Yang-Mills thermodynamics

Ralf Hofmann

Universität Karlsruhe (TH)

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Institute of Mathematics, National Academy of Science, Kyiv, Ukraine

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Universität Karlsruhe (TH)
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plan

- brief motivation and preview on phase diagram
  [hep-th: 0411214, 0504064, 0609033, 0609172, 0702027]
- deconfining ground-state physics:
  coarse-grained, interacting calorons
- coarse-grained excitations:
  Legendre-trafos and loop expansion
- preconfinement:
  cond. magn. monopoles, dual Meissner effect
- low temperatures:
  Hagedorn, flip of statistics, Borel summation
- summary, conclusions, mention of applications

Why nonpert. YMTD?

- infrared instability of PT even for $T \gg \Lambda$ in magnetic sector
  [Linde 1980]

- highly nonpert. ground-state physics even for $T \gg \Lambda$:

  – $\theta_{\mu\mu} \propto T$
  [Miller 1998]

  – spatial string tension: $\sigma \propto T^2$
  [Philipsen 1998, Korthals-Altes 1998, ...]

- no lattice control at low temperature:

  – correlation length larger than linear lattice size

  – analytical grasp $\Rightarrow$ equilibrium violated

preview: phase diagram SU(2)

confining

ground state:

Cooper−pair condensate of single center−vortex loops, pressure precisely zero

excitations:

massless (single) and massive (self−intersecting) center−vortex loops (spin−1/2 fermions)

preconfining

ground state:

condensate of magnetic monopoles, collapsing center−vortex loops, negative pressure

excitations:

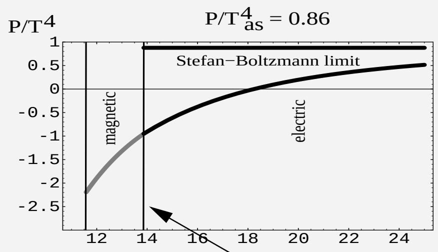
massive dual gauge modes

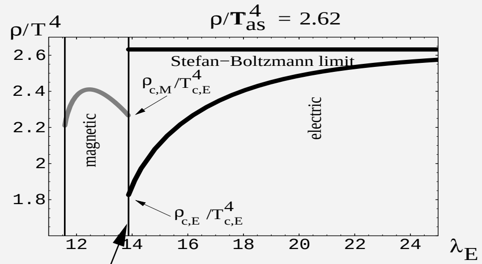
deconfining

ground state: interacting calorons and anticalorons, negative pressure

excitations: massless and massive gauge modes

power−like approach to Stefan Boltzmann limit





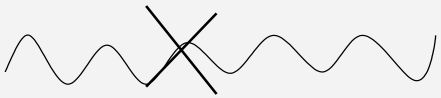
decoupling of V^+, V^-, no (anti) screening

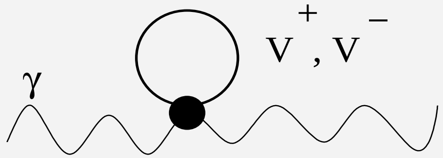
T_c

Hagedorn

2nd order like

T





(anti) screening

deconfining ground state

- coarse-grained (anti)calorons of $|Q| = 1$
  $\Rightarrow$ adjoint scalar field ϕ^a, $|\phi|$ spatially homogeneous
- strategy:

  – thermodynamics $\Rightarrow$ ϕ^a periodic in eucl. time in any admissible gauge $\Rightarrow$ phase $\hat{\phi}^a$ determined by *classical* configs.

  – stable configs.: $|Q| = 1$ HS (anti)calorons (BPS) of trivial holonomy (only these enter!)

– compute $\hat{\phi}^a$ $\in$(Kernel of $\mathcal{D}$) by respecting isotropy and $S_{\rm HS} = \frac{8\pi^2}{g^2} \neq f(T,\Lambda)$ in *inf.*-vol. average over magnetic-magnetic correlation mediated by *single* (anti)caloron

– fixes $\mathcal{D}$ uniquely $\Rightarrow$ winding number

– impose BPS $\Rightarrow$ $\hat{\phi}^a$

– average saturates rapidly

$\Rightarrow$ scale Λ and analyticity in ϕ^a

$\Rightarrow$ RHS of BPS eq. for ϕ^a

$\Rightarrow$ ϕ's potential and saturation scale $|\phi|$

$\Rightarrow$ inertness of $|\phi|$

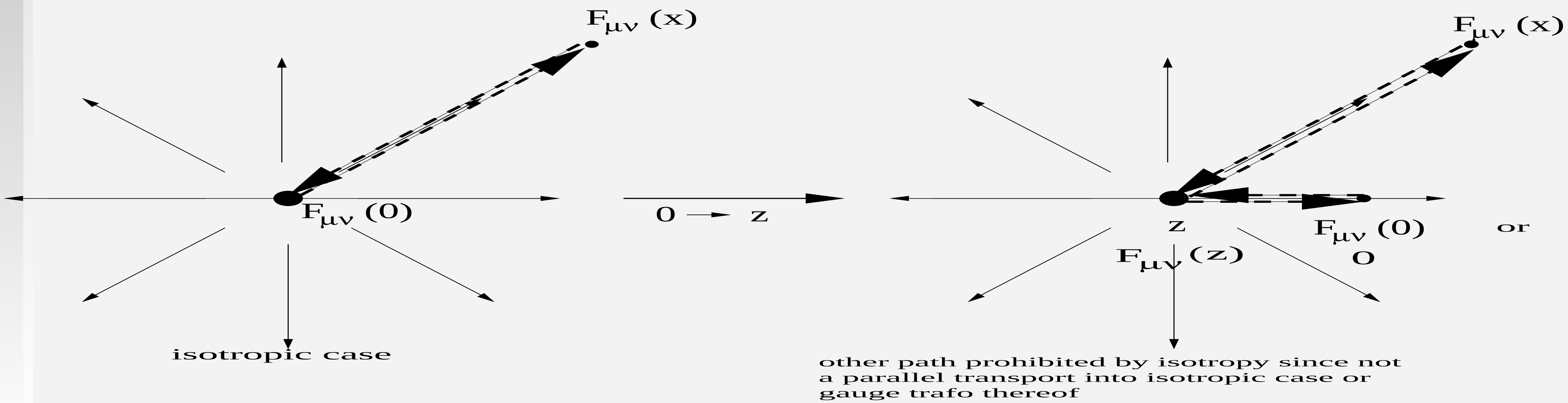
technically:

(integration over $S_3^{R=\infty}$)

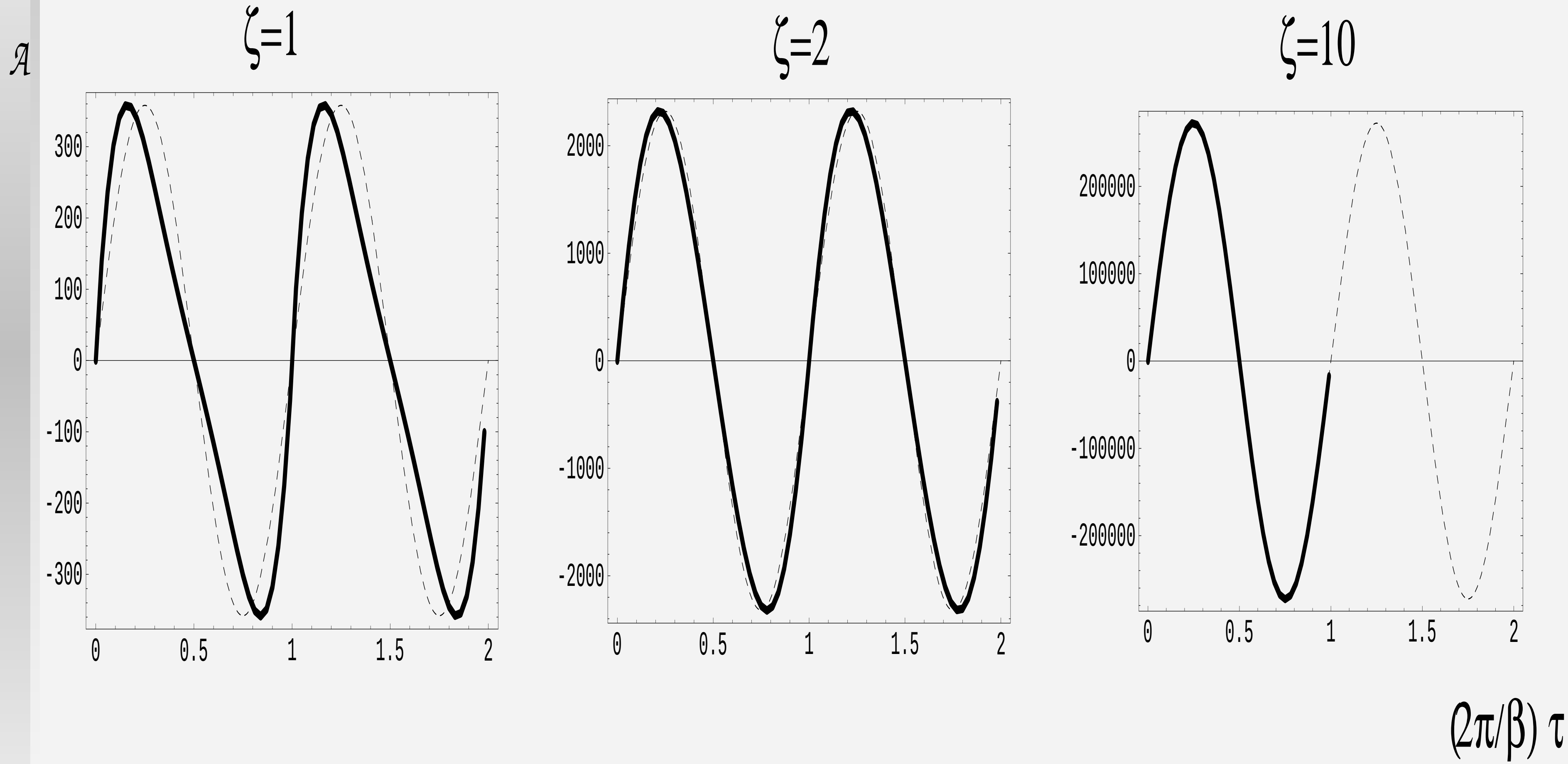
$$\hat{\phi}^a(\tau) \in \sum_{\text{HS (anti)caloron}} \text{tr} \int d^3x \int d\rho \, \frac{\lambda^a}{2} \times$$

$$F_{\mu\nu}((\tau,0)) \, \{(\tau,0),(\tau,\vec{x})\} \times$$

$$F_{\mu\nu}((\tau,\vec{x})) \, \{(\tau,\vec{x}),(\tau,0)\} \; .$$



saturation:



𝒜
ζ=1
ζ=2
ζ=10
300
200
100
0
-100
-200
-300
2000
1000
-1000
-2000
200000
100000
-100000
-200000
0
0.5
1
1.5
2
(2π/β) τ

$\Longrightarrow$

- $\mathcal{D} = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$

- $\partial_\tau \phi = \pm i\,\Lambda^3\,\lambda_3\,\phi^{-1}$, (fixed global gauge)
  where $\phi^{-1} \equiv \frac{\phi}{|\phi|^2}$

$\Rightarrow V(\phi) = \text{tr}\,\Lambda^6 \phi^{-2}$ by squaring RHS

$\Rightarrow |\phi| = \sqrt{\frac{\Lambda^3}{2\pi\,T}}$

$\Rightarrow$ unique, coarse-grained action for
$|Q| = 1$ HS (anticalorons)

$\Rightarrow \phi$'s inertness

What about $Q = 0$?

