\phi|^2} \left[ - \left( 3 - \frac{k^2}{m^2} \right) + \frac{k^1 k^1}{m^2} \right] \\ &\quad \times n_B(|k_0|/T) \delta(k^2 - m^2) \frac{d^4 k}{(2\pi)^4} \Big|_{p^2=G} \\ &\equiv H(T, \mathbf{p}, G)\end{aligned}$$

Via  $\delta$ -function, integration over  $k_0$  yields  $k_0 \rightarrow \pm\sqrt{\mathbf{k}^2 + m^2}$ .  
With  $p_0 = \pm\sqrt{\mathbf{p}^2 + G(p_0, \mathbf{p})}$  and  $\mathbf{p} \parallel \mathbf{e}_3$  constraint reads

$$\left| G + 2 \left( \pm\sqrt{\mathbf{p}^2 + G} \sqrt{\mathbf{k}^2 + m^2} - pk_3 \right) + m^2 \right| \leq |\phi|^2$$

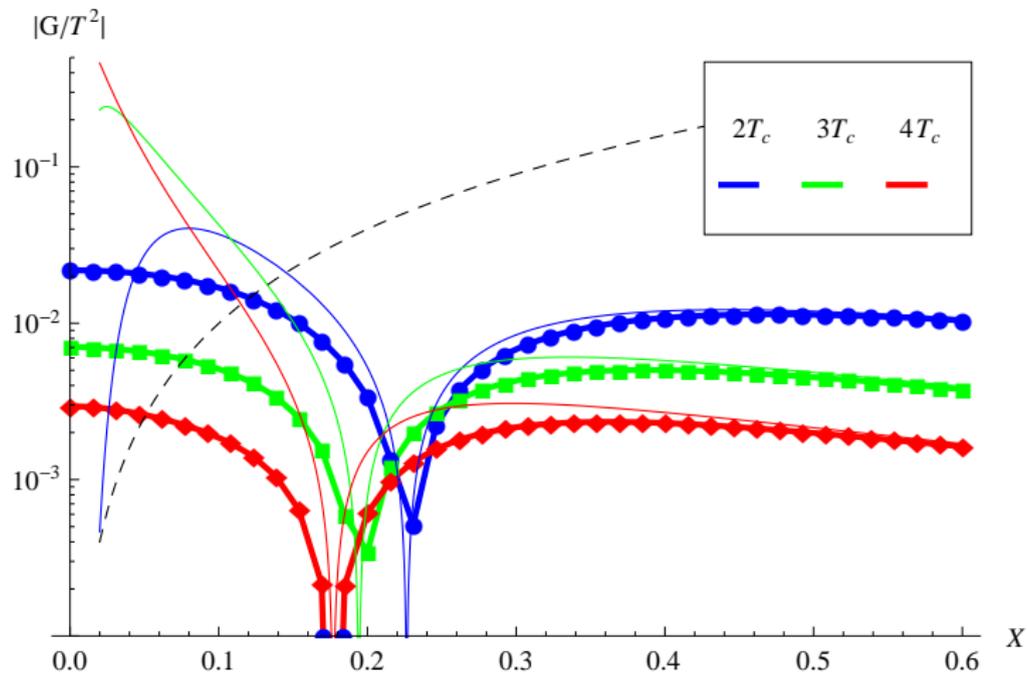
## Full calculation of $G$

Strategy to solve  $\text{Re}G(p_0, \mathbf{p}) = H[T, \mathbf{p}, G(p_0, \mathbf{p})]$

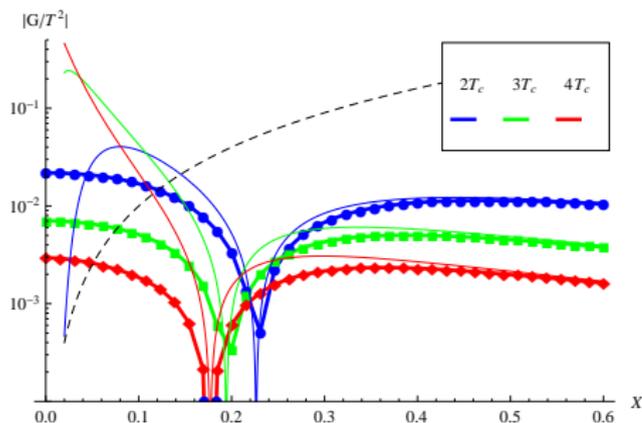
[Ludescher, Hofmann '08]:

1. fix  $T$  and  $\mathbf{p}$  (integrand in  $H$  independent of  $p_0 \Rightarrow G = G(\mathbf{p})$ )
2. prescribe any value of  $G_1$
3. calculate  $H(T, \mathbf{p}, G_1)$  using Monte-Carlo integration
4. repeat steps 2 and 3 to obtain  $H(T, \mathbf{p}, G)$
5. solve  $G = H(T, \mathbf{p}, G)$  numerically using Newton's method

