Karlsruher Institut für Technologie

The structure of the thermal ground state in SU(2) Quantum Yang-Mills theory

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$$S_\beta = \mathrm{tr}\,\frac{1}{2}\int_0^\beta d\tau \int d^3x\, F_{\mu\nu}F_{\mu\nu}$$

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- motivation for nonperturbative approach to Yang-Mills theory

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- $\mathrm{U}(1) \subset \mathrm{SU}(2)$ wave propagation/photonic excitation and deconfining thermal ground state
- Lorentz invariance and mixing SU(2)s
- some physics implications of deconfining SU(2) Yang-Mills gas

motivation

- Andrei Linde (1980):
  „Infrared Problem in the Thermodynamics of the Yang-Mills Gas“

  - soft magnetic sector screened weakly in perturbation theory (infrared instability)

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- issue of finite-volume constraints in lattice gauge theory

  - correlations mediated by soft magnetic sector insufficiently probed by available lattice sizes

nonperturbative Yang-Mills thermodynamics: SU(2)

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thermal ground state at high temperature:

- **Euclidean action:**

$$S = \frac{\mathrm{tr}}{2} \int_0^\beta d\tau \int d^3x \, F_{\mu\nu} F_{\mu\nu} \, , \qquad (\beta \equiv 1/T)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ [Schafer et Shuryak (1996)]

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$$F_{\mu\nu}[A] = \pm\tilde{F}_{\mu\nu}[A] \;\Rightarrow\; \theta_{\mu\nu}[A] \equiv 0\,.$$

(A_μ periodic)

field configs. stabilized by gauge-field winding: $\partial\mathbf{R}^4 = S_3 \to SU(2) = S_3$

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- **in particular:** (anti)calorons of winding number **unity** with action:

$$S = \frac{8\pi^2}{g^2} \quad\Longrightarrow$$

essential zero of weight $\exp[-S]$ in partition function $\Longrightarrow$ PT ignores these field configs.

Calorons of top. charge unity (selfdual field configs. on $S_1 \times \mathbf{R}_3$): (singular gauge)

['t Hooft, Rebbi & Jackiw (1977)]

Harrington-Shepard (1977): (trivial holonomy)

$$A_\mu = \bar{\eta}^a_{\mu\nu} t_a \partial_\nu \log \Pi(\tau, r)$$

with
$$\Pi = \begin{cases} \left(1 + \frac{1}{3}\frac{s}{\beta}\right) + \frac{\rho^2}{x^2} & (|x| \ll \beta) \\ 1 + \frac{s}{r} & (r \gg \beta) \end{cases}$$

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$\Rightarrow$ $F_{\mu\nu}$ that of singular-gauge instanton with $\rho'^2 = \dfrac{\rho^2}{1 + \frac{1}{3}\frac{s}{\beta}}$ $(|x| \ll \beta)$

(action: $S_c = \frac{8\pi^2}{g^2}\int_{S_3^\delta} d\Sigma_\mu K_\mu = \frac{8\pi^2}{g^2}$ localised about instanton center in $S_1 \times \mathbf{R}_3$)

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$$E^a_i = B^a_i = s\,\frac{\delta^a_i - 3\,\hat{x}^a \hat{x}^i}{r^3} \qquad (r \gg s)\,.$$

(static selfdual dipole-field with dipole moment: $p^a_i = s\,\delta^a_i$)

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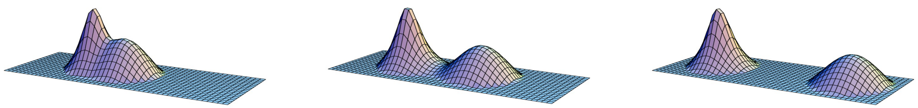
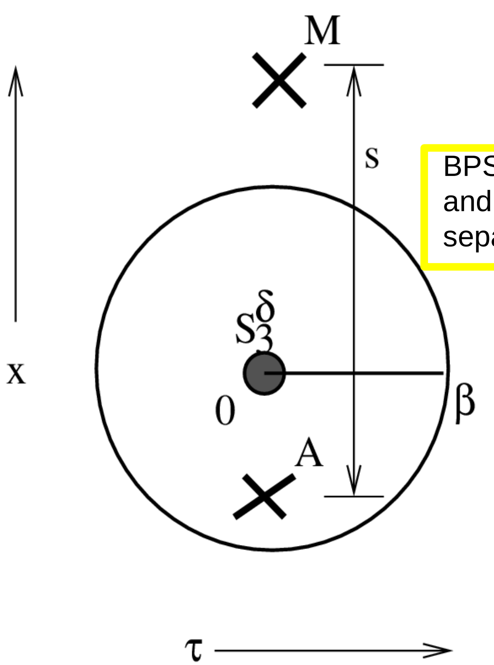
$$m_M = 4\pi u \,, m_A = 4\pi \left(\frac{2\pi}{\beta} - u \right)$$



(action density on spatial slice)



BPS monopole (M)
and antimonopole (A),
separated by distance s

(M-A separation, caloron center)

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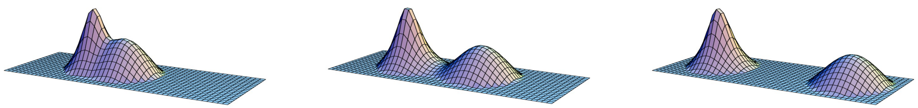

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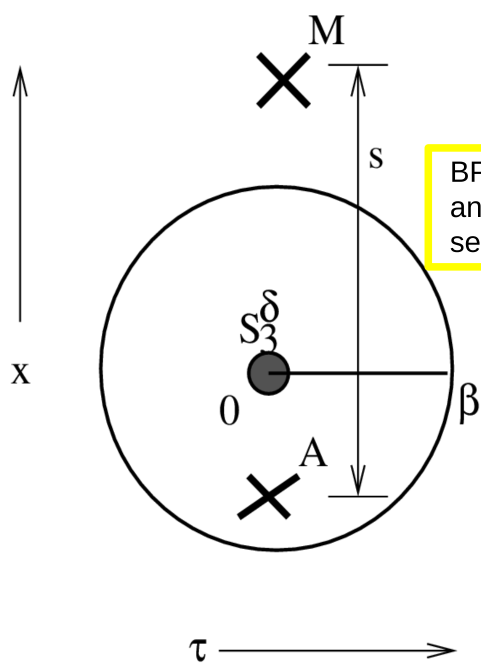

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  [Diakonov et al. (2004)]

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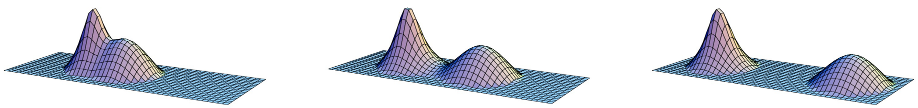

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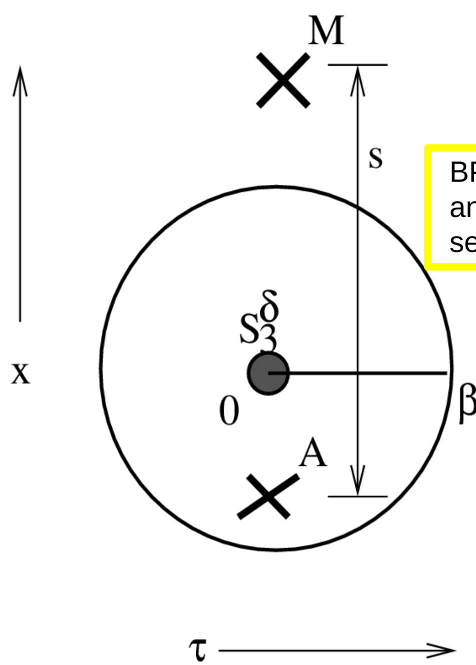