- perturbative renormalizability:

  ['t Hooft, Veltman 1971-73]

  $\Rightarrow$ coarse-graining yields
  same form as fundamental action

- gauge invariance glues $Q = 0$ to $|Q| = 1 \Rightarrow$

$$S = \mathrm{tr} \int_0^\beta d\tau \int d^3x \left(\frac{1}{2} G_{\mu\nu} G_{\mu\nu} + D_\mu \phi D_\mu \phi + \Lambda^6 \phi^{-2}\right)$$

- subject to offshellness constraints in unitary-Coulomb gauge (coarse-graining down to resolution $|\phi|$)

full ground state

- from $D_\mu G_{\mu\nu} = 2ie[\phi, D_\nu \phi]$:
  – pure gauge $a_\mu^{bg} = \frac{\pi}{e} T \delta_{\mu 4} \lambda_3$
  $\Rightarrow$ ground-state energy-density and pressure

  $$\rho^{g.s} = 4\pi \Lambda^3 T = -P^{g.s} \neq 0$$

- rotation to unitary gauge $a_\mu^{bg} = 0$:
  – gauge transformation singular but admissible
  (does not affect periodicity of fluct. δa_μ)

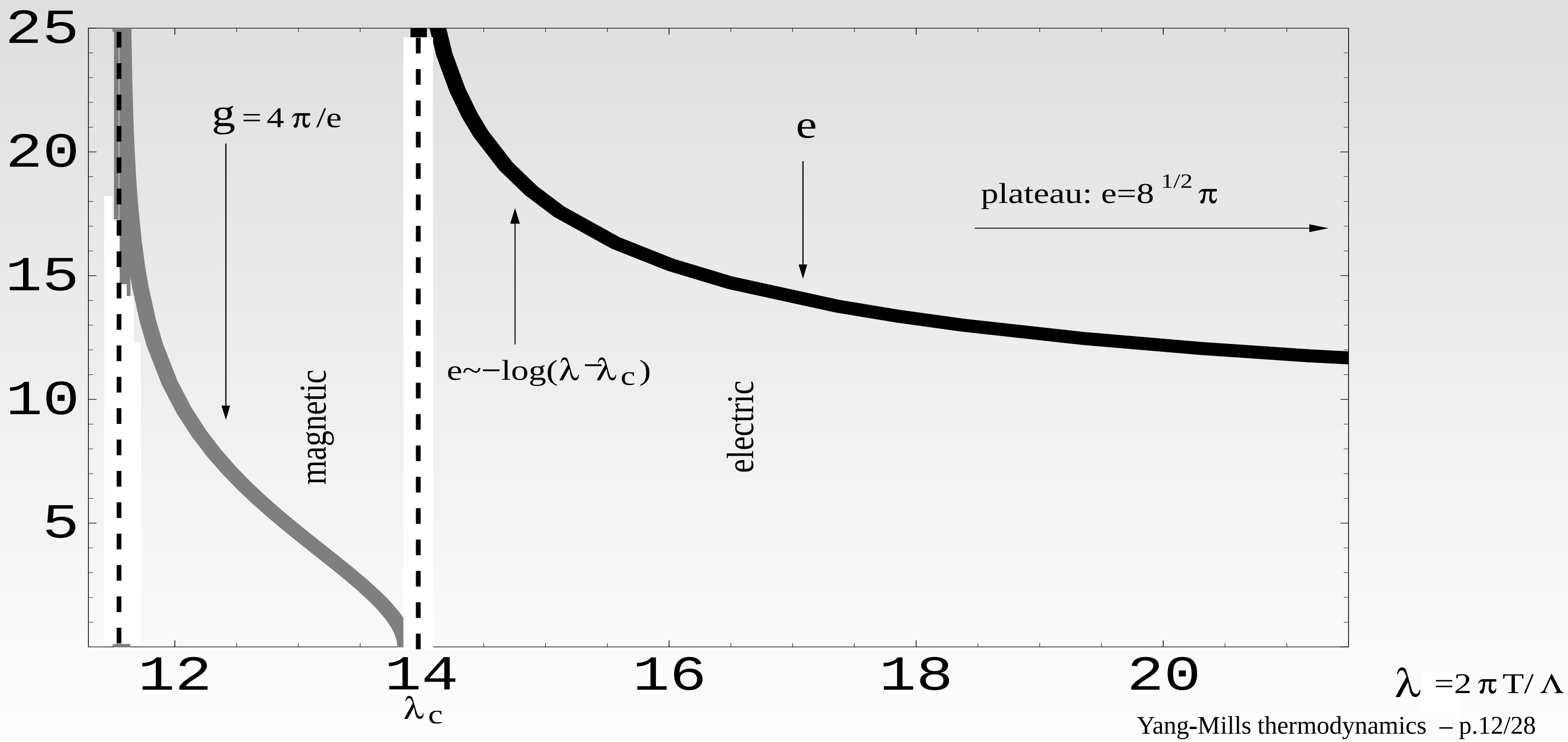
  – but: $\text{Pol}[a^{bg}] = -\mathbf{1} \stackrel{GT}{\rightarrow} \text{Pol}[a^{bg}] = +\mathbf{1}$
  $\Rightarrow$ Z_2^{el} degeneracy
  $\Rightarrow$ deconfinement

excitations and loop expansion

- adjoint Higgs mechanism:

  2 out of 3 directions massive with $m = e\sqrt{\frac{\Lambda_E^3}{2\pi T}}$

- T evolution of eff. coupl. e:
  requiring that P, ρ, ... from partition function



- counting of d.o.f.:

  fundamentally:

  3 species (gluons)$\times$2 pols.+
  1 species (monop)$\times$2 charges =8

  after coarse-graining:

  2 species (gluons)$\times$3 pols.+
  1 species (gluon)$\times$2 pols.=8

  $\Rightarrow$ 8 (fund)=8 (coarse-grained).

