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Overview

- (anti)selfdual gauge fields, stable and unstable (anti)calorons
- deconfining thermal ground state
 - sketch of a priori estimate
 - ground-state permittivity and permeability
 - excitations: waves vs. particles
- deconfining thermodynamics
 - evolution of coupling
 - pressure and energy density
- radiative corrections
 - loop expansion of pressure
 - polarisation tensor

- thermal photon gases: cosmic microwave background (CMB) and beyond
- postulate SU(2)_{CMB}
 - temperature (T) -redshift (z) relation in FLRW Universe: implications for high-z cosmological model, 3D Ising exponent
 - high-z $-\Lambda$ CDM interpolation: **Planck-scale axion** and its vortices, **dark-sector** physics
 - angular power spectra, resolving the trouble with ${\cal H}_0$ and early-reionisation puzzle; low baryon density

The gauge group SU(2), (anti)selfdual gauge fields at finite temperature T_{\parallel} :

 J. Schwinger and R. P. Feynman argued in 1953 (see also A. Migdal) that finite temperature is introduced by reducing Euclidean spacetime to a cylinder,

$$\mathbf{R}^4 \longrightarrow S_1 \times \mathbf{R}^3 \longrightarrow S_\beta \equiv \frac{1}{2g^2} \int_0^\beta dx_4 \int dx^3 \operatorname{tr} F^2$$
.

and by demanding periodictiy of field configurations:

$$A_{\mu}(x_4, \vec{x}) = A_{\mu}(x_4 + \beta, \vec{x}), \quad \text{where} \quad \beta \equiv rac{1}{T}$$
 .



(Anti)calorons: trivial vs. non-trivial holonomy

• behavior of adjoint "Higgs field" A_4 at spatial infinity determines magnetic substructure: non-trivial holonomy

[Atiyah, Drinfeld,Hitchin,Manin 1978; Nahm 1983; Lee & Lu 1998; Kraan & Van Baal 1998]



- Harrington-Shepard caloron: special case where monopole delocalized and zero mass and antimonopole localized and finite mass
- "integrating out" Gaussian quantum fluctuations about non-trivial holonomy caloron:

[Diakonov 2004 along lines of 't Hooft 1976 for instanton]

- small holonomy (likely): fall-back to **trivial holonomy** \rightarrow **stable**
- large holonmy (unlikely): dissociation of caloron into its

monopole-antimonopole constituents \rightarrow unstable

Anatomy of (stable) HS caloron: [Gross & Pisarski & Yaffe 1983]

$$A_{\mu} = \bar{\eta}^{a}_{\mu\nu} t_{a} \partial_{\nu} \log \Pi(\tau, r)$$
$$\Pi = \begin{cases} \left(1 + \frac{1}{3} \frac{s}{\beta}\right) + \frac{\rho^{2}}{x^{2}} & (|x| \ll \beta) \\ 1 + \frac{s}{r} & (r \gg \beta) \end{cases} \qquad \left(s \equiv \frac{\pi \rho^{2}}{\beta}\right)$$

$$E_i^a = B_i^a \sim -\frac{\hat{x}^a \hat{x}_i}{r^2} \quad (\beta \ll r \ll s) ,$$

(static selfdual monopole-field)

$$E_{i}^{a} = B_{i}^{a} = s \frac{\delta_{i}^{a} - 3 \hat{x}^{a} \hat{x}^{i}}{r^{3}} \quad (r \gg s) \,.$$

(static selfdual **dipole-field** with dipole moment: $p_i^a = s \, \delta_i^a$)

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- Strategy (ground-state estimate): [Herbst & Hofmann 2004, Hofmann 2005, Giacosa & Hofmann 2007]
 - (i) perform spatial coarse graining over Euclidean time dependence of single HS caloron and anticaloron to render this time dependence a "choice of gauge" for phase of an adjoint and inert scalar field ϕ
- (ii) find e.o.m.s (1st **and** 2nd order) for ϕ
- (iii) from (ii) find e.o.m. for $V(\phi)$ densely packed (anti)caloron centers