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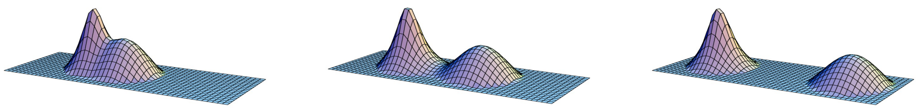

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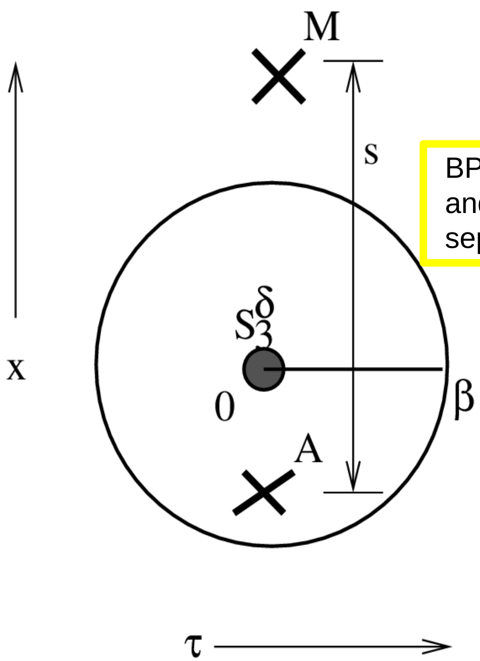

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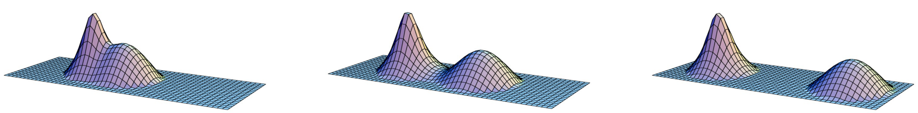

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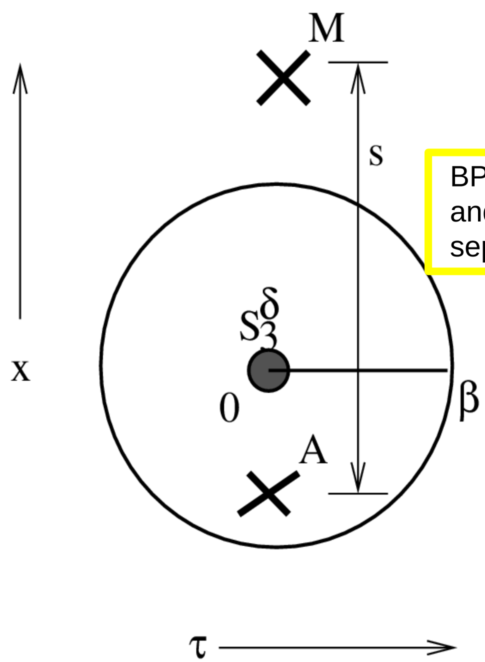

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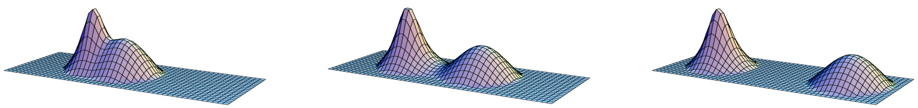

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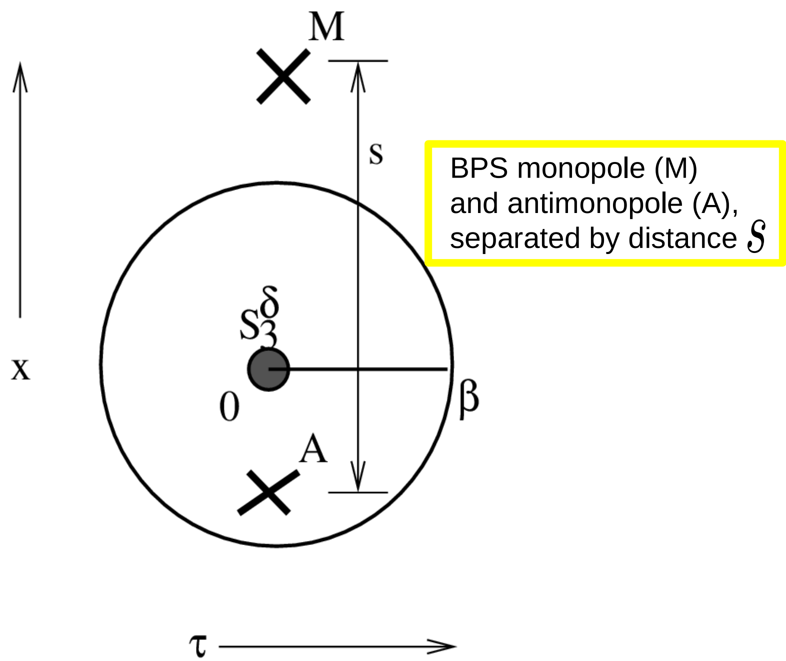

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- locus of action within S_3^δ $(\delta \to 0)$

- trivial-holonomy limit:
  M massless, A still massive, stable

spatial coarse-graining over pair of trivial-hol. (anti-)calorons: inert, adjoint scalar field ϕ

[Herbst & RH (2004)]

$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \mathrm{tr} \int d^3x \int d\rho\, t^a\, F_{\mu\nu}(\tau,\vec{0}) \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} F_{\mu\nu}(\tau,\vec{x}) \left\{(\tau,\vec{x}),(\tau,\vec{0})\right\}$$

- unique, dimensionless definition of **family of phases**, where

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- uniquely determined, annihilating operator: $D = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$

- $\{\hat{\phi}^a\}$ sharply dominated by cut-off for ρ integration (integral cubically dependent on cut-off)

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$$V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}$$

(Yang-Mills scale constant of integr.)

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- BPS equation:

$$\partial_\tau\phi = \pm 2i\,\Lambda^3\,t_3\,\phi^{-1} = \pm i\,V^{1/2}(\phi)$$

$\Rightarrow$ no **additive** ambiguity in V !

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- [dense packing of (anti)caloron centers only affects (anti)caloron peripheries, packing voids (inhomogeneities) reflected by small imaginary radiative corrections to pressure]

[Bischer, Grandou, RH (2017)]

effective action (deconfining phase), thermal ground state

- $$\mathcal{L}_{\rm eff}[a_\mu] = {\rm tr}\left(\frac{1}{2}\,G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)$$

(i) perturbative renormalizability (G^2 highest power in effect. action, propagating part of a_μ adiabatic excitation of thermal ground state)
(iiI) ϕ 's inertness – no higher dim., mixed operators to mediate 4-momentum transfer between ϕ and a_μ
(iii) gauge invariance

[see also RH (2016)]

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- effective YM equation $D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]$ has ground-state solution:

$$a^{\rm gs}_\mu = \mp\delta_{\mu 4}\frac{2\pi}{e\beta}t_3 \qquad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0)$$

(centers of HS (anti)calorons packed densely, static peripheries overlap to form $a^{\rm gs}_\mu$)

$$\Longrightarrow \quad P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T\,.$$

← interacting small and transient-holonomy (anti)calorons, (collapsing monopole-antimonopole pairs)

(vanishing entropy density of ground state!)