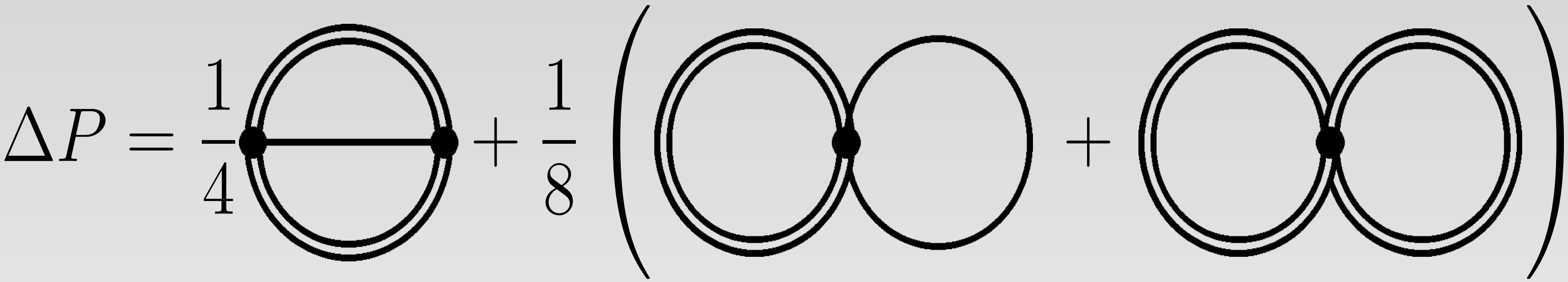
  same way for SU(3):

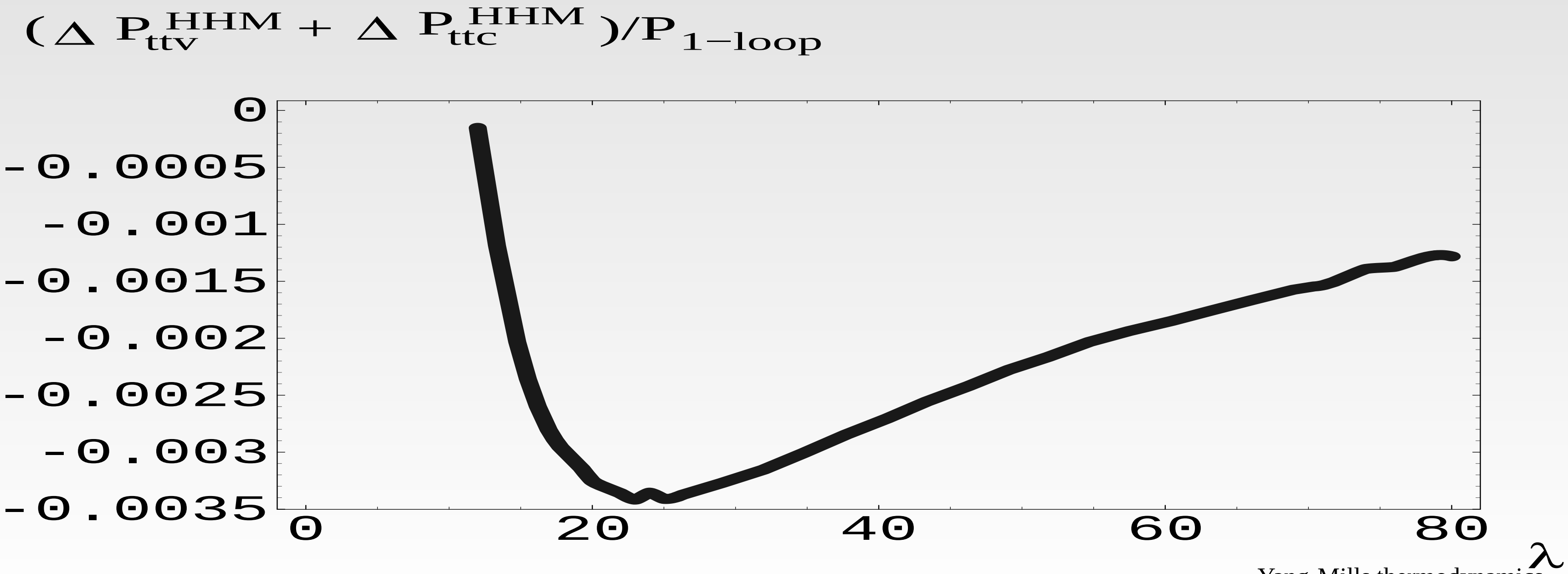
  $\Rightarrow$ 22 (fund)=22 (coarse-grained)

- **loop expansion:**

  2-loop:

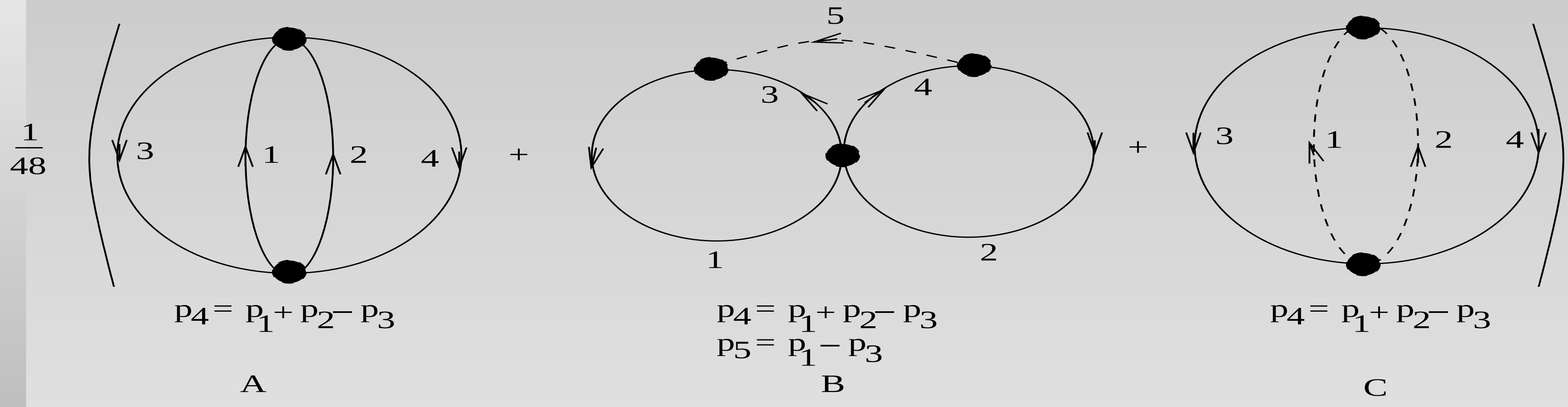
  [Rohrer,Herbst,RH 2004; Schwarz,RH,Giacosa 2006]

