SU(2)$_{\text{CMB}}$ at high redshifts and the value of $H_0$

Steffen Hahn | March 22, 2017 | 5th Winter Workshop on Nonpertubative Quantum Field Theory, IN ΦNI
Outline

1. Motivation
   - tension between $H_0$ values
   - CMB anomalies

2. $H_0$ from high-$z$ $\Lambda$CDM
   - sound horizon $r_s$
   - $\Lambda$CDM model

3. $H_0$ from high-$z$ SU(2)$_{\text{CMB}}$
   - differences between SU(2)$_{\text{CMB}}$ and $\Lambda$CDM
   - straight-forward calculation of $r_s$ in SU(2)$_{\text{CMB}}$
   - reinterpretation of $v_b$ freeze out condition

4. Speculative interpolation of high- and low-$z$ models
   - Planck-scale axion
   - PSA vortices: percolation/depercolation model

5. Summary and outlook
Motivation

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Tension between $H_0$ values

Figure 1: CMB (red, [AAA$^+16$]) vs. local cosmological observation (gray, [RMH$^+16$]).
Tension between $H_0$ values

Figure 2: H0LiCOW measurement of $H_0$ (blue, [BCS$^{+16}$]).
What is $H_0$?

**Definition: Hubble parameter**

$$H_0 = \left. \frac{\dot{a}(t)}{a(t)} \right|_{t_0}, \quad ds^2 = dt^2 - a^2(t) \, dr^2 \quad (\text{FLRW metric, } a_0 = a(t_0) = 1) \quad (1)$$

- current expansion rate of the universe
- measure for the age of the universe
- important for cosmologically local distance calibrations

**Definition: cosmological redshift**

$$z = \frac{1}{a} - 1, \quad z(t_0) = 0, \quad z(0) = \infty \quad (2)$$

- redshift due to cosmological expansion (the earlier the higher)
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**Figure 3**: Different Rayleigh-Jeans line temperature fits [FKL+11].
CMB anomalies: early reionization

What is reionization?
- late time effect due to non-linear structure growth
- ignition of star-like objects (e.g. quasars...)
- ionizing spectral components of radiation ⇒ reionization

Detection using quasar light
- quasars are very old and have a very high luminosity
- emission during reioniz. implies Gunn-Peterson trough in spectrum
  ⇒ \( z_i \sim 6 \) ([BFW⁺01])

Calculation out of CMB anisotropies
- CMB photons scatter off free electrons (Thomson)
- fit of optical depth to TT angular power spectrum of CMB
  ⇒ \( z_i \sim 8.8 \) ([AAA⁺16]), \( z_i \sim 11 \) ([AAAC⁺14])

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- \( H_0 \) from high-z \( \Lambda \)CDM
- \( H_0 \) from high-z SU(2)_{CMB}
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$H_0$ from high-z $\Lambda$CDM

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CMB anomalies: large angles

Figure 4: Large angle suppression in TT($\theta$) [SH08], [CHSS10]. (Low variance of temperature fluctuations in ecliptic northern hemisphere.)
CMB anomalies: large angles

Figure 5: CMB cold spot (non-gaussianity of temperature fluctuations) [Vie10].
CMB anomalies: large angles

Figure 6: Alignment low-\(l\) CMB multipoles [TOCH03, OCTZH04, CHSS06]
**$H_0$ from high-$z$ $Λ$CDM**

1. **Motivation**
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2. **$H_0$ from high-$z$ $Λ$CDM**
   - sound horizon $r_s$
   - $Λ$CDM model

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   - differences between SU(2)$_{CMB}$ and $Λ$CDM
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5. **Summary and outlook**
### Sound horizon $r_s$

**Definition: sound horizon**

\[
    r_s(z) = \int_z^{\infty} dz' \frac{c_s(z')}{H(z')}, \quad c_s(z) = \frac{1}{\sqrt{3(1 + R(z))}}
\]  

- Computable in high-$z$ model
- $c_s$ sound velocity that propagates baryonic acoustic oscillations

### Definition

\[
    R(z) = \frac{3}{4} \frac{\rho_{b,0}}{\rho_{\gamma,0}} \cdot \frac{(z + 1)^3}{(z + 1)^4} = 111.019 \eta_{10} \cdot \frac{(z + 1)^3}{(z + 1)^4}, \quad \eta_{10} = \frac{n_{b,0}}{n_{\gamma,0}} 10^{-10}
\]

- $n_{\gamma,0}$ out of $T_0$
- $\eta_{10}$ $z$-independent in $\Lambda$CDM (no longer $z$-independent if CMB photons subject to SU(2)$_{\text{CMB}}$)

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Nearly model independ. extract. of $r_s H_0$

Figure 7: $r_s - H_0$ relation (yellow) [BVR16].