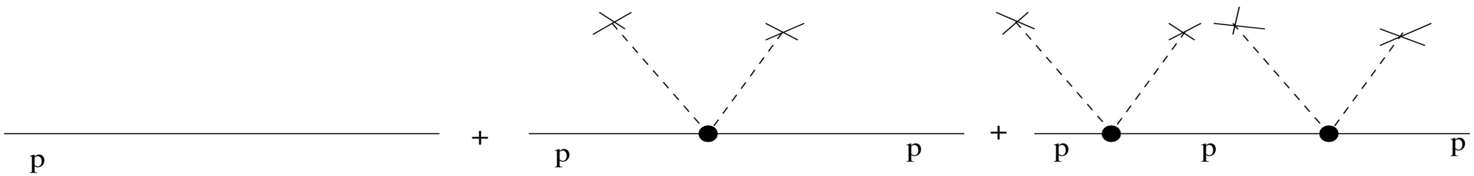
adjoint Higgs mechanism (deconfining phase)

(SU(2) → U(1))

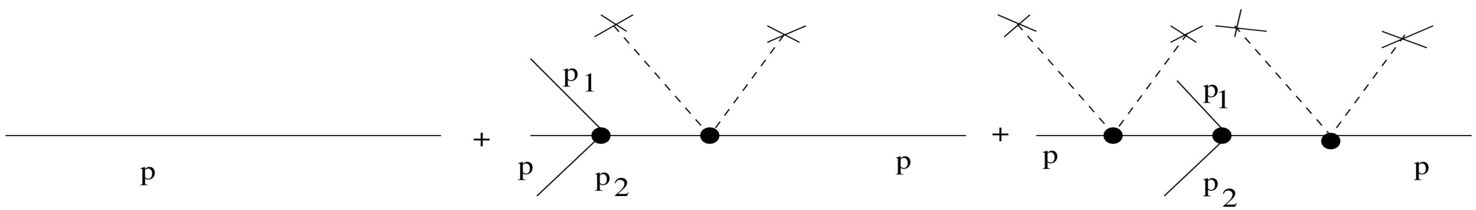
- from effective action:

$$m_a^2 = -2e^2 \mathrm{tr}\,[\phi, t_a][\phi, t_a] \quad \xrightarrow{\text{unitary gauge}} \quad m_1^2 = m_2^2 = 4e^2 \frac{\Lambda^3}{2\pi T}\,, \quad m_3 = 0$$

- no momentum transfer to ϕ , **but this infinitely often**
  (Dyson series for mass generation):



(a)

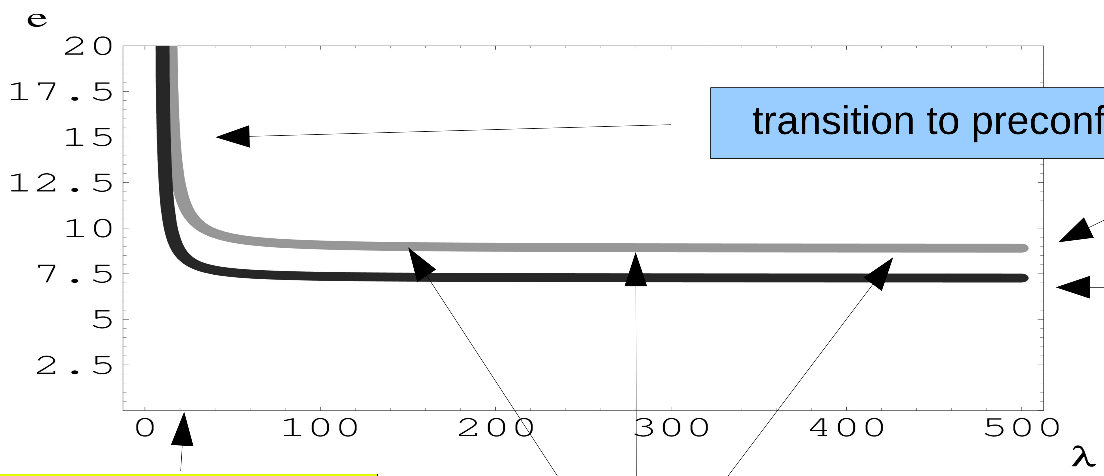


(b)

- no off-shell propagation of massive modes
  (otherwise: momentum transfer to ϕ !)

effective gauge coupling

e ← thermodynamical consistency



transition to preconfining phase

small-mass expansion, pressure
[Dolan et Jackiw (1974)]

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

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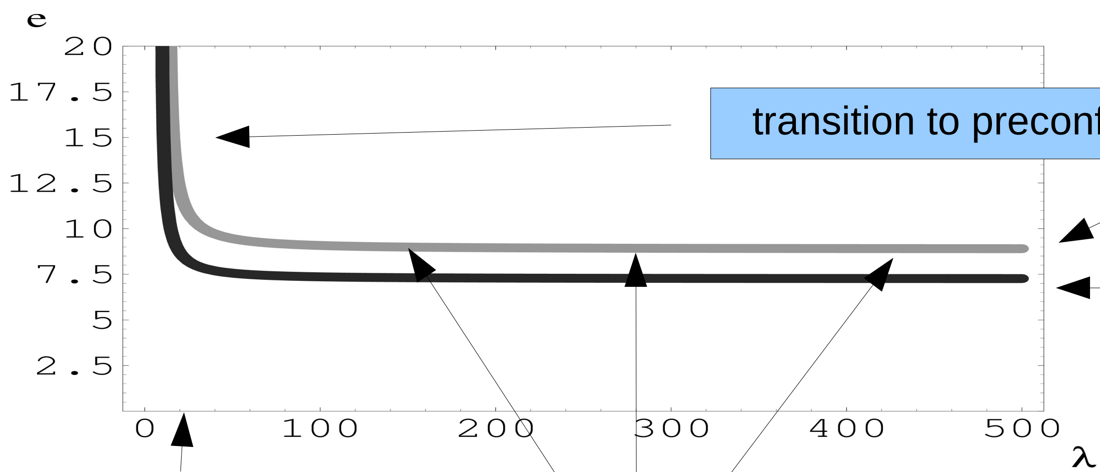


transition to preconfining phase

SU(2)

SU(3)