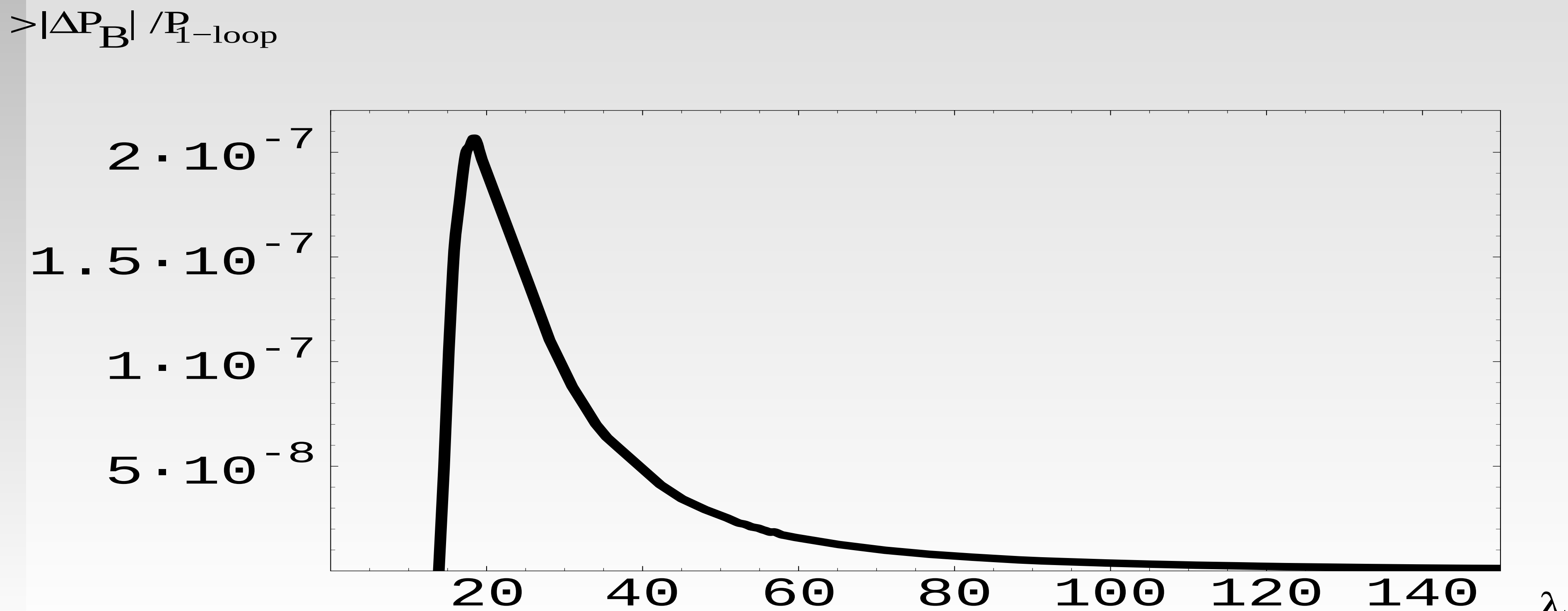




irreducible 3-loop:

[Kaviani,RH 2007]





arguments on loop expansion in general:
[RH 2006]

- resummation of 1PI diagrams
  $\Rightarrow$ **no pinch singularities**

- irreducible diagrams **terminate**
  at finite loop order

(Euler characteristics for spherical polygon,
constraints on loop momenta in effective theory
$\Rightarrow$
number of constraints **exceeds**
number of independent radial loop variables
at **sufficiently large number of loops**)

preconfining phase

- condensation of monopoles:
  - phase of complex scalar = magnetic flux through $S_2^{R=\infty}$ of M-A pair at rest ($e \to \infty$)
  - modulus as in dec. phase
  - no change of form of action for free dual gauge modes by coarse-graining

  $\Rightarrow$ **unique effective action**
  - Polyakov loop always unique
  - pressure exact at one loop
  - evol. of magnetic coupling g by requiring der. of pressure from fund. partition function

- counting of d.o.f.:

  fundamentally:

  1 species ('photon')$\times$2 pols.+
  1 species (center-vortex loop) =3

  after coarse-graining:

  1 species (massive 'photon')$\times$3 pols.=3

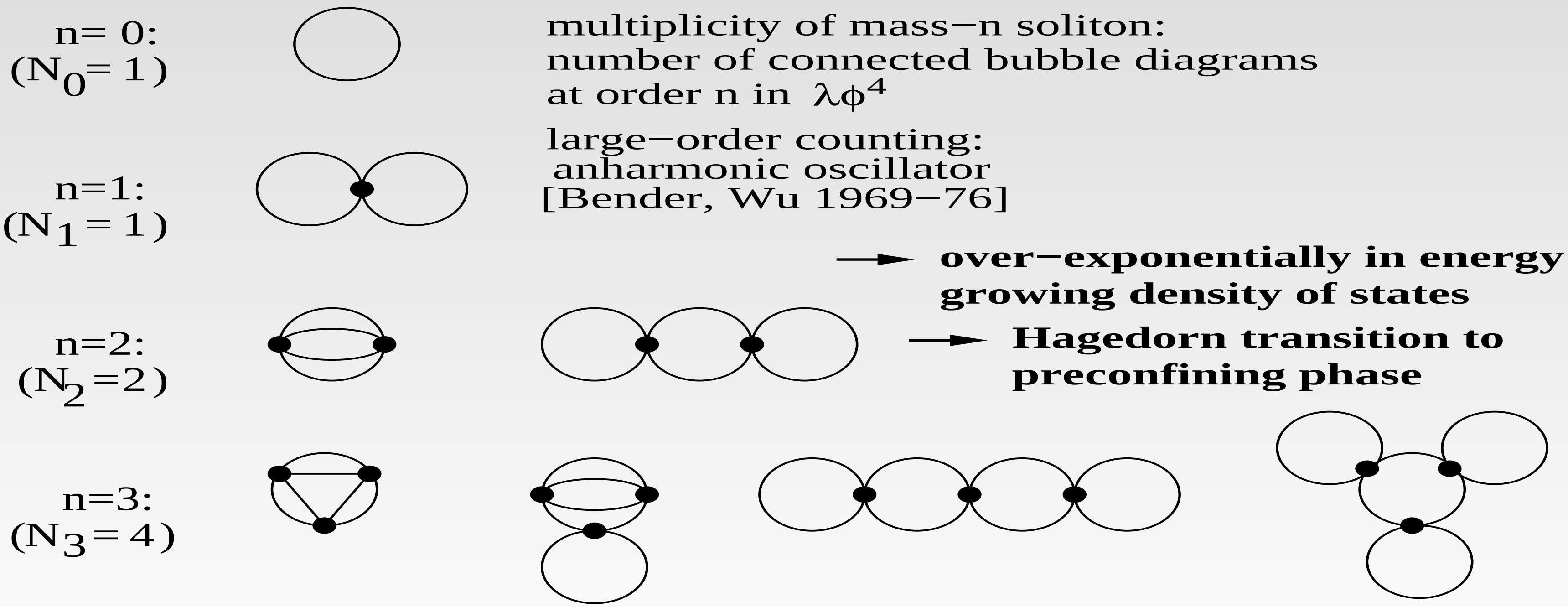
  $\Rightarrow$ 3 (fund)=3 (coarse-grained).

  same way for SU(3):

  $\Rightarrow$ 6 (fund)=6 (coarse-grained)

low temperature

- condensation of center-vortex loops (CVL's)
  – discrete values of phase of complex scalar field = center flux through $S_1^{R=\infty}$ of spin-0 vortex pair at rest ($g \longrightarrow \infty$)
  – spectrum: single and selfintersecting CVL's



- counting of d.o.f.:

fundamentally:

1 species ('very massive photon')$\times$3 pols.=3

after coarse-graining:

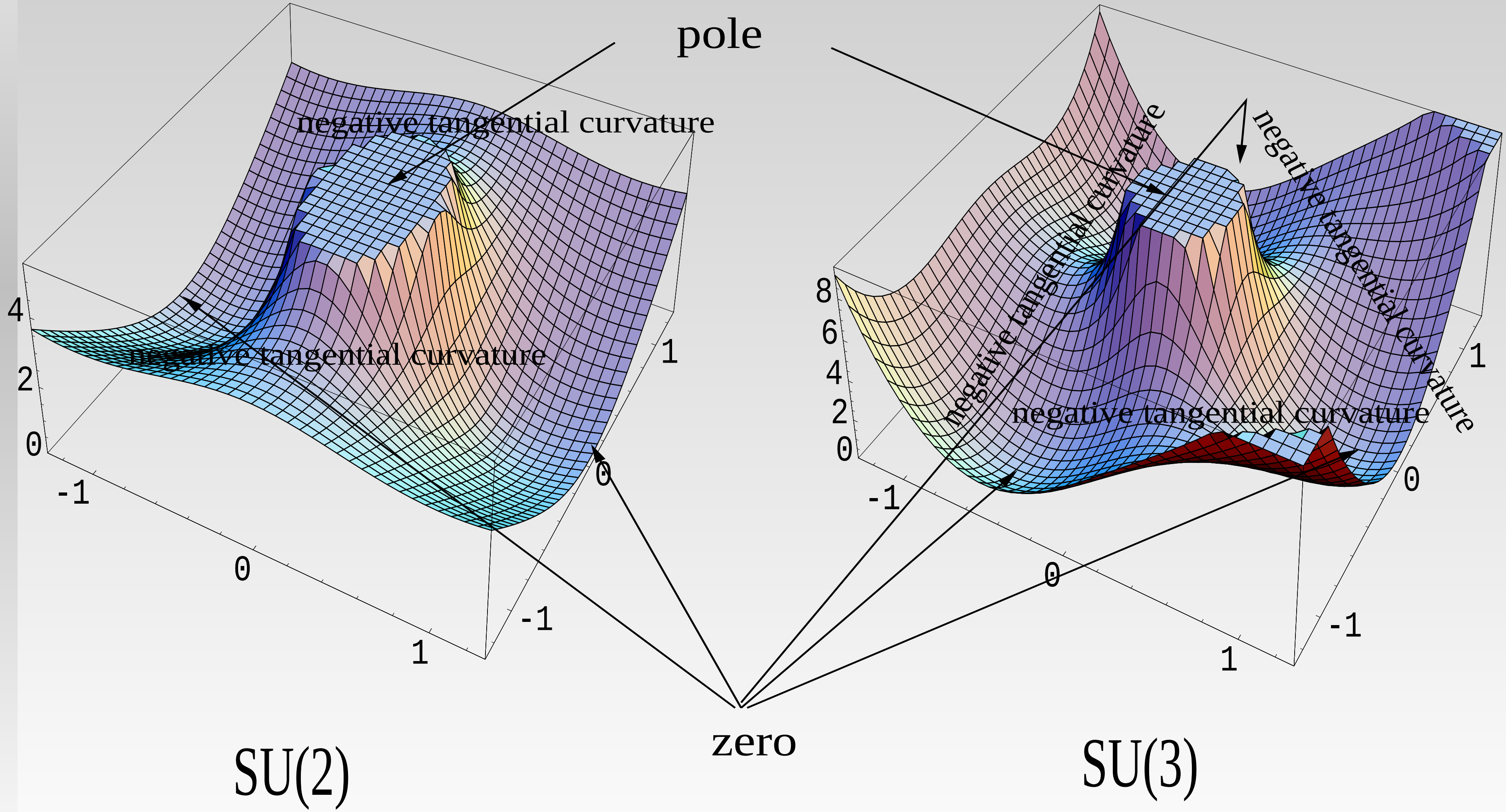
1 species (massless CVL)+
1 species (massive CVL)$\times$ 2 charges=3