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Which value of decoup. \( z \) determines \( r_s \)?

**Definition: optical depth**

\[
\tau (z^*) = \int_{t(z^*)}^{t_0} dt \dot{\tau} = \sigma T \int_0^{z^*} dz \frac{\chi_e (z) n_e^b (z)}{(z + 1) H(z)} = 1
\]

- \( \dot{\tau} \) from Thomson scattering (without reionization!)
- decoupling of photons at recombination

**Definition: drag depth**

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\tau_d (z_d) = \int_{t(z_d)}^{t_0} dt \dot{\tau}_d = \sigma T \int_0^{z_d} dz \frac{\chi_e (z) n_e^b (z)}{(z + 1) H(z) R(z)} = 1
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- baryon velocity freeze out, end of drag epoch (Compton drag)
- corresponding \( r_s \) visible in today's matter correlation function
Which value of decoup. $z$ determines $r_s$?

**Definition: optical depth**

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\tau(z_*) = \int_{t(z_*)}^{t_0} dt \frac{\dot{\tau}}{\dot{\tau}} = \sigma_T \int_0^{z_*} dz \frac{\chi_e(z) n^b_e(z)}{(z + 1) H(z)} \equiv 1
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Clarification

Definition: electron number density

\[ n_e^b = (1 - Y_p) n_{b,0} (z + 1)^3 \text{ cm}^{-3} \]  (7)

- electrons before recombination II (hydrogen)
- \( Y_p \) Helium mass fraction in baryons

Definition: ionization fraction

\[ \chi_e (z) = \frac{n_e (z)}{n_e^b} \]  (8)

- \( \chi_e \) is computed with the recfast [Sco] (Boltzmann code)
Definition: electron number density

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Figure 8: $\chi_e$ marks recombination epoch.
ΛCDM model

Definition: critical density (today)
\[ \rho_{C,0} = \frac{3}{8\pi G} H_0^2 \]  \hspace{1cm} (9)

- out of Hubble equation in limit of flat universe
- \( G \) denotes Newton’s constant

Definition: \( z \) dependence of \( H(z) \)
\[ \frac{H(z)}{H_0} = \sqrt{\Omega_{\Lambda,0} + (\Omega_{b,0} + \Omega_{DM,0}) (z + 1)^3 + \Omega_{r,0} (z + 1)^4} \]  \hspace{1cm} (10)

- \( \Omega_{x,0} \): proportion of stuff \( x \) normalized to critical density \( \rho_{C,0} \)
- matter scaling: \((z + 1)^3\), radiation scaling: \((z + 1)^4\)
**ΛCDM model**

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What is $\Omega_{r,0}$? High-z approximation.

**Definition: radiative fraction ($\Lambda$CDM)**

$$\Omega_{r,0} = \Omega_{\gamma,0} + \Omega_{\nu,0} = \left(1 + \frac{7}{8} \left(\frac{4}{11}\right)^{\frac{4}{3}} N_{\text{eff}}\right) \Omega_{\gamma,0} \quad (11)$$

- $7/8$ correction due to neutrinos being Fermions and Photons Bosons
- $4/11$ can be obtained out of entropy conserv. of $e^+ e^-$ annihilation
- $N_{\text{eff}}$ fit parameter (represents effective number of massless neutrinos)

**High-z approximation**

$$\frac{H(z)}{H_0} \approx \sqrt{(\Omega_{b,0} + \Omega_{DM,0}) (z + 1)^3 + \Omega_{r,0} (z + 1)^4} \quad (12)$$

- since $\Omega_{\Lambda,0} < 1$ it can be neglected for $100 < z$
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Calculation of $r_s$ in $\Lambda$CDM

### Parameters with errors ([AAA$^+$16])

- $\Omega_{b,0} h^2 = 0.0222 \pm 0.0002$
- $\Omega_{DM,0} h^2 = 0.1199 \pm 0.0022$
- $N_{\text{eff}} = 3.15 \pm 0.23$
- $Y_p = 0.252 \pm 0.041$