small-mass expansion, pressure **[Dolan et Jackiw (1974)]**

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8}\pi$$

coarse-graining dominated by $\rho \sim |\phi|^{-1}$

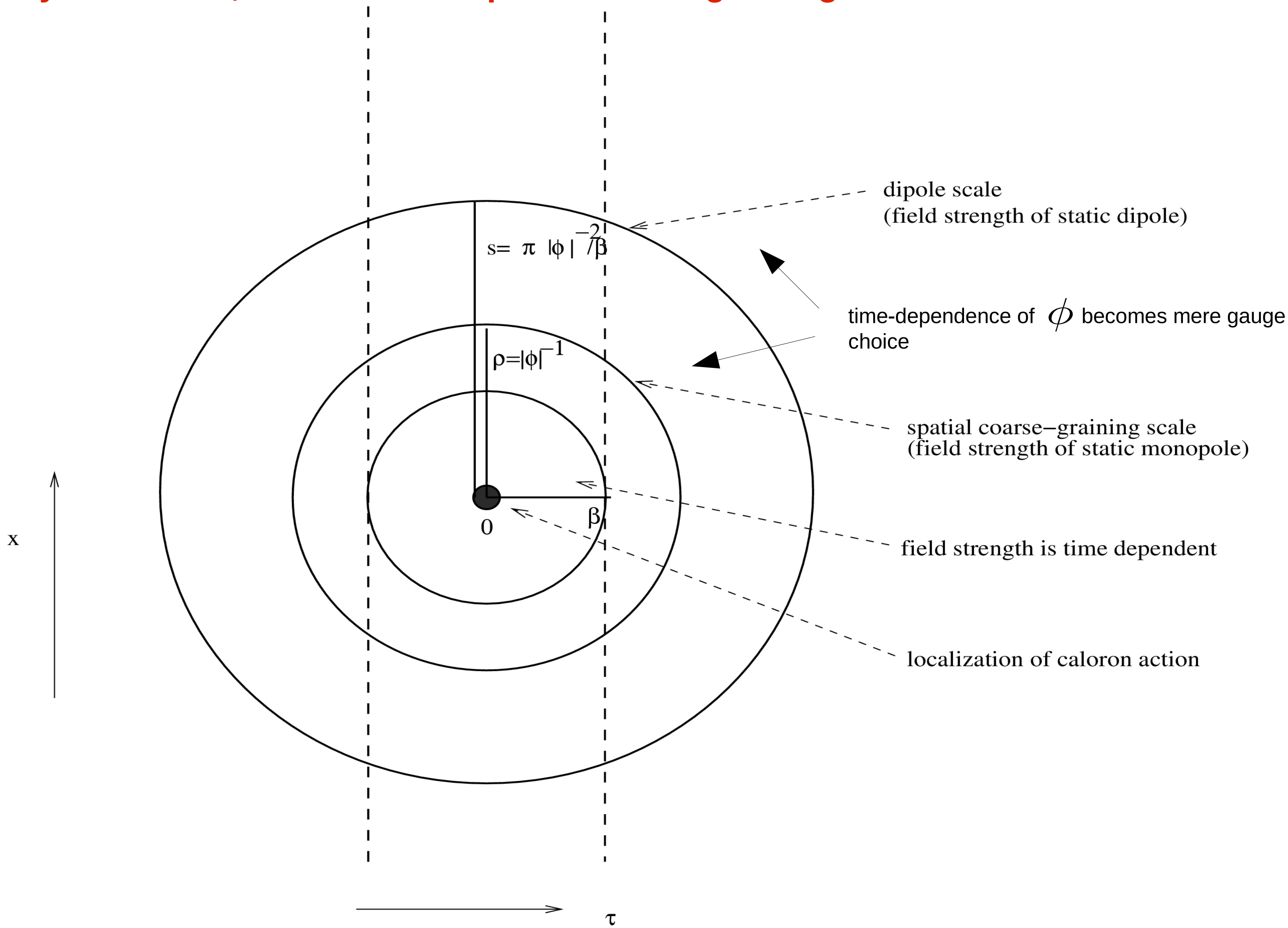
- restore $\hbar$

$$e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}} \,.$$

$$S_{C/A} = \hbar \,.$$

[Brodsky et al. (2011); Kaviani & RH (2012), RH (2012,2013)]

anatomy of caloron, inferred after spatial coarse-graining:



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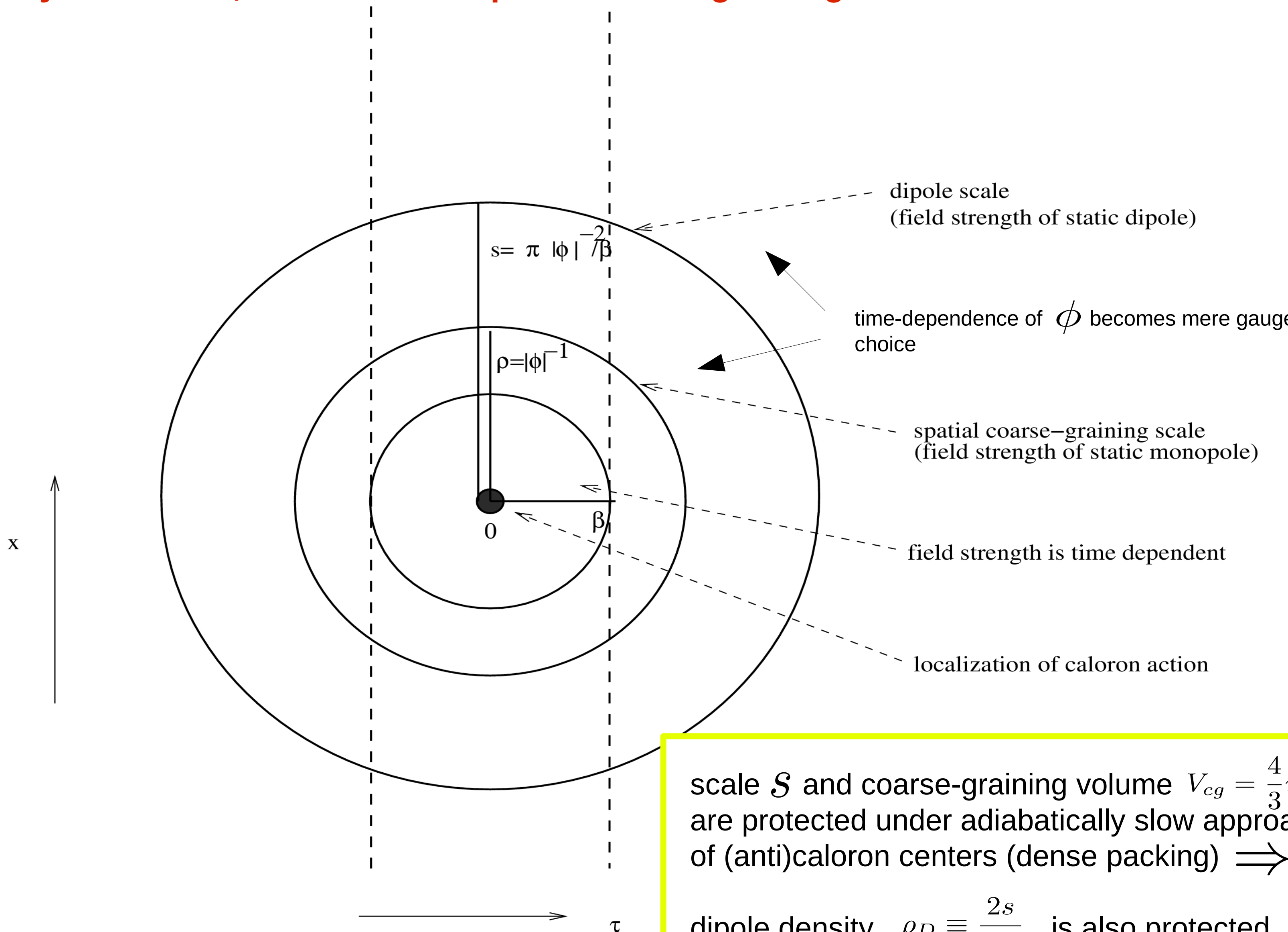

scale S and coarse-graining volume $V_{cg} = \frac{4}{3}\pi|\phi|^{-3}$ are protected under adiabatically slow approach of (anti)caloron centers (dense packing) $\Longrightarrow$

dipole density $\rho_D \equiv \frac{2s}{V_{cg}}$ is also protected

summary: induced, effective thermal QFT; control of radiative corrections

defining Yang-Mills action: classical, Euclidean gauge-field theory on $S_1 \times \mathbf{R}_3$

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small-holonomy (anti)calorons of action $\hbar$ **constitute** effective **thermal ground state, mediate interactions** (vertices) between effectively propagating modes (BE distributed QF – massiv; low-frequency waves, high-frequency BE distr. QF - massless) [Kaviani & RH (2012), Krasowski & RH (2013), Grandou & RH (2015), RH (2016)]

