Heidelberg University WS 2024/2025

Quantum Gravity and the Renormalization Group

Assignment 4 – Nov 15+18

Exercise 9: Approaches to Quantum Gravity

Motivation: To get a small overview of the field of Quantum Gravity, we will discuss some specific approaches and how they (aim to) solve the predictivity problem of perturbative renormalisation. The different approaches are expected to give rise to different physical effects. However, it is not yet understood whether they really do, because most quantum-gravity approaches are quite involved and it is very difficult to extract physical predictions. In addition, because the Planck scale is so far removed from scales that experiments can probe directly, it is difficult to check any predictions.

Read up on one (or more, if you like) approach to Quantum Gravity. What is the respective starting point/idea? How would it solve the issue that in a perturbative quantisation, predictivity breaks down? Is predictivity restored? If so, how? What is the current state of the approach, and what are the major open questions? Pick out of these approaches:

- String Theory
- Loop Quantum Gravity
- Causal Set Theory
- Causal Dynamical Triangulations
- Group Field Theory

Do **not** pick Asymptotic Safety — this will be discussed during this course!

Here are a few literature pointers to get an idea about the different approaches:

- String theory: 2406.09508
- Loop Quantum Gravity: 2303.18172
- Causal Set Theory: 1903.11544
- Causal Dynamical Triangulations: 1905.08669
- Group Field Theory: gr-qc/0607032

If you want learn even more (read: too much) about the field of quantum gravity, have a look at the Handbook of Quantum Gravity. Most of the chapters are freely available on the arXiv.

Exercise 10: Effective field theory reasoning and the cosmological constant

Motivation: To get some more feeling about what the scale of new (gravitational) physics could be, we will have a look at the action from the point of view of effective field theory.

Based on the standard effective field theory treatment, what would you expect the size of the cosmological constant to be if the action is written in the form

$$\Gamma = \frac{1}{16\pi G_N} \int \mathrm{d}^4 x \sqrt{-\det g}\, (-2\Lambda + R + ...)? \tag{10.1}$$

Compare to the observational result that the vacuum energy density is $\rho_\Lambda = \frac{\Lambda}{8\pi G} \approx 10^{-47}\,\mathrm{GeV}^4$.

Discuss, based on this result, where you would expect the "scale of new physic" to be. Is there actually new physics there?

In this exercise, we are talking about the "cosmological constant problem". The value for the cosmological constant (aka the vacuum energy) induced by quantum fluctuations is proportional to the fourth power of the cutoff, Λ_{UV}^4, by simple mass dimension arguments. The cutoff for quantum gravity would be the Planck mass, so that a "natural" estimate would be

$$\frac{\Lambda}{G_N} \sim M_{\mathrm{Pl}}^4 \sim 10^{80}\mathrm{GeV}^4\,. \tag{10.2}$$

This naive estimate is off by a cool 120 orders of magnitude. Keep however in mind that this is a quantum correction to the bare cosmological constant. If the latter is a free parameter, there is a priori no issue.