(iv) complete action density for "dynamics" of ϕ by appeal to inertness, gauge invariance, and perturbative renormalizability of k=0 part

['t Hooft 1972, 't Hooft & Veltman 1973]

– Solve (2nd order) e.o.m. for curvature-free configuration a_{μ}^{gs}

overlapping (anti)caloron peripheries

Inert and adjoint scalar field from HS (anti)caloron centers:

- family of phases

$$\{\hat{\phi}^a\} \equiv \sum_{C,A} \operatorname{tr} \int d^3x \int d\rho \, t^a \, F_{\mu\nu}(\tau,\vec{0}) \, \{(\tau,\vec{0}),(\tau,\vec{x})\} \times \, F_{\mu\nu}(\tau,\vec{x})\{(\tau,\vec{x}),(\tau,\vec{0})\},$$

where
$$\{(\tau, \vec{0}), (\tau, \vec{x})\} \equiv \mathcal{P} \exp\left[i \int_{(\tau, \vec{0})}^{(\tau, \vec{x})} dz_{\mu} A_{\mu}(z)\right], \{(\tau, \vec{x}), (\tau, \vec{0})\} \equiv \{(\tau, \vec{0}), (\tau, \vec{x})\}^{\dagger}$$
.

unique definition:

- no higher n points
- no higher k
- no curvature of lines
- no shiftability of base point







- leads to: $\{\hat{\phi}^a\}$ as kernel of **uniquely determined differential operator**

$$\mathcal{D} \equiv \partial_{\tau}^2 + \left(\frac{2\pi}{\beta}\right)^2$$

(harmonic oscillator)



- but: \mathcal{D} exhibits **explicit** β **dependence** which must not be there; absorb β dependence into **potential** $V(\phi)$



full effective action density, ground-state solution:

– inertness of ϕ , renormalizability, gauge invariance:

$$\blacktriangleright \quad \mathcal{L}_{\text{eff}}[a_{\mu}] = \text{tr} \left(\frac{1}{2} G_{\mu\nu} G_{\mu\nu} + (D_{\mu}\phi)^2 + \frac{\Lambda^6}{\phi^2}\right) \,.$$

(
$$G_{\mu\nu} = \partial_{\mu}a_{\nu} - \partial_{\nu}a_{\mu} - ie[a_{\mu}, a_{\nu}] \equiv G^{a}_{\mu\nu} t_{a}$$
, $D_{\mu}\phi = \partial_{\mu}\phi - ie[a_{\mu}, \phi]$.)
effective coupling

– Euler-Lagrange for a_{μ} :

$$D_{\mu}G_{\mu\nu} = ie[\phi, D_{\nu}\phi]$$

pure-gauge solution:

$$a^{\rm gs}_{\mu} = \mp \delta_{\mu 4} \frac{2\pi}{e\beta} t_3 \quad (D_{\nu}\phi \equiv G_{\mu\nu} \equiv 0) \longrightarrow P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T \,.$$

Overlapping, small and transient-holonomy (anti)calorons implying collapsing monopole-antimonopole pairs.

– **interpretation** of ground-state estimate $\phi, a_{\mu}^{\mathrm{gs}}$:



 $a^{\rm gs}_\mu$ describes the collective overlap of all peripheries at a given center when centers are densely packed.

This is not yet the entire truth since there are packing voids and slight overlaps of centers \rightarrow effective radiative corrections, later.