### Parameters without errors (calculated out of $T_0 = 2.725$ K)

- $\Omega_{\gamma,0} h^2 = 2.468 \times 10^{-5}$

### Definition: $h$

\[ H_0 = h \cdot 100 \text{ km/s/Mpc} \quad (13) \]
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**Motivation**: $H_0$ from high-$z$ $\Lambda$CDM  $H_0$ from high-$z$ SU(2)$_{\text{CMB}}$ Speculative interpolation of high- and low-$z$ models  Summary and outlook

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Calculation of $r_s$ in $\Lambda$CDM

Figure 9: $\Lambda$CDM, $r_s(z_*)$ (orange, lower), $\Lambda$CDM, $r_s(z_d)$ (green, upper)
**Error estimation**

1. Generate gaussian distributed random value
   \[ \{\Omega^{(i)}_{b,0}, \Omega^{(i)}_{DM,0}, N^{(i)}_{\text{eff}}, Y^{(i)}_p\} \]

2. Calculate \( z^{(i)}_x \)

3. Calculate \( r^{(i)}_s \)

4. Enough values?
   - Yes
     - Use \( \{z_x\}, \{r_s\} \) for histogram, fit gaussian
   - No
     - Repeat \( i \rightarrow (i + 1) \)

---

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Summary and outlook
Table 1: Cosmological high-z models: $\Lambda$CDM versus $SU(2)_{CMB}$.

<table>
<thead>
<tr>
<th></th>
<th>$\Lambda$CDM</th>
<th>$SU(2)_{CMB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T/T_0$</td>
<td>$z+1$</td>
<td>$0.63(z+1)$</td>
</tr>
<tr>
<td>$\Omega_{DM}$</td>
<td>$\Omega_{DM}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$N_{\nu}$</td>
<td>$N_{eff}$</td>
<td>$3$</td>
</tr>
<tr>
<td>$T_\nu/T$</td>
<td>$\left(\frac{4}{11}\right)^{1/3}$</td>
<td>$\left(\frac{16}{23}\right)^{1/3}$</td>
</tr>
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</table>
Figure 10: $\Lambda$CDM behaviour (blue, dashed), SU(2)\textsubscript{CMB} behaviour (red solid, [Hof15])
**T(z)-scaling**

**Definition: high z behaviour**

\[
\frac{T(z)}{T_0} \xrightarrow{z \gg 10} 0.63 (z + 1)
\]

1. fundamental different \(T(z)\) scaling (curvature in \(T\) divided by \(z + 1\) which reflects presence of Yang Mills scale \(\Lambda_{\text{CMB}} \sim 1 \times 10^{-4} \text{ eV}\))
2. recovery of linear relation at high \(z\) albeit subject to lower slope
3. \(\text{SU}(2)_{\text{CMB}}\) gas has 8 instead of 2 relativistic degrees of freedom

**Today's checks of \(T(z)\)**

\[
T(z) = T_0 (z + 1)^{1 - \beta}
\]
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**Today's checks of \( T(z) \)**

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T(z) = T_0 (z + 1)^{1-\beta}
\]  

(15)

- thermal Sunyaev-Zeldovich effect [LGSM\(^+\)15]
- molecular rotation spectra [MBB\(^+\)13]
\[ \Delta l_{\text{tSZ}} = \frac{T_0^3}{2\pi^2 (e^x - 1)^2} x^4 e^x \tau (\theta f(x) - v_r + R(x, \theta, v_r)) , x = \omega / T \] (16)

- electrons of hot plasma scatter off CMB photons
- first order approximation (deviation of Planck spectrum)
  \[ \Rightarrow \beta \approx 0 !? \]
- adiabatically slow expansion implies that photon spectra depend on one mass scale only: \( T \)
  \[ \Rightarrow \omega \text{ scales as } T \text{ does (prejudice of } \omega \text{ implies prejudice of } T) \]
- analogous argumentation for rotation spectra

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**Motivation**
- \( H_0 \) from high- \( z \) \( \Lambda \)CDM
- \( H_0 \) from high- \( z \) \( SU(2)_{\text{CMB}} \)
- Speculative interpolation of high- and low- \( z \) models

**Summary and outlook**
- Steffen Hahn – \( SU(2)_{\text{CMB}} \) at high redshifts and the value of \( H_0 \)
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$T(z)$-scaling?! 

