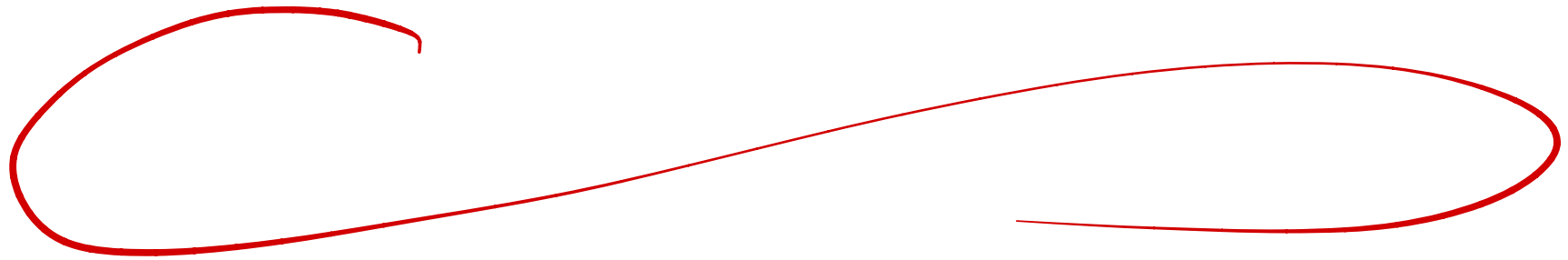


Quantum gravity and the renormalisation group



45 min + 15 min break
↓ + 45 min

lectures: Fr 11:15 - 13:00

Phil 12 KLS

tutorials: Mo 14:15 - 16:00

Phil 12 EG 059

central question: what is the quantum structure
of spacetime?

how: Functional Renormalisation Group (FRG)

≅ "mathematical microscope for quantum
field theories
(QFTs)"

prerequisites:

- General Relativity (GR)
- QFT I (helpful: II)

you can try QFT I in
parallel but it will be
challenging

also helpful:

- QFT on curved spacetimes
- (but optional) • Standard Model / Particle Physics / ...
- Intro to FRG

- content:
- perturbative quantisation of GR and breakdown of predictivity
 - quadratic gravity and stability challenges
 - asymptotic safety in QFTs
 - FRG for gravity
 - advanced topics (maybe: swampland, positivity bounds)

- useful material:
- lecture notes of last year  significant overlap

- Lectures in Quantum Gravity

[SciPost Phys. Lect. Notes 98,

YT recordings @Quantumgravity.

nordita]

partial overlap, 

includes many more details + ST, quantum BHs, EFT, ...

two great
books with
many details

- Introduction to Covariant Quantum Gravity
and Asymptotic Safety
[Percacci 2017]
- Quantum Gravity and the FRG: The Road
towards Asymptotic safety
[Reuter, Saueressig 2019]
- Handbook of Quantum Gravity [2024]

→
the QG bible, "handbook" = 4300 pages

tutorials: Mo 14:15-16:00 with Fabian Wagner

→ where you really learn the material

→ weekly exercises + occasional presence exercises

no hand-in 😊

exercises, lecture notes + other info will be posted

on the course webpage

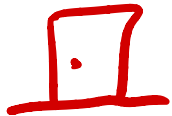
→ thphys → people → research associates → BK

- ask **all** your **questions** - there are **no** stupid questions!
- provide **feedback** if something about the lecture is not ideal for you to learn the material



knorr @ thphys.uni-heidelberg.de

wagner-f @ - - -



Phil 12 3056

(might change soon)

Why quantum gravity?

→ GR describes classical gravity...

... but matter is quantum!

↳ Einstein's equations:

$$\rightarrow R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$$

$$\underline{G_{\mu\nu}} = 8\pi G_N \underline{T_{\mu\nu}}$$

functions on
spacetime

operators

note:

we use natural
units, i.e.

$$c=1$$

$$\hbar=1 \text{ (mostly)}$$

→ mathematical inconsistency

→ physical inconsistency: RHS exhibits

uncertainty (e.g. position vs. momentum), but LHS
has no uncertainty

⇒ what is the gravitational field of a
"fuzzy" source?

Semiclassical gravity "let's close our eyes and pretend there
is no issue here"

idea: maybe gravity couples to expectation value of
stress-energy

$$T_{\mu\nu} \rightarrow \langle T_{\mu\nu} \rangle$$

which stat?

finite? still tensor?

$$\hookrightarrow G_{\mu\nu} \stackrel{?}{=} 8\pi G_N \langle T_{\mu\nu} \rangle$$

corresponds to: matter described by QFT on curved spacetime + gravitational field sourced by $\langle \rangle$ of energy-momentum

Q: is this enough to consistently describe quantum matter on classical spacetime?

or: does it work?