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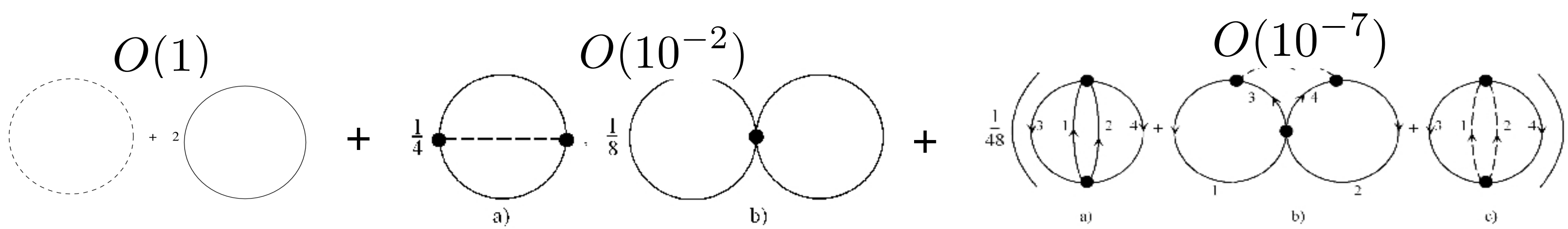
kinematic constraints in (totally fixed) unitary-Coulomb gauge imply that radiative corrections are extremely **well controlled**
[Schwarz, Giacosa, & RH (2006), Ludescher & RH (2008), Bischer, Grandou, & RH (2017)]

higher 2PI bubbles, resummation
[RH (2006), Bischer, Grandou, & RH (2017)]



real-world implications

electric-magnetically dual interpretation of U(1) charge:

if SU(2) something to do with photons [RH (2005), Grandou & RH (2015), etc]
then **electric-magnetically dual** interpretation required:
in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

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But: magnetic coupling in SU(2)

$$g = \frac{4\pi}{e}\,.$$

$\Rightarrow$ SU(2) to be interpreted in an **electric-magnetically dual way**.
(e.g., magnetic monopole $\longleftrightarrow$ electric monopole, etc.)

electric/magnetic dipole density (permittivity/permeability of vacuum):

[temperature a fictitious quantity]

$$|\mathbf{D}_e| = \frac{2s}{V_{\rm cg}} \propto T^{1/2}$$

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$$\Longrightarrow \quad \epsilon_0[Q(\mathrm{Vm}^{-1})] \equiv \frac{|\mathbf{D}_e|}{|\mathbf{E}_e|} = \frac{9}{32\pi^2}\frac{\Lambda[\mathrm{m}^{-1}]}{\Lambda[\mathrm{eV}]}(\xi Q)^2 \neq f(T)$$

$(\xi = 19.56)$

similarily for magnetic permeability μ_0 .

$\Longrightarrow$ Lorentz invariance of thermal ground state.

[Grandou & RH (2015)]

electric/magnetic dipole density (permittivity/permeability of vacuum):

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However:

$$\mathbf{E}_e^4\nu \ll 8\Lambda^9$$

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$\Longrightarrow$

- if em wave propagation indeed occurs by undulating repolarisations of dipole densities in SU(2) deconfining thermal ground state then nature must make use of **several SU(2) YM factors of hierarchical YM scales**

  e.g.: $\Lambda_{\rm CMB} \sim 10^{-4}$ eV, $\Lambda_e \sim 5 \times 10^5$ eV, etc.

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[temperature a physical quantity]

In thermal situation, wave propagation only in Rayleigh-Jeans regime.
One then shows [RH (2016)]

$$\epsilon_0[Q(\mathrm{Vm}^{-1})] \equiv \frac{|\mathbf{D}_e|}{|\mathbf{E}_e|} = \frac{9}{64\pi^2}\frac{\Lambda[\mathrm{m}^{-1}]}{\Lambda[\mathrm{eV}]}(\tilde{\xi}Q)^2 \times \frac{\Lambda^3}{\nu^2\Delta\nu}\left[1\right]$$

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$\Longrightarrow$

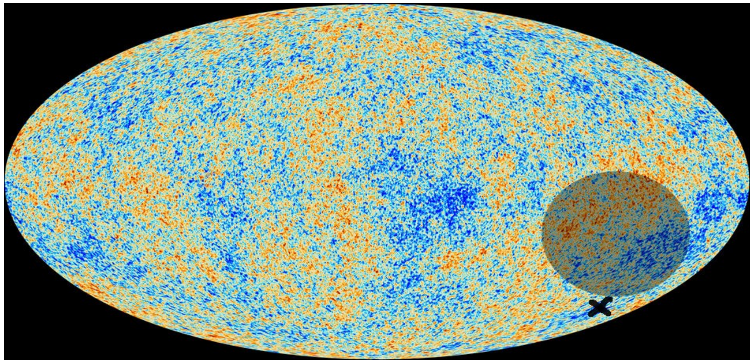
(temperature independence of ϵ_0)

$$\tilde{\xi}^2 = 2\xi^2\frac{\nu^2\Delta\nu}{\Lambda^3}$$

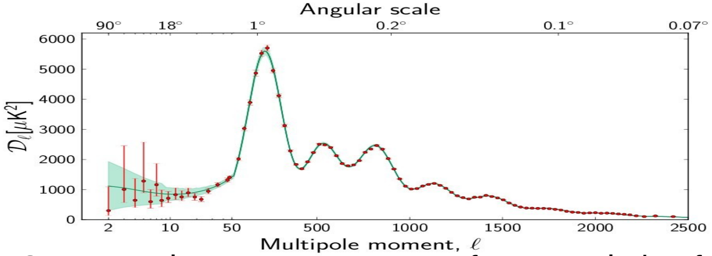
Increased screening of dipole charges with decreasing frequency.

Some other physics implications of the deconfining SU(2) Yang-Mills gas

thermal photon gases, fixing of an SU(2) YM scale:



Cosmic Microwave Background (CMB) as seen by the Planck satellite mission [ang. res. 5', E and B mode polarisation maps, etc.]



CMB angular power spectrum of TT correlation function

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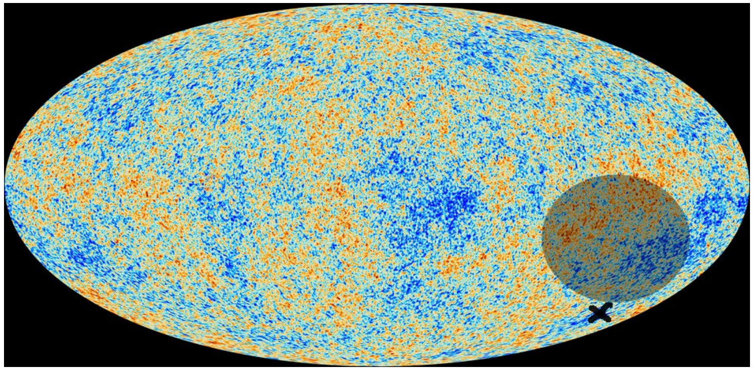

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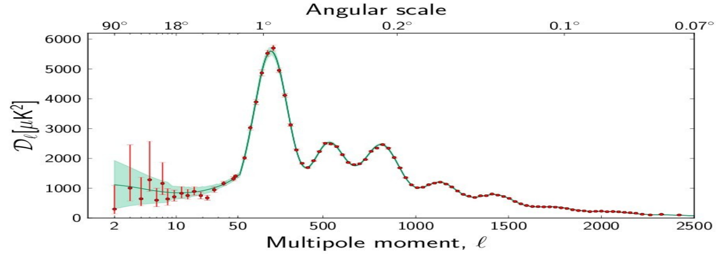

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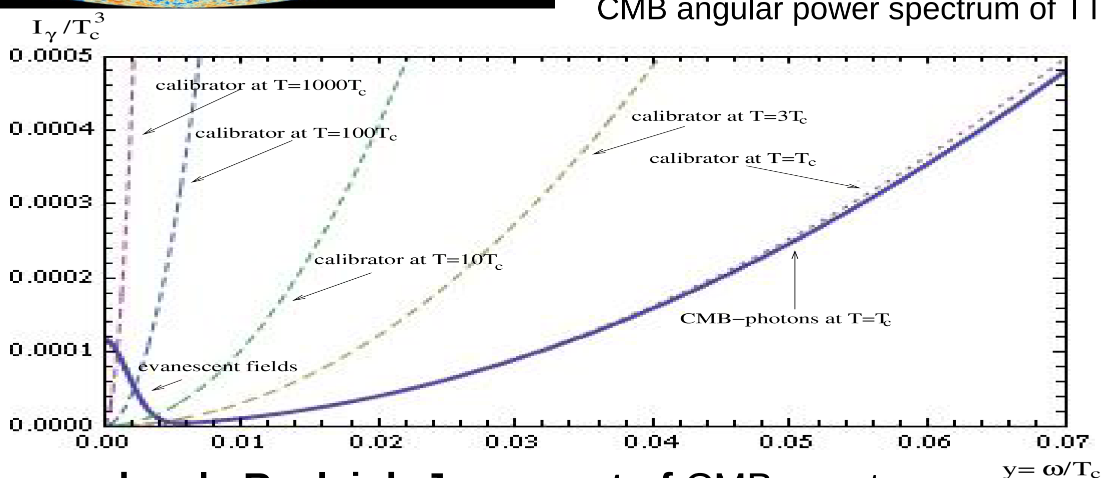


