Quantum Gravity and the Renormalization Group

Assignment 2 – Oct 25

Exercise 4: Alternative gravitational theories

Motivation: We will now investigate alternative theories for gravity. This has both theoretical and experimental motivations: on the theoretical side, we know that General Relativity breaks down, e.g., at the centre of black holes, and a quantum theory has to take over. On the experimental side, with recent observations of gravitational waves and black hole shadows, we will soon be able to test deviations from General Relativity. We will discuss these theories in some more detail in the lectures soon.

Let us study the simplest extensions of General Relativity by adding terms to the action with four derivatives of the metric, and investigate what impact these have on gravitational waves.

a) Find a basis of all independent terms with (in total) four derivatives that are constructed purely in terms of the metric, and that respect diffeomorphism invariance. *Hints:* This should involve curvatures and covariant derivatives only. How many derivatives of the metric does the Riemann tensor have, and what are its symmetries? Also, how many uncontracted indices can terms in the action have?

b) Derive the equations of motion from the generalised action that you have found.

c) **[hard question]** Let us consider one specific (and phenomenologically very important) model, the Starobinsky theory. Its action is that of General Relativity with an R^2-term added,

$$S^{\text{Staro}} = \int \mathrm{d}^4x \, \sqrt{-\det g} \, \left[\frac{R - 2\Lambda}{16\pi G_N} + \alpha \, R^2 \right] , \tag{4.1}$$

so it is a special case of the theory above. From the equations of motion that you derived in b), find out if solutions to Einstein's equations are still solutions to the Starobinsky theory. If yes, where is the "new physics" in this theory?

d) **[hard question]** Back to the full system of b): linearise the equations of motion in Minkowski space. Do gravitational waves propagate differently now? Can these differences be removed by a suitable gauge fixing?

Exercise 5: Evidence for Quantum Gravity?

In the lecture, we discussed an argument for the quantisation of gravity that relies on what the gravitational field would be of a superposition of a massive object at different positions.

In 1981, Page and Geilker wrote a paper about an experiment they conducted (Phys. Rev. Lett. 47 (1981) 979-982). Read the paper and critically comment on whether their experiment realises the above argument.