# Selfconsistent result for $G$ , real part



## Selfconsistent result for $G$ , real part



- ▶  $X \gtrsim 0.2$ :  $G < 0$   
(anti-screening)
- ▶ Dip:  $G = 0$
- ▶  $X \lesssim 0.2$ :  $G > 0$   
(screening)

Comparison with approximate result:

- ▶ Zeros of  $G$  agree (must be)
- ▶ For  $X \gtrsim 0.2$ , approximate agrees with selfconsistent result (expected)
- ▶ Results different when  $G \gtrsim X^2$  (not surprising)

## Selfconsistent result for $G$ , imaginary part

Imaginary part:  $\text{Im}G \propto$  

At left vertex: particle with mass  $\sqrt{G}$  decaying into two on-shell particles with mass  $m$  only possible if

$$\frac{G}{T^2} \geq 4 \frac{m^2}{T^2} = 64\pi^2 \frac{e^2}{\lambda^3} \quad (2)$$

- ▶  $G \leq 0$ : condition (2) never satisfied
- ▶  $G > 0$ :

$$\frac{G(X=0, T)}{T^2} \propto \frac{1}{\lambda^3} \ll 64\pi^2 \frac{e^2}{\lambda^3} \sim 5 \times 10^4 / \lambda^3$$

condition (2) never satisfied

Diagram  $A = 0$ , hence no imaginary part of  $G$ , hence  $\gamma = 0$  and assumption  $\gamma \ll \omega$  satisfied trivially.

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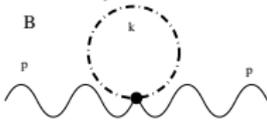
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## Full Calculation of $F$

Assume  $F \in \mathbb{R}$  (turns out to be selfconsistent)

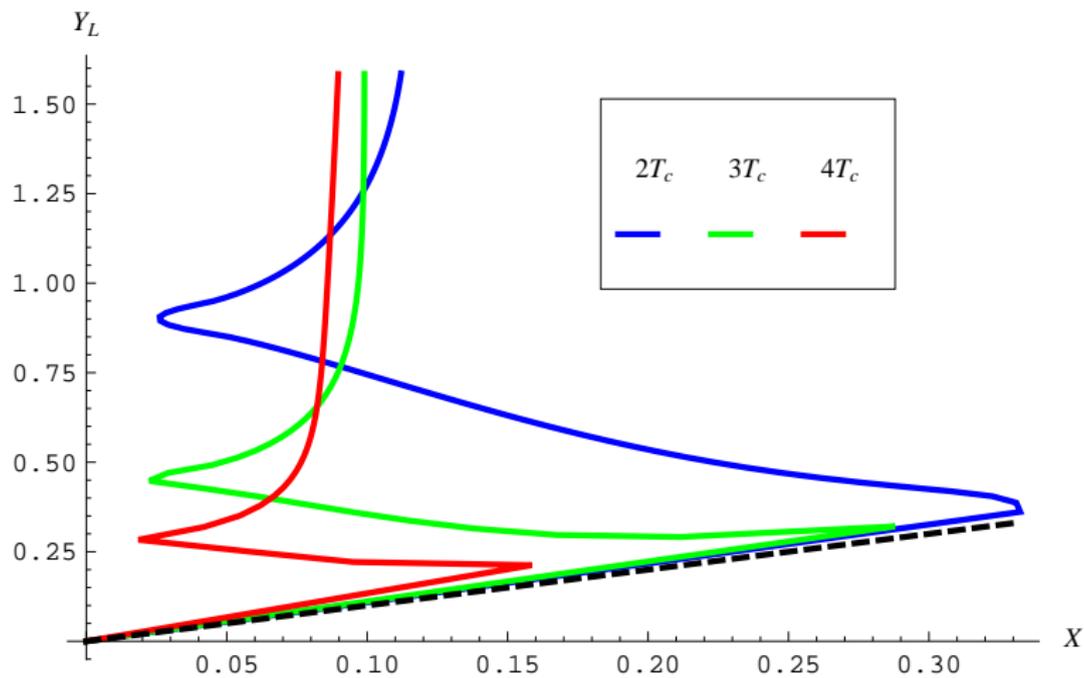
Apply Feynman rules to  $\mathbf{p}^2 = \text{Re}\Sigma^{00} =$    
yields gap equation:

$$\mathbf{p}^2 = \Sigma_{\text{B}}^{00}(p) = 8\pi e^2 \int_{|(p+k)^2| \leq |\phi|^2} \left[ \left( 3 - \frac{k^2}{m^2} \right) + \frac{k^0 k^0}{m^2} \right] \times n_{\text{B}}(|k_0|/T) \delta(k^2 - m^2) \frac{d^4 k}{(2\pi)^4} \Big|_{p^2=F} \quad (3)$$

Strategy to find  $F$  similar to that of finding  $G$  [Falquez, Hofmann, Baumbach '11].