**Sunyaev-Zeldovich effect**

\[
\Delta l_{tSZ} = \frac{T_0^3}{2\pi^2} \frac{x^4 e^x}{(e^x - 1)^2} \tau (\theta f(x) - v_r + R(x, \theta, v_r)) , \quad x = \omega / T 
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Motivation $H_0$ from high-$z$ ΛCDM $H_0$ from high-$z$ SU(2)$_{\text{CMB}}$ Speculative interpolation of high- and low-$z$ models Summary and outlook

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Figure 11: New scaling can be fitted by even function:

\[ y \approx 0.2\pi + 0.1x^2 + 0.9x^4 - 1.4x^6 + 1.1x^8 - 0.3x^{10} \] (red solid), checked scaling (cyan, dashed, \( \beta \approx 0.6 \))
With recombination \( T_\ast \sim 3000 \text{ K} \)

\[
1800 \sim z_{\text{dec}}^{(\text{SU}(2)_{\text{CMB}})} > z_{\text{dec}}^{(\Lambda \text{CDM})} \sim 1100
\]  

\[
\left(\frac{1100}{1800}\right)^3 \sim \frac{\Omega_{b,0}}{\Omega_{b,0} + \Omega_{\text{DM},0}}
\]

- matter domination, radiation doesn’t play a role at decoupling

\( \Omega_{\text{DM}} \) at high \( z \)
Neutrino $N_\nu$ and $T_\nu$

- here not a fit parameter ($N_{\text{eff}}$)
- $N_\nu = 3$ (missing width in $Z_0$ decay)

### Conversion neutrino to photon $T$

$$
\left( \frac{T_\nu}{T} \right)^3 = \frac{g_1}{g_0} = \begin{cases}
\frac{4}{11}, & g_1 = 2, g_0 = 2 + \frac{7}{8}4 & (\Lambda \text{CDM}) \\
\frac{16}{23}, & g_1 = 8, g_0 = 8 + \frac{7}{8}4 & (\text{SU}(2)_{\text{CMB}})
\end{cases}
$$

- change in relativistic degrees of freedom
- $g_1$ relativistic degrees after, $g_0$ relativistic degrees before $e^+ e^-$ annihilation
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SU(2)$_{\text{CMB}}$ model

High-z Hubble parameter

$$\frac{H(z)}{H_0} \approx \sqrt{\Omega_{b,0} (z + 1)^3 + \Omega_{\gamma,0} \frac{8}{2} \left(1 + \frac{7}{32} \left(\frac{16}{23}\right)^{\frac{4}{3}} N_{\nu}\right)} (z + 1)^4 \quad (19)$$

Parameters with errors [AAA$^+16$]

- $\Omega_{b,0} h^2 = 0.0222 \pm 0.0002$
- $Y_p = 0.252 \pm 0.041$

Parameters without errors (calculated out of $T_0 = 2.725$ K)

- $\Omega_{\gamma,0} h^2 = 2.468 \times 10^{-5}$, out of $T_0 = 2.725$ K
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- $\Omega_{\gamma,0} h^2 = 2.468 \times 10^{-5}$, out of $T_0 = 2.725$ K
Straight-forward calc. of \( r_s \) in \( \text{SU}(2)_{\text{CMB}} \)

Figure 12: \( \text{SU}(2)_{\text{CMB}}, r_s(z_\ast) \) (blue, upper), \( \text{SU}(2)_{\text{CMB}}, r_s(z_d) \) (pink, lower)
Look at baryon freeze out

Baryonic Euler equation [PW68, HS96]

\[
\frac{d\nu_b}{dz} = -\frac{1}{a} \frac{da}{dz} \nu_b + \frac{k}{H(z)} \psi + \frac{1}{H(z)} \sigma T n_e^b \chi_e a (\Theta_1 - \nu_b) / R
\]  

- describes baryon velocity \( \nu_b \) of perfect baryon-photon fluid
- \( \Theta_1 \) dipole in temperature via Doppler effect
- \( \psi \) gravitational potential
Look at baryon freeze out