$\Rightarrow$ 3 (fund)=3 (coarse-grained).

same way for SU(3):

$\Rightarrow$ 6 (fund)=6 (coarse-grained)

– potential unique up to inessential, U(1) invariant rescaling



– **asymptotic-series** representation of pressure:

$$P_{\rm as} = \frac{\Lambda^4}{2\pi^2}\hat{\beta}^{-4}\times$$
$$\left(\frac{7\pi^4}{180} + \sqrt{2\pi}\,\hat{\beta}^{\frac{3}{2}}\sum_{l=0}^{L} a_l \sum_{n\geq 1}(32\lambda)^n\, n!\, n^{\frac{3}{2}+l}\right)\ ,$$

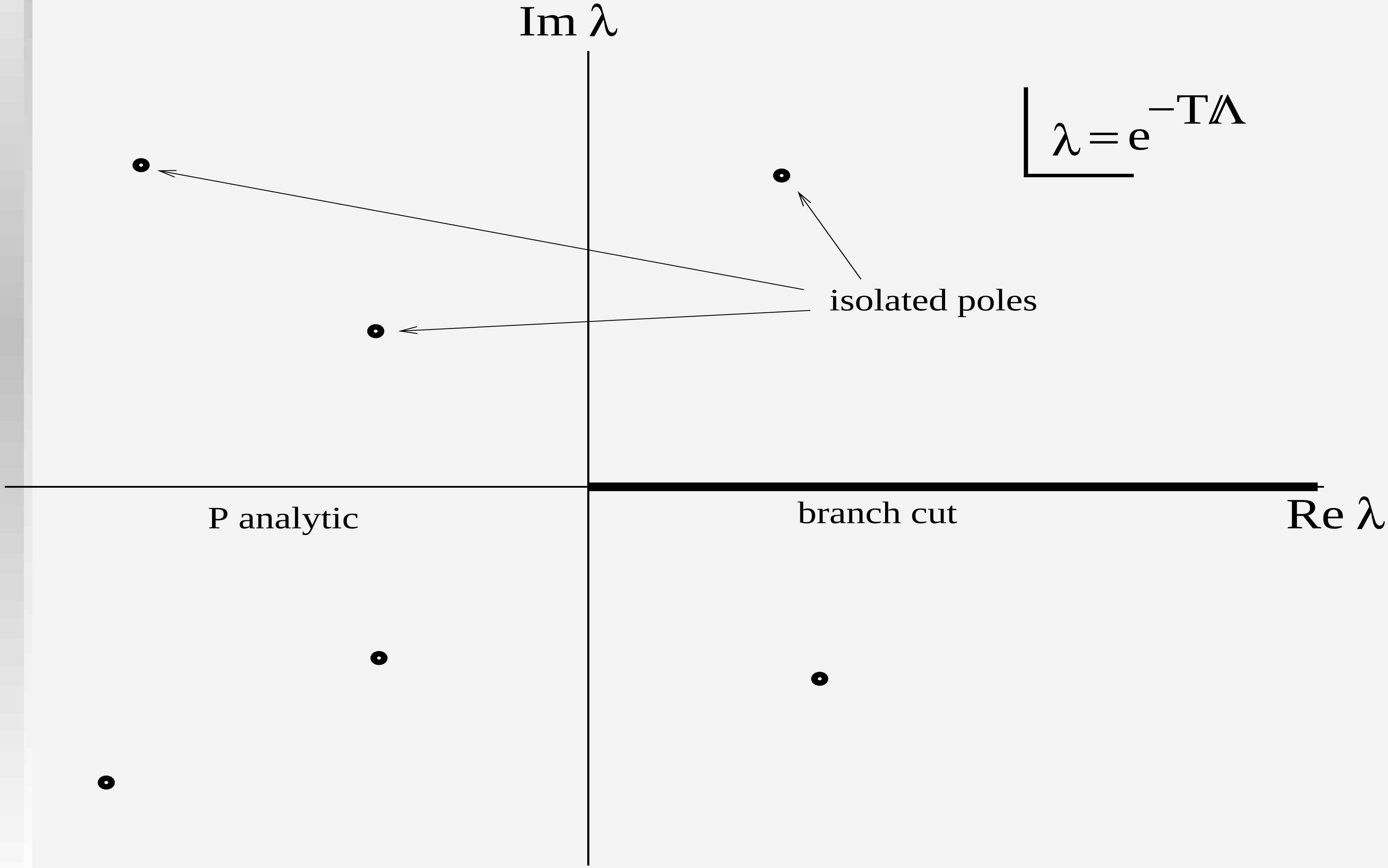
where $\hat{\beta} \equiv \Lambda/T$ and $\lambda \equiv \exp[-\hat{\beta}]$.