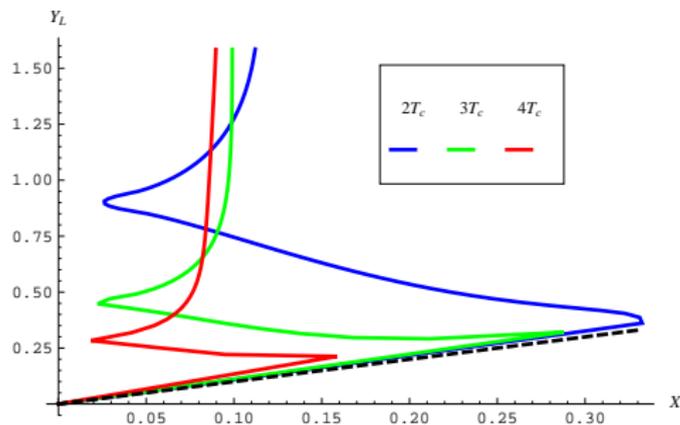
## Selfconsistent Result for $F$

$$Y_l \equiv \frac{\omega_l(\mathbf{p}_l, T)}{T} = \sqrt{\frac{F(p_l^2, T)}{T^2} + \frac{\mathbf{p}_l^2}{T^2}}, \quad X \equiv |\mathbf{p}_l|/T$$



## Selfconsistent Result for $F$

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- ▶ 3 branches
- ▶  $Y_L$  defined only for  $X \lesssim 0.34$
- ▶ superluminal group velocity

# Selfconsistent Result for $F$ , Interpretation

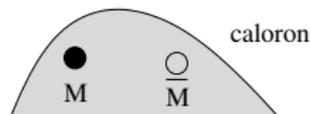
Interpretation in terms of magnetic monopoles

[Falquez,Hofmann,Baumbach '11]

- ▶ longitudinal modes due to charge density waves
- ▶ light like propagation:
  - ▶ stable (yet unresolved) monopoles released by large holonomy caloron dissociation [Diakonov et al. '04]
  - ▶ density disturbance can only be propagated by radiation field, which propagates at the speed of light



- ▶ superluminal propagation:
  - ▶ unstable monopoles contained in small holonomy caloron
  - ▶ extended calorons provide instantaneous correlation between monopoles, leading to superluminal propagation



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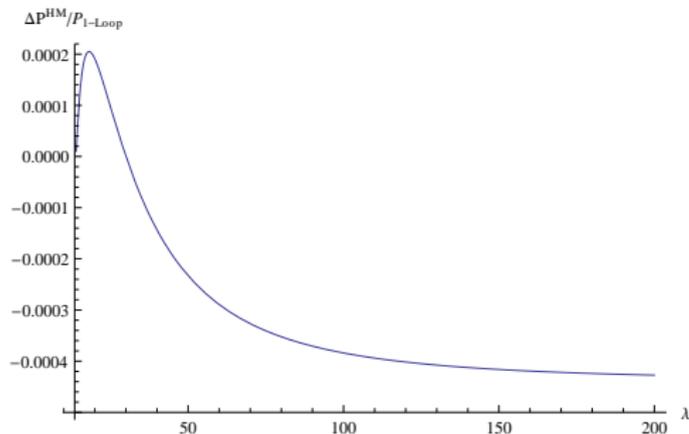
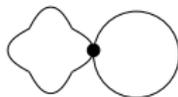
Monopole Properties from a 2-Loop Correction to the Pressure

Summary and Outlook

## 2-Loop Corrections

- ▶ “Bubble diagrams” yield pressure (cmp. talk by Dariush)

- ▶ For  $T \gg T_c$  only relevant diagram:



- ▶  $\Delta P \propto -4 \times 10^{-4} T^4$
- ▶ Temperature of TLM gas reduced!

# Monopole Properties

## Explanation

Energy used to break up calorons, creating monopole anti-monopole pairs [Schwarz,Giacosa,Hofmann '06].

Detailed analysis shows [Ludescher,Keller,Giacosa,Hofmann '08]:

- ▶ average monopole-antimonopole distance  $\bar{d} < |\phi|^{-1}$   
⇒ monopoles unresolved in effective theory
- ▶ screening length  $l_s$  due to small-holonomy calorons:  $l_s = 3.3\bar{d}$   
⇒ magnetic flux of monopole and antimonopole cancel (no area law for spatial Wilson loop)

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# Summary and Outlook

- ▶ Coarsegrained YMT ground state described by scalar field  $\phi(T)$ ,  
 $SU(2) \xrightarrow{\phi} U(1)$
- ▶  $|\phi|$  constrains loop momenta
- ▶ Calculated polarization tensor  $\Sigma^{\mu\nu}$  for TLM mode on 1-loop level
- ▶ Approximation  $p^2 = 0$  in constraint
  - ▶ Constraint solvable analytically
  - ▶ Dispersion relation for transverse mode;  
no propagating longitudinal mode
- ▶ Selfconsistent calculation
  - ▶ Constraint implemented numerically
  - ▶ Dispersion relation for transverse- and longitudinal mode
- ▶ In YMT monopoles are unresolvable and screened

## Question

Is 1-loop calculation sufficient?

## Answer

See talk by Dariush!

Thank you.



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