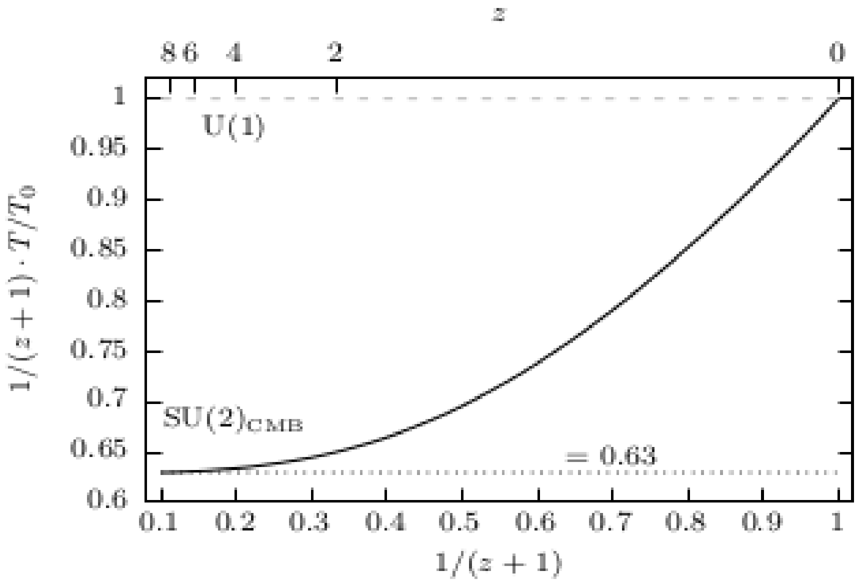
deeply Rayleigh-Jeans part of CMB spectrum:

- cosmic radio background (UEGE),
  [terrestial observ. (1981-1999), Arcade 2 (2009), RH (2009)]
- in SU(2) YMTD **critical** onset of Meissner effect at deconfining-preconfining phase boundary

  evanescent modes at low frequencies

-sharp fixation of $T_c = T_0 = 2.725$ K or $\Lambda = 10^{-4}$ eV → $\mathrm{SU(2)_{CMB}}$

CMB, temperature redshift relation in $\mathrm{SU(2)_{CMB}}$:



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$$\frac{d\rho}{da} = -\frac{3}{a}(P + \rho)$$

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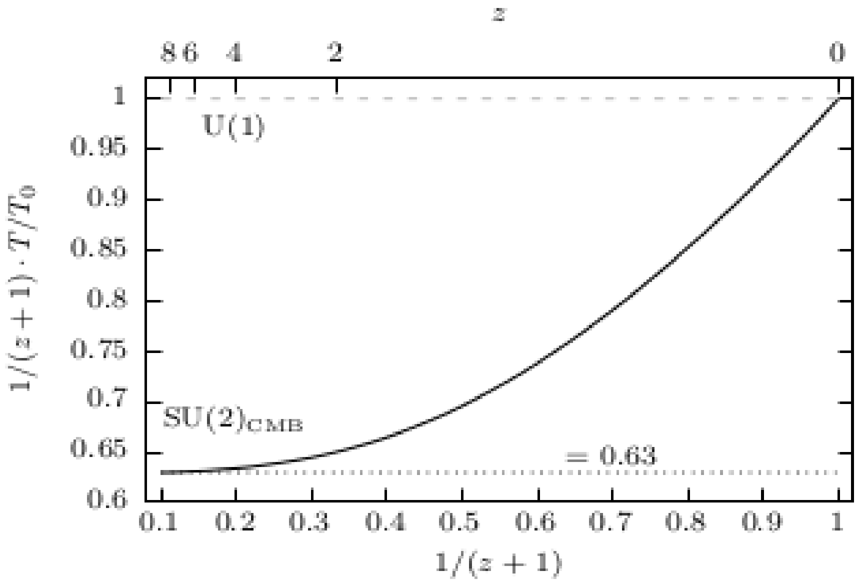

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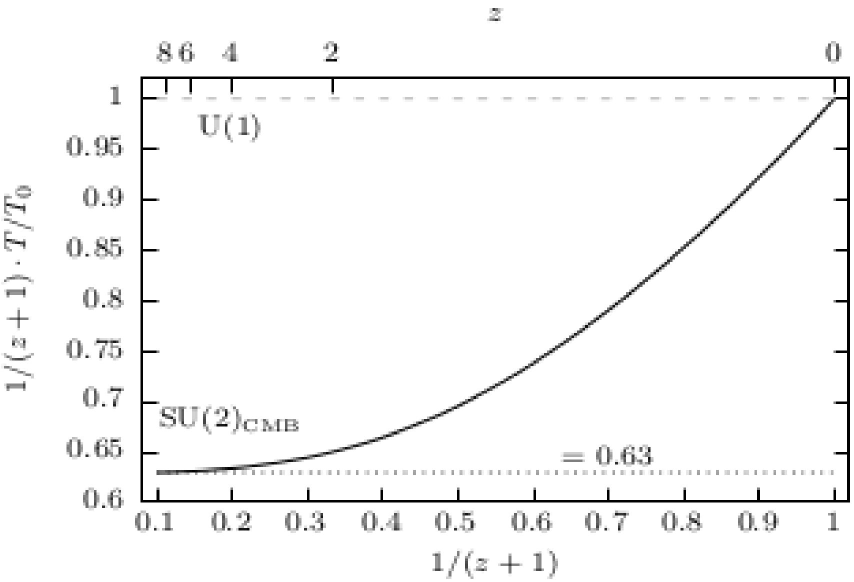

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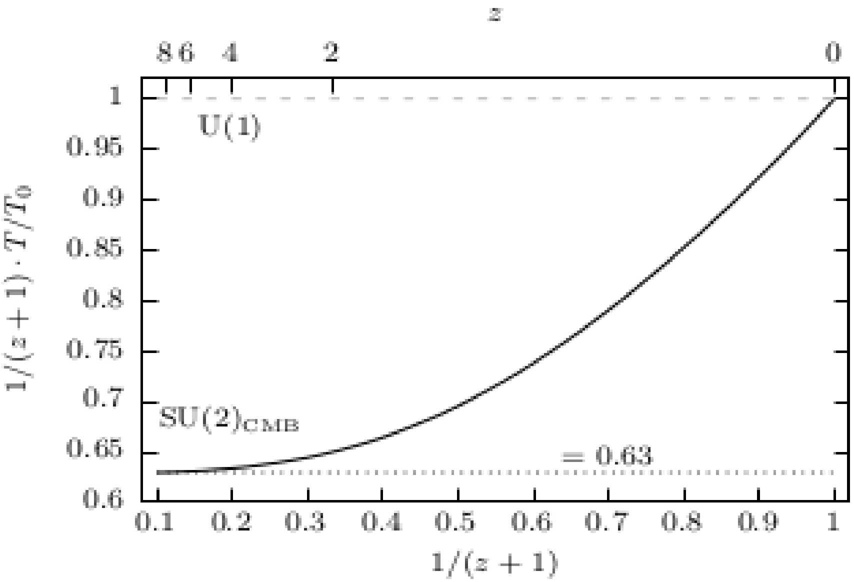

