Quantum Gravity and the Renormalization Group

Assignment 3 – Nov 08+11

Exercise 6: Superficial degree of divergence

Motivation: In this exercise, we discuss the superficial degree of divergence of a scalar theory that mimics the divergence count of General Relativity. The goal is to reinforce your understanding of why the perturbative quantisation fails.

As we have discussed in the lecture, General Relativity generates new divergences at every loop order. This concretely means that the loop divergences that are created come in a form that *cannot* be absorbed by a renormalisation of the Einstein-Hilbert terms in the action. To understand this in more depth, we will first consider a scalar toy model, and then we'll try to translate what we understand to the case of gravity.

a) Consider a ϕ^4 theory in four-dimensional Minkowski space with a microscopic action

$$S^{\phi^4} = \int \mathrm{d}^4x \left[\frac{1}{2} \left(\partial_\mu \phi\right) \left(\partial^\mu \phi\right) - \frac{\lambda_4}{4!} \phi^4 \right] . \tag{6.1}$$

At a structural level, i.e. *without explicitly calculating/evaluating any diagram*, argue what the superficial degree of divergence is of the loop diagrams contributing to the renormalisation of the four-scalar-vertex. Argue that at every loop order, you can absorb the divergences in a renormalisation of the coupling λ (and potentially a wave-function renormalisation of the field and a mass term).

b) Now consider ϕ^6 theory, also in four-dimensional Minkowski space, with the microscopic action

$$S^{\phi^6} = \int \mathrm{d}^4x \left[\frac{1}{2} \left(\partial_\mu \phi\right) \left(\partial^\mu \phi\right) - \frac{\lambda_6}{6!} \phi^6 \right] . \tag{6.2}$$

Once again at a structural level, discuss the superficial degree of divergence for the four- and six-scalar vertex *at one-loop order*. Can this theory be renormalised perturbatively?

c) **[hard question]** Next, consider a scalar field theory with a derivative interaction:

$$S^{\partial\phi} = \int \mathrm{d}^4x \left[\frac{1}{2} \left(\partial_\mu \phi\right) \left(\partial^\mu \phi\right) - \frac{\lambda_\partial}{(2!)^2} \left(\partial_\mu \phi\right) \left(\partial^\mu \phi\right) \phi^2 \right] . \tag{6.3}$$

What is the one-loop divergence structure in this case? Can you absorb all divergences of the four-scalar vertex?

d) **[hard question]** Finally, let us discuss the degree of divergence in General Relativity. In a similar way as for the scalar field theories above, argue what the divergence structure is at one and two loops. For this, consider the divergences of the two-graviton vertex. Can the divergences be absorbed, or do we have to introduce structurally new counterterms? If the latter, how do they look like?

Exercise 7: *[Presence]* Computing the one-loop divergence in gravity with xAct

Motivation: In this exercise, we will explicitly compute the one-loop divergence in General Relativity with the help of xAct. You do not have to bring your own laptop, the Mathematica file will be sent around afterwards.

Exercise 8: *[Presence]* Counting degrees of freedom

Motivation: In this exercise, we will go into some more detail on how we can count the number of physical degrees of freedom in a gauge theory.