Solution: $\psi \approx 0$

$$\frac{\nu_b(z)}{z + 1} \sim \lim_{z \to \infty} \int_z^Z dz' \frac{e^{-\tau_d(z', z)}}{H(z')(z' + 1)} \dot{\tau}_d(z') \Theta_1(z'),$$

justified by absence of dark matter

Definition

$$D_d(z', z) = \frac{e^{-\tau_d(z', z)}}{H(z')(z' + 1)} \dot{\tau}_d(z')$$

analogous in the photon case $\tau_d \to \tau$

Motivation $H_0$ from high-z ΛCDM $H_0$ from high-z SU(2)$_{\text{CMB}}$ Speculative interpolation of high- and low-z models Summary and outlook

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Look at baryon freeze out

Solution: $\Psi \approx 0$

\[
\frac{\nu_b(z)}{z+1} \sim \lim_{z \to \infty} \int_z^Z dz' \frac{e^{-\tau_d(z',z)}}{H(z')(z'+1)} \dot{\tau}_d(z') \Theta_1(z') ,
\]  

justified by absence of dark matter

Definition

\[
D_d(z', z) = \frac{e^{-\tau_d(z',z)}}{H(z')(z'+1)} \dot{\tau}_d(z')
\]

analogous in the photon case $\tau_d \to \tau$
Look at baryon freeze out

Figure 13: The lf in $z_{lf,d}$ denotes left flank. Optical depth definition at maximum.
Figure 14: SU(2)_{CMB}, r_s(z_{lf,d}) (cyan).
Figure 15: $z_{lf,d}$ of $\Lambda$CDM.
Final result

Figure 16: $\Lambda$CMB, $r_s(z_{lf,d})$ (magenta).

Motivation

$H_0$ from high-$z$ $\Lambda$CDM

$H_0$ from high-$z$ SU(2)$_{\text{CMB}}$

Speculative interpolation of high- and low-$z$ models

Summary and outlook

Steffen Hahn – SU(2)$_{\text{CMB}}$ at high redshifts and the value of $H_0$

March 22, 2017
Speculative interpolation of high- and low-z models

1. Motivation
   - tension between $H_0$ values
   - CMB anomalies

2. $H_0$ from high-z $\Lambda$CDM
   - sound horizon $r_s$
   - $\Lambda$CDM model

3. $H_0$ from high-z SU(2)$_{\text{CMB}}$
   - differences between SU(2)$_{\text{CMB}}$ and $\Lambda$CDM
   - straight-forward calculation of $r_s$ in SU(2)$_{\text{CMB}}$
   - reinterpretation of $\nu_b$ freeze out condition

4. Speculative interpolation of high- and low-z models
   - Planck-scale axion
   - PSA vortices: percolation/depercolation model

5. Summary and outlook
Planck-scale axion

Definition: axion energy density, axion pressure

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) \quad (23) \]

- dynamical chiral symmetry breakdown induced by gravitational torsion at Planck scale ([FHSW95, GH07, GHN08])

Axion potential (Peccei-Quinn)

\[ V(\phi) = (\kappa \Lambda_{\text{CMB}})^4 \cdot \left(1 - \cos \left(\frac{\phi}{m_P}\right)\right), \quad m_P = \frac{1}{\sqrt{8\pi G}} \quad (24) \]

- anomalous breaking of symmetry $U_A(1) \rightarrow 1$ induced by thermal ground states of Yang Mills theories
- $\kappa$: dimensionless fudge factor, $\Lambda_{\text{CMB}} \sim 10^{-4}\text{eV}$
- spatially homogeneous field: frozen to slope of $V$ at high $z$, damped oscillations at low $z$
Planck-scale axion

**Definition:** axion energy density, axion pressure

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]  

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- Anomalous breaking of symmetry \( U_A(1) \rightarrow 1 \) induced by thermal ground states of Yang Mills theories
- \( \kappa \): dimensionless fudge factor, \( \Lambda_{\text{CMB}} \sim 10^{-4}\text{eV} \)
- Spatially homogeneous field: frozen to slope of \( V \) at high \( z \), damped oscillations at low \( z \)
The Planck-scale axion (PSA)

Definition: equation of motion (minimal coupling to gravity)

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{d}{d\phi} V(\phi) = 0 \]  

Definition

\[ H^2 = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_{\text{DM},e} + \rho_b + \rho_r \right) \]  