– Borel transformation and analytic continuation $\Rightarrow$ **analytic** dependence on Borel parameter t (polylogs) for $\lambda < 0$

– inverse Borel trafo:
$\Rightarrow$ analytic dependence for $\lambda < 0$ and

meromorphic in entire λ-plane except for $\lambda \geq 0$

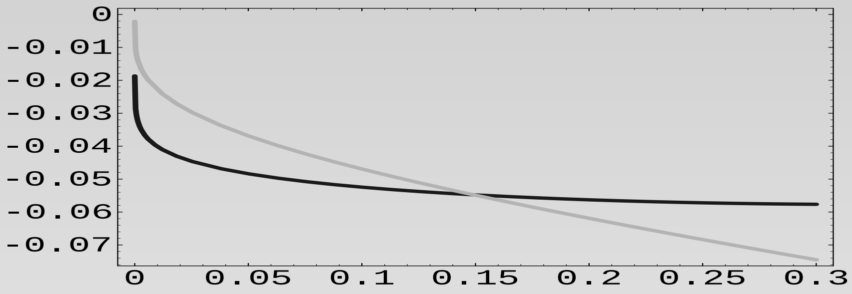
analyticity structure of physical pressure P:



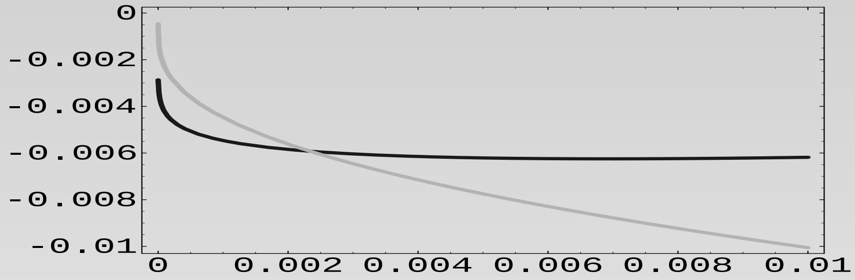
– Re P continuous across cut:

– sign-ambiguous Im P grows slower than Re P

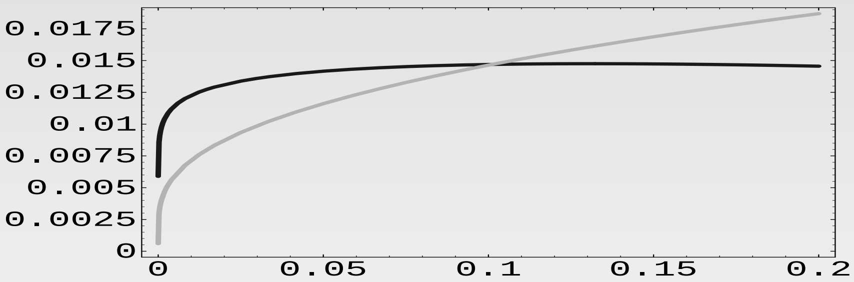
– turbulences become relevant for sufficiently high T only



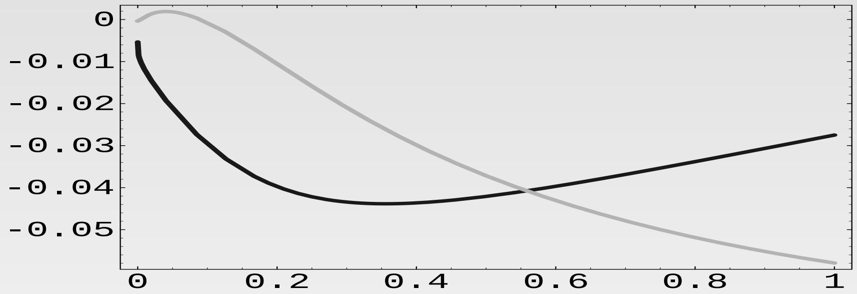
(a)



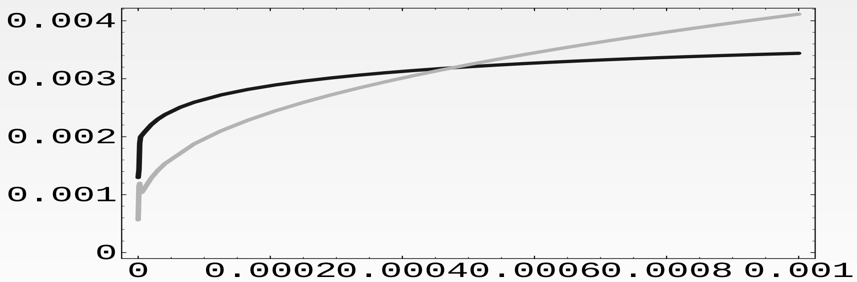
(b)



(c)



(d)



(e)

summary and conclusions

- **deconf. phase:**
  - magnetic-magnetic correlations in (anti)calorons generate adj. Higgs field
  - rapid saturation of average
  - negative ground-state pressure by microscopic holonomy shifts (annihilating M-A pairs)
  - thermal quasiparticles on tree level (adj. Higgs mech.)
  - very small radiative corrections, termination of expansion in terms of irreducible loops

- **preconf. phase:**
  - averaged magnetic flux of M-A pair through S_2^∞ generates phase of complex scalar
  - dual gauge field Meissner massive
  - loop expansion trivial

- **conf. phase:**
  - averaged center flux of CVL pair through min. surface spanned by S_1^∞ generates discrete values of phase of complex scalar
  - excitations are single or selfintersecting CVL's of factorially growing multiplicity
  - asymptotic-series representation of pressure
  - Borel summability for complex values of T
  - analytic continuation: rapidly (slowly) rising modulus of real (imaginary) part
  - interpretation: growing relevance of turbulences with increasing T

- physics applications:
  - CMB
  - late-time cosmology (axion + SU(2))
  - electroweak symmetry breaking

Thank you.