Quantum Gravity and the Renormalization Group

Assignment 1 – Oct 18

Exercise 1: Counting gravitons

Motivation: In this exercise, we estimate the number of gravitons emitted in a binary merger. This gives us an idea whether the detection of single gravitons from such events is feasible.

Consider a binary merger of two solar mass ($m \sim 2 \times 10^{30}$kg) black holes. Estimate the number of gravitons N_h emitted via gravitational waves.
For this, assume that about $r = 5\%$ of the total mass is radiated away in mono-"chromatic" gravitational waves with a frequency of about 100 Hz. Given this number and keeping in mind the huge experimental effort in detecting gravitational waves, do you find it likely that we will detect single gravitons in the near future?

Exercise 2: Einstein's equations

Motivation: We review the derivation of Einstein's equations from an action. In this exercise, we will do so with the background field method. The goal is to get some (more) practice with computing perturbations, which will come in very handy very soon.

Einstein's equations are the equations of motion of General Relativity, and can be derived from an action principle. The action of General Relativity is the Einstein-Hilbert action, given by

$$S_{\text{EH}}[g] = \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-\det g}\,(R - 2\Lambda)\,, \tag{2.I}$$

where G_N is Newtons constant, R is the Ricci scalar and Λ the cosmological constant. Derive Einstein's equations from the requirement of a stationary action,

$$\frac{\delta S[g]}{\delta g_{\mu\nu}(x)} = 0\,. \tag{2.II}$$

Exercise 3: Linearised Einstein's equations

Motivation: This short exercises builds upon exercise 2 to derive the wave equation for linear metric perturbations about Minkowski space — a.k.a. gravitational waves.

Linearise Einstein's field equations about Minkowski space, that is, consider

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 0\,, \tag{3.I}$$

with

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3.II}$$

to linear order in h. Why do we have to set Λ to zero for this?