- 3H \dot{\phi} damping "force"
- \( \frac{d}{d\phi} V(\phi) \) driving "force"

\[ \rho_{\text{DM},0} = \lim_{z \to 0} \left( \dot{\phi}^2 + \rho_{\text{DM},e} \right) \]

\[ \Omega_{\Lambda,0} \rho_{C,0} = \lim_{z \to 0} \left( V(\phi) - \frac{1}{2} \dot{\phi}^2 \right) \]

- not conserved separately

Motivation  \( H_0 \) from high-z ΛCDM  \( H_0 \) from high-z SU(2)\text{CMB}  Speculative interpolation of high- and low-z models  Summary and outlook

Steffen Hahn – SU(2)\text{CMB} at high redshifts and the value of \( H_0 \)  

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The Planck-scale axion (PSA)

Definition: equation of motion (minimal coupling to gravity)

\[ \ddot{\phi} + 3H\dot{\phi} + \frac{d}{d\phi} V(\phi) = 0 \] (25)

- \( 3H\dot{\phi} \) damping "force"
- \( \frac{d}{d\phi} V(\phi) \) driving "force"

Definition

\[ H^2 = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_{\text{DM},e} + \rho_b + \rho_r \right) \] (26)

- \( \rho_{\text{DM},0} = \lim_{z \to 0} \left( \dot{\phi}^2 + \rho_{\text{DM},e} \right) \)
- \( \Omega_{\Lambda,0} \rho_{\text{C},0} = \lim_{z \to 0} \left( V(\phi) - \frac{1}{2} \dot{\phi}^2 \right) \)
- not conserved separately

Motivation

\( H_0 \) from high-z \( \Lambda \)CDM

\( H_0 \) from high-z SU(2)\(_{\text{CMB}}\)

Speculative interpolation of high- and low-z models

Summary and outlook

Steffen Hahn – SU(2)\(_{\text{CMB}}\) at high redshifts and the value of \( H_0 \)

March 22, 2017
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Fitting:

1. critical density $\rho_{C,0}$
2. dark energy $\Omega_\Lambda = 0.7$
3. zero of deceleration parameter $q_0$ at $z_q \sim 0.7$

- 3 fits to local cosmological data (parameters $\Omega_{DM,e,0}, \kappa, \phi_{in}$)
- $q_0$ out of supernovae Ia, luminosity distance redshift relation, standard ruler
- spatially homogeneous PSA model falsified by $z_q > 1$
### PSA vortices: percolation/depercolation model

**Definition: Ansatz**

\[
\frac{H(z)}{H_0} = \sqrt{\Omega_{DS}(z) + \Omega_{b,0}(z + 1)^3 + \Omega_{r,0}(z + 1)^4}
\]  

(27)

- \(\Omega_{r,0}\) is the radiation part in SU(2)\(_{\text{CMB}}\)
- \(\Omega_{DS}\) represents dark sector composed of percolated/depercolated PSA vortices
- Presumably PSA vortices abundantly generated across Hagedorn phase transitions in early universe due to Yang Mills theories going confining
- Perculation of these PSA vortices in the sense of Kosterlitz-Thouless transition
- \(\Omega_{DM,0} + \Omega_{\Lambda,0} = \Omega_{DS,0}\) equals the \(\Lambda\)CDM
- Deperculation at \(0 < z_p < z_\ast\)
Fitting of $z_p$

Definition: instantaneous phase transition

$$\Omega_{DS}(z) = \Omega_{\Lambda,0} + \Omega_{DM,0} \left[ (z + 1)^3 \theta (z_p - z) + (z_p + 1)^3 \theta (z - z_p) \right]$$  \hspace{1cm} (28)

Definition: angular size of sound horizon

$$\theta_* = \frac{r_s \left( z_{\text{lf},*} \right)}{\int_{0}^{z_{\text{lf},*}} \frac{dz}{H(z)}}$$  \hspace{1cm} (29)

- angle of first acoustic peak in TT angular power spectrum
Fitting of \( z_p \)

**Definition: instantaneous phase transition**

\[
\Omega_{DS}(z) = \Omega_{\Lambda,0} + \Omega_{DM,0} \left[ (z + 1)^3 \theta(z_p - z) + (z_p + 1)^3 \theta(z - z_p) \right]
\] (28)