- nature of excitations

electric/magnetic dipole density:

$$|\mathbf{D}_e| = rac{2s}{V_{
m cg}} \propto T^{1/2}$$

classical external electric (or magnetic, selfduality!) field strength squared to match ground-state energy density:

$$\rho_{\rm gs} = 4\pi T \Lambda^3 = \rho_{\rm EM} = \epsilon_0 \overline{|\mathbf{E}|}_e^2 \Rightarrow \overline{|\mathbf{E}_e|} \propto T^{1/2} -$$

$$\epsilon_0[Q(\mathrm{Vm}^{-1})] \equiv \frac{|\mathbf{D}_e|}{|\mathbf{E}_e|} = \frac{9}{32\pi^2} \frac{\Lambda[\mathrm{m}^{-1}]}{\Lambda[\mathrm{eV}]} (\xi Q)^2 \neq f(T)$$

similarly for μ_0

speed of light independent of intensity and, because of Doppler, independent of Lorentz frame

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(${\cal V}\,$ frequency of a monochromatic wave, back to natural units)

- \Rightarrow for thermal ground state to be excited in a wavelike way (electromagnetic spectrum) Yang-Mills scale Λ must be sufficiently large.
- \Rightarrow in a thermodynamical situation, however, Λ must be small to avoid ultraviolet catastrophe in BB radiation.

If nature indeed makes use of the SU(2) **thermal ground state** to **progagate em disturbances** then at least **two** such **theories** are required, subject to a **thermalization dependent mixing angle for their Cartan subalgebras**.

(*)

Deconfining thermodynamics

– go to entirely fixed and physical unitary-Coulomb gauge:

$$\phi^a = \delta^{a3} |\phi| \,, \quad \partial_i a_i^3 = 0 \,.$$

→ mass spectrum (adjoint Higgs mechanism) :

$$m_{1,2} = 2e|\phi| \equiv m , \quad m_3 = 0 .$$

$$\text{yet undetermined}$$
off-Cartan massive vector modes, thermal quasi-particle fluctuations} Cartan massless "photons"

– to draw same Legendre trafos from effective as from fundamental partition function one demands for pressure $P\,$:

 $\frac{\partial P}{\partial m} = 0$

(on one-loop thermodynamical selfconsistency)

Corrections from higher loops (hopefully) well under control.

evolution equation for coupling $\,e\,$

Deconfining thermodynamics



Deconfining thermodynamics: pressure and energy density



A (anti)caloron center localises Planck's quantum of action \hbar .

Such centers therefore must be interpreted as effective vertex induces (scattering of a_{μ} -fields) or as originators of all massive fluctuations or high-frequency massless fluctuations

real-time Feynman rules in unitary-Coulomb gauge.

(completely fixed gauge)

To not resolve a center in 2-2 scattering, momentum transfer in all Mandelstam variables s, t, u is bounded by $|\phi|^2$.

To not resolve a center in 2-1 scattering, off-shellness of massless mode is bounded by $|\phi|^2$.

Radiative corrections are **infrared** (masses) and **ultraviolet** (bounds on momenta transfers) **finite**.

Deconfining thermodynamics: radiative corrections

- loop expansion of pressure:



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Deconfining thermodynamics: radiative corrections

- polarisation tensor of massless mode

[Schwarz, Giacosa, RH 2007; Ludescher, RH 2008; Falquze, RH, Baumbach 2011]



 \rightarrow gap equations for transverse and longitudinal screening functions G and F

upon resummation: radiatively corrected/invoked dispersion laws

$$\omega_T^2 = \vec{p}^2 + G$$

$$\omega_L^2 = \vec{p}^2 + F$$

Transverse modes:

[Falquez, RH, Baumbach 2011]



(Yang-Mills scale or T_c fixed by CMB observation at low frequencies, later)

Deconfining thermodynamics: radiative corrections

Longitudinal modes:

[Falguez, RH, Baumbach 2012]

various low-momentum branches, physics implications: later

3.0 $2\lambda_{c}$ 2.52.0Υ 1.5 $1.2 \lambda_{c}$ $1.06 \lambda_{c}$ $3\lambda_c$ 1.0 0.5 Y=X 0.0 ⊾ 0.0 0.1 0.2 0.3 0.4X

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The Cosmic Microwave Background as seen by Planck and WMAP