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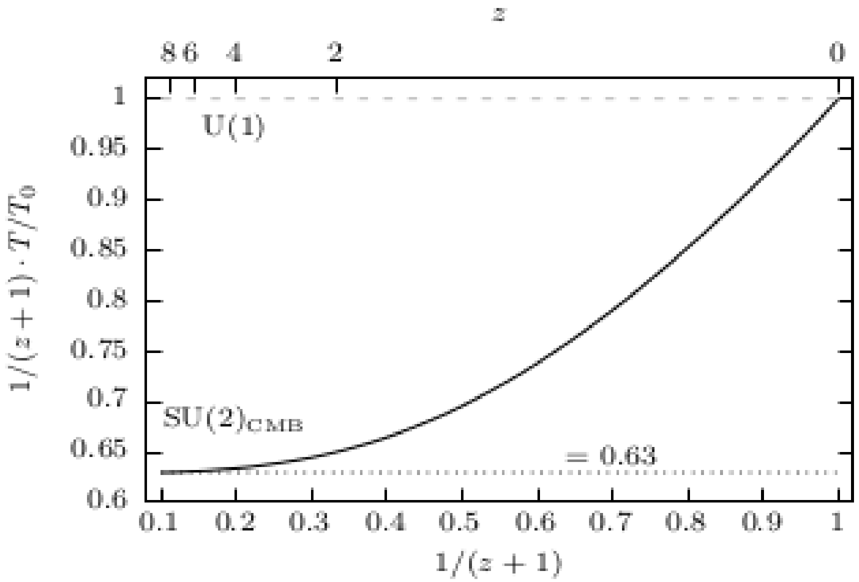

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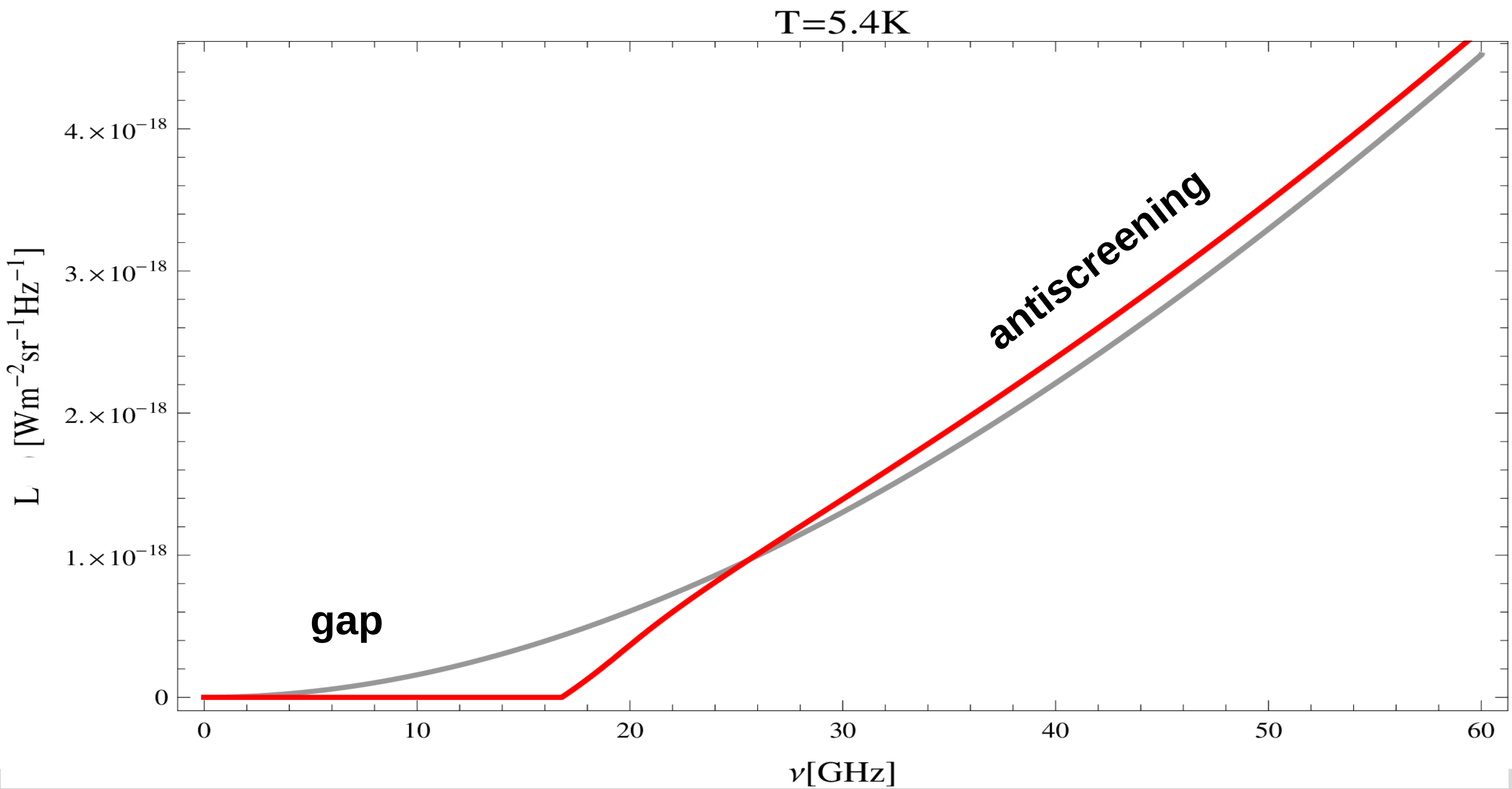
  → role of **baryons+dark matter** in $\Lambda\mathrm{CDM}$ played by **baryons only** in new high-z model subject to $\mathrm{SU(2)_{CMB}}$

$\mathrm{SU}(2)_{\mathrm{CMB}}$ radiative effects: blackbody spectral anomaly

max. gap in Rayleigh-Jeans reg. at T=5.4 K
massless mode – transverse polarizations

[Schwarz, Giacosa & RH (2006), Ludescher & RH (2008), Falquez, RH & Baumbach (2010,2011)]