**Definition: angular size of sound horizon**

\[
\theta_* = \frac{r_s(z_{lf,*})}{\int_0^{z_{lf,*}} \frac{dz}{H(z)}}
\] (29)

- angle of first acoustic peak in TT angular power spectrum
Figure 17: Angle of first peak for different peculation redshifts (solid). Horizontal line (dashed) represents real value.
Selfconsistency of $SU(2)_{\text{CMB}}$ high-$z$ model

\[
1 \gg \frac{\Omega_{\text{DM},0}}{\Omega_{b,0}} \frac{(z_p + 1)^3}{(z_{l_{f,*}} + 1)^3}
\]  

(30)

<table>
<thead>
<tr>
<th>Motivation</th>
<th>$H_0$ from high-$z$ LCDM</th>
<th>$H_0$ from high-$z$ SU(2)$_{\text{CMB}}$</th>
<th>Speculative interpolation of high- and low-$z$ models</th>
<th>Summary and outlook</th>
</tr>
</thead>
</table>

Yes.

\[
\frac{\Omega_{\text{DM},0}}{\Omega_{b,0}} \frac{(z_p + 1)^3}{(z_{l_{f,*}} + 1)^3} \sim 0.6\% \ll 1
\]  

(31)
Summary and outlook

1. Motivation
   - tension between $H_0$ values
   - CMB anomalies

2. $H_0$ from high-z $\Lambda$CDM
   - sound horizon $r_s$
   - $\Lambda$CDM model

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   - Planck-scale axion
   - PSA vortices: percolation/depercolation model

5. Summary and outlook
Summary and outlook

Motivation: $H_0$ from high-z $\Lambda$CDM

$H_0$ from high-z $SU(2)_{CMB}$

Speculative interpolation of high- and low-z models

Steffen Hahn – $SU(2)_{CMB}$ at high redshifts and the value of $H_0$

March 22, 2017
Summary and outlook

Motivation $H_0$ from high-z ΛCDM $H_0$ from high-z SU(2)$_{CMB}$ Speculative interpolation of high- and low-z models

Steffen Hahn – SU(2)$_{CMB}$ at high redshifts and the value of $H_0$

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Motivation

- $H_0$ from high-z $\Lambda$CDM
- $H_0$ from high-z SU(2)$_{\text{CMB}}$
- Speculative interpolation of high- and low-z models

Summary and outlook

Sufficiently high redshifts and the value of $H_0$
Summary and outlook

Motivation

$H_0$ from high-$z$ ΛCDM

$H_0$ from high-$z$ SU(2)$_{\text{CMB}}$

Speculative interpolation of high- and low-$z$ models

Steffen Hahn – SU(2)$_{\text{CMB}}$ at high redshifts and the value of $H_0$

Summary and outlook

$r_s (\text{Mpc})$

$\Lambda$CDM, $r_s(z_d)$

$\Lambda$CDM, $r_s(z_*)$

SU(2)$_{\text{CMB}}$, $r_s(z_*)$

SU(2)$_{\text{CMB}}$, $r_s(z_d)$

$H_0$ (Mpc$^{-1}$km/s)

65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
Summary and outlook

[Graph showing the relationship between $H_0$ (Mpc$^{-1}$ km/s) and $r_s$ (Mpc) for different models: $\Lambda$CDM, SU(2)\textsubscript{CMB}, and SU(2)\textsubscript{CMB'} at high redshifts ($z_{d,lf}$).]

Motivation

$H_0$ from high-$z$ $\Lambda$CDM

$H_0$ from high-$z$ SU(2)\textsubscript{CMB}

Speculative interpolation of high- and low-$z$ models

Steffen Hahn – SU(2)\textsubscript{CMB} at high redshifts and the value of $H_0$

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Interpolating model: high-\(z\) SU(2)$_{\text{CMB}}$ with low-\(z\) ΛCDM

- slow-roll dynamics of Planck-scale axion field falsified (\(z_q\) too high)
- however percolation/depercolation model for PSA vortices is promising: self consistent computation of angular size of sound horizon

Outlook

- Can such a model reproduce TT angular power spectrum?
- Can PSA interpolating model be made responsible for anomalous rotation curves in spiral galaxies (Tully-Fisher relation, elliptical galaxies, etc. ...)?
- Can radiative effects in SU(2)$_{\text{CMB}}$ explain large angle anomalies?
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