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Low-temperature photon gases: Fixation of Yang-Mills scale



interpretation as onset of deconfining-preconfining phase transition through
 Meissner mass — evanescence of low-frequency waves ;
 sharply fixes [RH 2009]

$$T_c = T_0 = 2.725 \,\mathrm{K} \quad \Rightarrow \Lambda = \frac{2\pi T_c}{13.87} \sim 10^{-4} \,\mathrm{eV} \qquad \longrightarrow \quad \mathrm{SU}(2)_{\mathrm{CME}}$$

CMB: temperature (T)-redshift (z) relation



follows from energy conservation in FLRW universe upon deconfining-phase SU(2) equation of state $P = P(\rho)$: [RH (2015)]

$$\frac{d\rho}{da} = -\frac{3}{a}(P+\rho)$$

immediate consequences:

- discrepancy addressed between re-ionisation redshifts as extracted from

(i) fit to **TT angular power spectrum** of CMB [Planck coll. 2013, 2015]

(ii) **Gunn-Peterson trough** in high-z quasar spectra [Becker et al 2001]

modification of high-z cosmological model, possible

- explanation of discrepancy in H_0 from ΛCDM fits to CMB power spectra and local observation, later

[Planck coll . 2013,2015; Riess et al 2016; HoliCow 2016]

CMB: temperature (T)-redshift (z) relation: 3D Ising exponent

[Hahn & RH (2017)]



CMB: temperature (T)-redshift (z) relation: 3D Ising exponent



- exponentiation of (*) under consideration of (**) yields

$$\exp(a) = (\tau - 1)^{-\left(\frac{1}{4}\right)^{\frac{1}{3}}}$$

– interpretation of $\exp(a)$ as l/l_0 (system size a where l_0 a suitable reference length)

– re-combination $\,\mathcal{Z}_{*}$ at a $\sim 1/0.63\,$ times higher redshift compared to $\,\Lambda {
m CDM}\,$

reduced **dark matter** at z_*

[Hahn, RH 2017; Hahn, RH, Kramer 2018]

 \rightarrow predicts (comov) sound horizon (at baryon drag) such that H_0 in agreement with local observation, later

But: requires interpolation to low-z ΛCDM Boi

[Bernal et al 2016; Riess et al 2018; Bonvin et al 2016]

(de-percolation of Planck-scale axion vortices)



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 $SU(2)_{
m CMB}$ and a Planck-scale axion (PSA): dark sector

– ultralight pseudo-scalar field φ first proposed by Frieman et al. 1995 and revived by Wilczek et al 2004 to serve as quintessence

[Peccei, Sola ,Wetterich 1987; Wetterich 1988; Peebles, Ratra 1988]

 - conceptual underpinning: radiative protection of a rather strongly determined potential arising from an explicit, quantum-anomaly induced breaking (topological charge!) of a dynamically broken global U(1) symmetry

[Adler,Bardeen,Bell,Jackiw1969; Fujikawa 1979;Peccei, Quinn 1977]



 ${
m SU(2)}_{
m CMB}$ and PSA: percolated and depercolated PSA vortices

 –transitions from deconfining to confining phases in SU(2) YM are highly nonthermal (Hagedorn)

[RH 2007]



U(1) phase φ may wind around S_1 — \blacktriangleright **PSA vortices**

- PSA percolate in Berezinski-Kosterlitz-Thouless transition subsequent to Hagedorn (not unreasonable to assume DE e.o.s. for percolate)
- de-percolation at some redshift $\mathcal{Z}_{\mathcal{P}}$, DE e.o.s. transforms into DM e.o.s.

$$\hat{\rho}_{\mathrm{DS}} = \hat{\rho}_{\Lambda} + \hat{\rho}_{\mathrm{CDM},0} \cdot \begin{cases} (z + 1)^3 & (z < z_p) \\ (z_p + 1)^3 & (z \ge z_p) \end{cases}$$

(interpolation of high-z model to ΛCDM)

$SU(2)_{CMB} \, \text{and} \, \text{PSA:}$ fits to angular power spectra



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${ m SU(2)}_{ m CMB}$ and PSA: H_0 and $r_{ m re}$