$\mathrm{SU(2)_{CMB}}$ radiative effects: longitudinal magnetic modes

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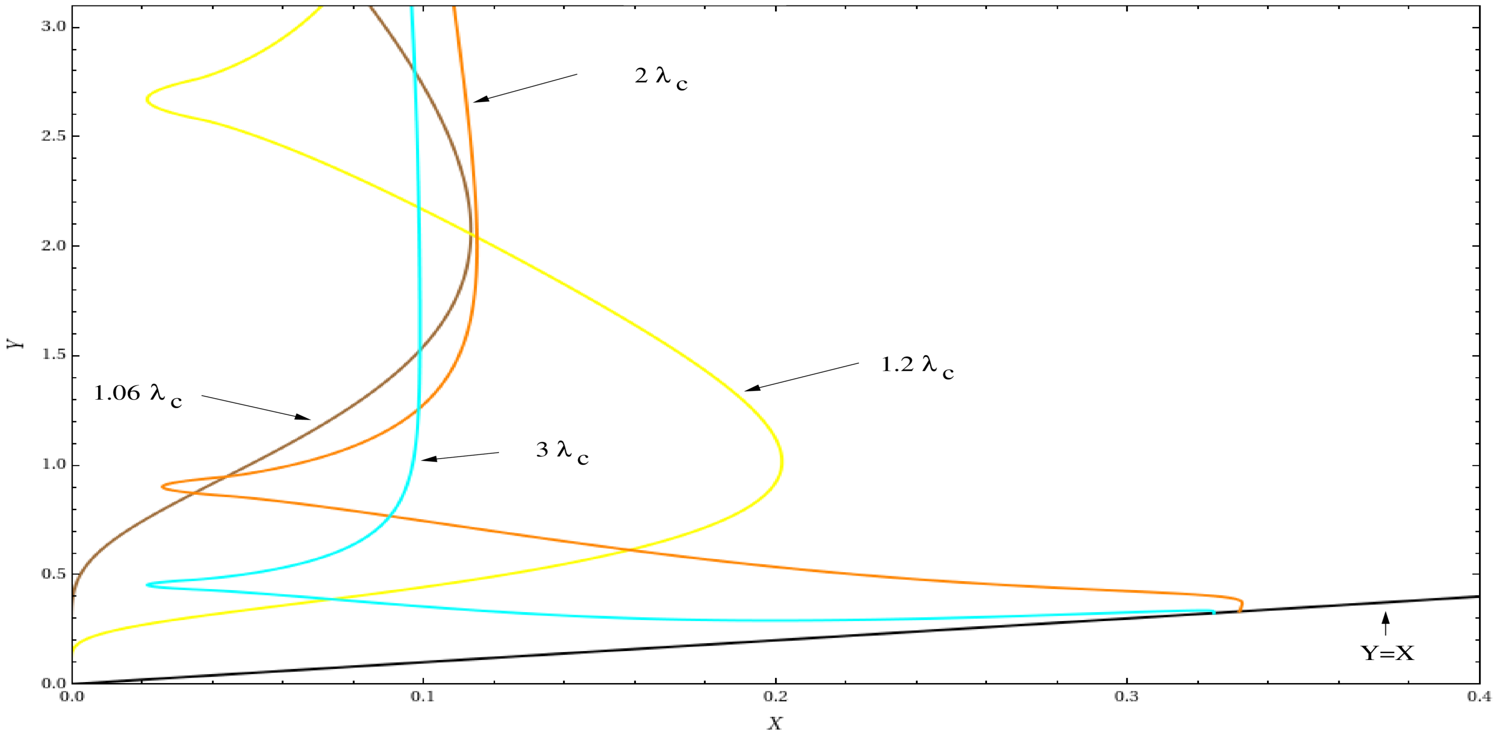

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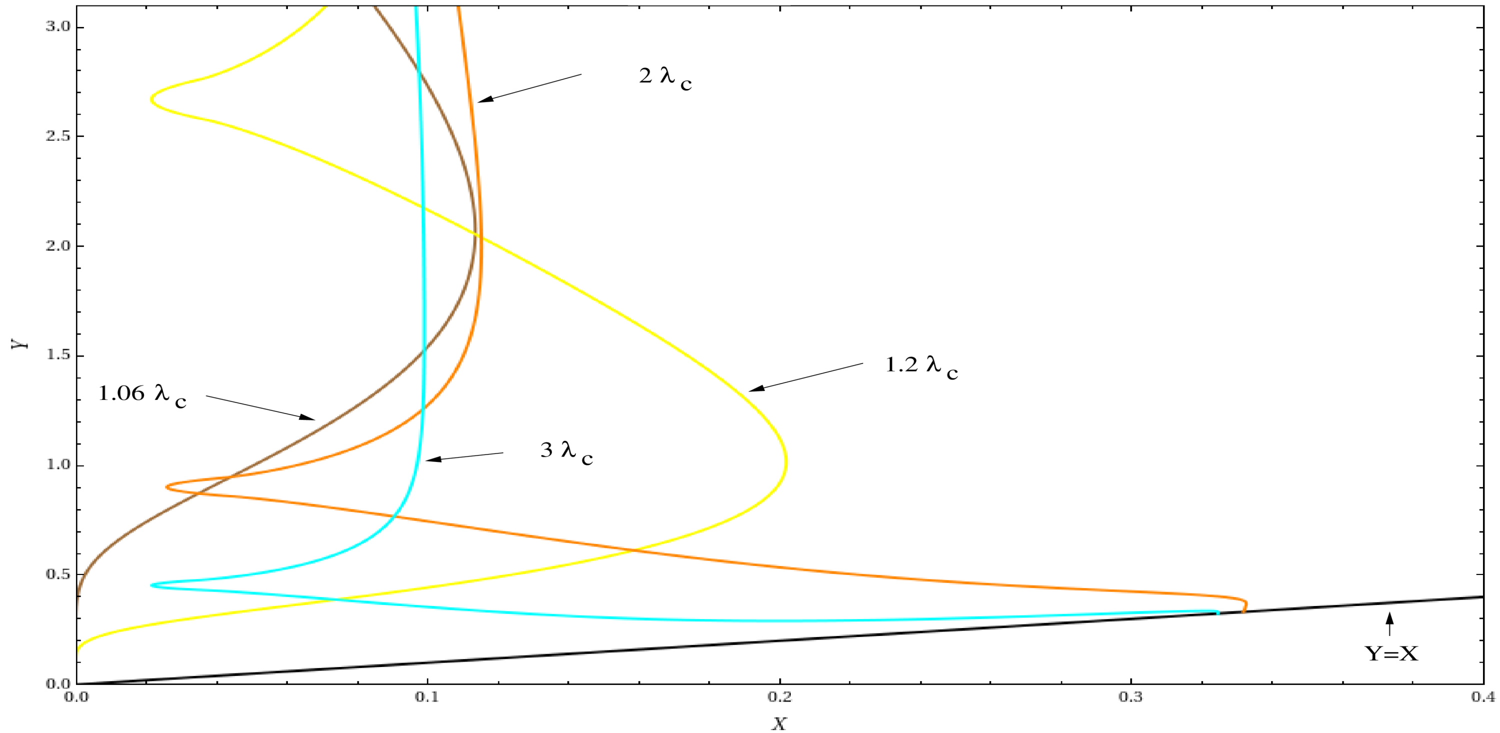

$\overset{?}{\Longrightarrow}$ intergalactic magnetic fields,
seed fields for galactic dynamos [Falquez et al. (2011)]

(astrophysical/cosmological coherence lengths through local breaking of isotropy by biasing negative temperature fluctuations of CMB through blackbody anomaly) [RH, Nature Physics (2013)]

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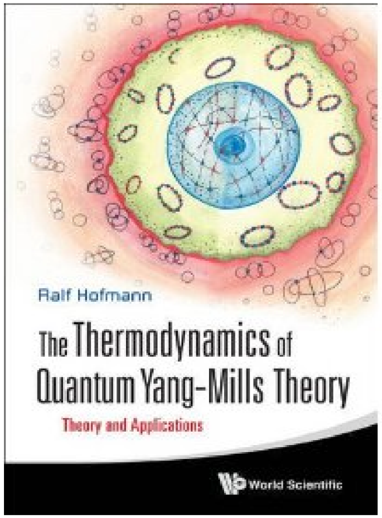
- other important cosmological implications: value of H_0 and nature of DM, DE
  (see talk by S. Hahn)

Theory:



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T. Grandou & RH, Adv. Math. Phys. (2015);
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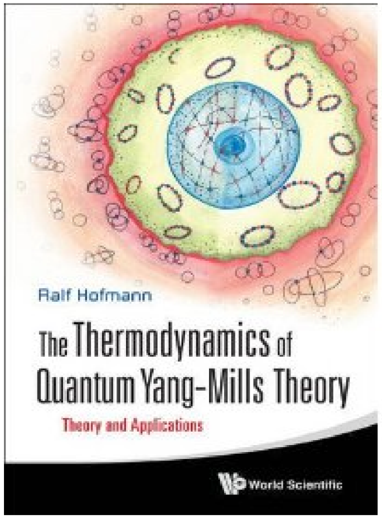

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Cosmological implications (CMB photons):

F. Giacosa and RH, Eur. Phys. J. C (2005);
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Thank you !