 $SU(2)_{CMB}\,$ and thermal photon dispersion law: CMB at large angles

– TT correlation function C(heta) (PLANCK) [courtesy: Schwarz, Copi, Huterer, Starkman 2015]



$SU(2)_{CMB}\,$ and thermal photon dispersion law: CMB at large angles



at around z~1: rapid formation of temperature depression

[Szopa, RH 2007; Ludescher, RH, 2008]

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- nonabelian gauge principle: group SU(2)
- (anti)selfdual gauge fields, stable and unstable (anti)calorons
- sketch of a priori estimate
- properties of stable (anti)calorons
- ground-state permittivity and permeability
- excitations: waves vs. particles
- evolution of **coupling**
- pressure and energy density
- loop expansion of pressure
- polarisation tensor

- applications to CMB: ${\rm SU(2)}_{\rm CMB}$ fixation of Yang-Mills scale; T-z relation; 3D Ising exponent; re-visiting the cosmological model at high-z; low-z induction of large-angle anomalies

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Collaborators

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Low-temperature photon gases: electric-magnetically dual interpretations of $U(1) \subset SU(2)$

 – if SU(2) something to do with photons [RH (2005), Grandou & RH (2015), etc] then electric-magnetically dual interpretation required:

in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\alpha = \frac{Q^2}{4\pi\hbar} \,,$$

– for Ω to be unitless:

$$\left(e = \frac{\sqrt{8\pi}}{\sqrt{\hbar}} \cdot\right)$$

$$Q \propto rac{1}{e}$$
 .

But: magnetic coupling in SU(2)

$$g = \frac{4\pi}{e} \,.$$

SU(2) to be interpreted in an **electric-magnetically dual way**. (e.g., magnetic monopole $\leftarrow \rightarrow$ electric monopole, etc.)

 $(2)_{\rm CMB}$ and a Planck-scale axion (PSA): dark sector

– interesting coincidence:
$$\,m_a\sim H_0$$

(in view of

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\mathrm{d}}{\mathrm{d}\varphi}V\left(\varphi\right) \sim \ddot{\varphi} + 3H\dot{\varphi} + m_a^2\varphi = 0$$

energy density in **damped oscillations** (DM eos) **comparable** to potential energy density (DE eos)

Notice: in ΛCDM

 $\Omega_{\Lambda} = 0.7 \sim \Omega_{\rm DM,0} = 0.25$

However: deacceleration parameter

Universe accelerates too early for viable structure formation ($z_0 > 3$ as opposed to $z_0 \sim 0.7$)

Homogeneously oscillating PSA field falsified.



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 ${
m SU(2)}_{
m CMB}$ and thermal photon dispersion law: CMB at large angles

– to address these in $SU(2)_{CMB}$ CMB Boltzmann hierarchies plus evolution of curvature perturbations subject to new cosmological model with modified T-z relation must be solved [under investigation]

- first-shot approach:

treat T as a scalar field, introduce kinetic term, and take potential from integrated BB anomaly _____ e.o.m. (spherical symm. + linear fluct.)

$$0 = \partial_{\tau}\partial_{\tau}\delta T - \left(\frac{\mathrm{d}a}{a\,\mathrm{d}\tau}\right)^{2} \left[\partial_{\sigma}\partial_{\sigma}\delta T + \frac{2}{\sigma}\,\partial_{\sigma}\delta T\right] - \frac{3}{\bar{T}}\,\partial_{\tau}\bar{T}\,\partial_{\tau}\delta T + \frac{\bar{T}_{0}^{2}}{kH_{0}^{2}} \left[\frac{1}{2}\left.\frac{\mathrm{d}^{2}\hat{\rho}}{\mathrm{d}T^{2}}\right|_{T=\bar{T}}\,\delta T + \frac{1}{2}\left.\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}T}\right|_{T=\bar{T}}\right]$$

to be determined from Doppler inferred discrepancy between measured and predicted dipole

[Szopa, RH 2007; Ludescher, RH 2009]

Anatomy of (stable) HS caloron:

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modified spectral radiance in black-body (BB) radiation