spoils: maybe not

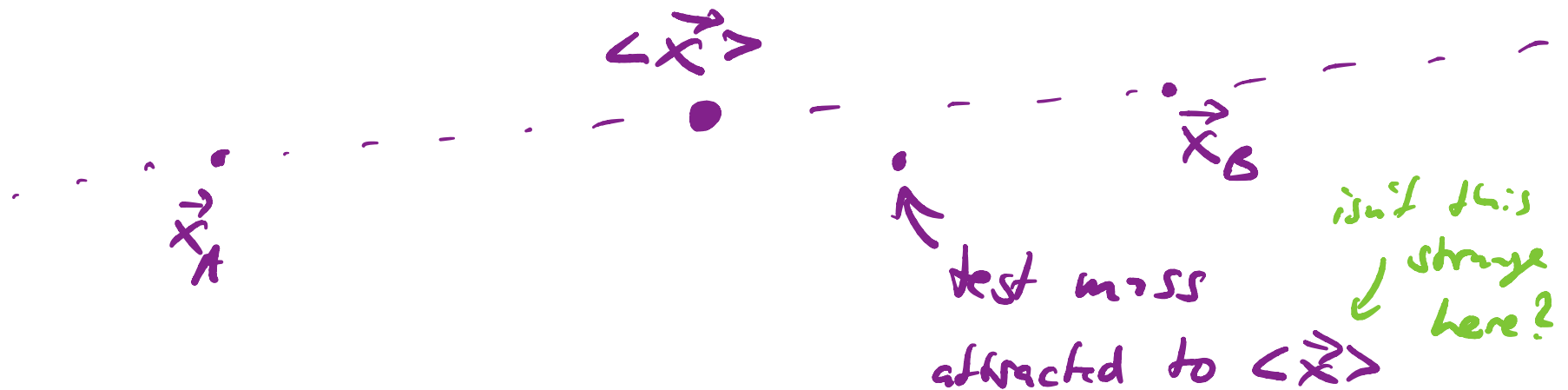
Example: grav. potential of a massive body that is in
a superposition of states centred about two locations

$$|\psi(x,t)\rangle \simeq \frac{1}{\sqrt{2}} (\delta(\vec{x} - \vec{x}_A) + \delta(\vec{x} - \vec{x}_B))$$

you can use
wavepackets if
you don't like
the δ 's

→ expected position $\langle \vec{x} \rangle = \frac{\vec{x}_A + \vec{x}_B}{2}$

↳ gravitational field sourced at $\langle \vec{x} \rangle$



↳ Upon measurement, wave function collapses

↳ either $\vec{x}_A$ or $\vec{x}_B \rightarrow$ instantaneous change of gravitational field?

- this thought experiment suggests that semiclassical gravity breaks down in some situations

See sheet 2  (Page + Geilker 1981 were the first to attempt an actual experiment — challenging to obtain QM superposition of an object that is massive enough to exert measurable grav. field)

- in analogy to QED, we expect the gravitational field of an object in a superposition is also a superposition of fields

Classical curvature singularities

- even GR signals its own breakdown, no QFT needed
 - curvature singularities
 - incomplete geodesics
- black holes, big bang

Example: Schwarzschild BH

we use the correct signature ☺

$$ds^2 = - \left(1 - \frac{2G_N M}{r}\right) dt^2 + \left(1 - \frac{2G_N M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

is a vacuum solution of GR, with $R_{\mu\nu} = 0$

however: $R_{\mu\nu\sigma\rho} \neq 0$, and in particular

$$K = R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} = 48 \frac{G_N^2 M^2}{r^6}$$

Kretschmann
scalar

→ infinite curvature at $r=0$

⇒ infinite tidal forces

similarly, the proper time τ_0 of a massive particle to reach the centre when starting at $r=r_0$ is

$$\tau_0 = \pi \sqrt{\frac{r_0^3}{8M}} < \infty$$

→ geodesics terminate in finite time

→ more complete theory need not be quantum per se,
but it is a minimal assumption

everything else is quantum, why not gravity?

Perturbative QG

- try what worked for the Standard Model

↳ quantise free theory (gravitational waves)

↳ decompose into gravitons
(similar to photons in QED)

then add interaction perturbatively

→ learn why we are still working on QG despite many
decades of research

Recap of gravitational waves

recall Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

highly non-linear in $g_{\mu\nu}$

• GW = solutions to linearised equations

→ use background field method:

we split the metric $g_{\mu\nu}$ into an arbitrary background metric $\bar{g}_{\mu\nu}$ and a perturbation $h_{\mu\nu}$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

variations exist,
for us this is
enough

→ you can define background curvatures,
cov. derivatives etc.

→ indices are raised & lowered with $\bar{g}$

→ for GW, we consider $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$ (flat background)

→ as long as $|h_{\mu\nu}| \ll 1$ (= grav. field created by perturbation is small, i.e. self-interactions can be neglected)

linearisation should work well

↳ linearise Christoffel symbol & curvature tensors: **sheet 1**

means equality up to terms outside of our interest

$$\Gamma^\mu_{\alpha\beta} \simeq \frac{1}{2} \eta^{\mu\nu} (\partial_\alpha h_{\nu\beta} + \partial_\beta h_{\nu\alpha} - \partial_\nu h_{\alpha\beta})$$

defined in context

$$R_{\mu\nu} \simeq \frac{1}{2} [\partial_\mu \partial^\alpha h_{\nu\alpha} + \partial_\nu \partial^\alpha h_{\mu\alpha} - \partial^2 h_{\mu\nu} - \partial_\mu \partial_\nu \underline{h}]$$

$\underline{h} = h^\alpha_\alpha$

$$R \simeq \partial^\mu \partial^\nu h_{\mu\nu} - \partial^2 h$$

note: expanding to higher order gives interaction terms of $h_{\mu\nu}$

→ important in **strong-field regime**, e.g. when GWs are created via black hole mergers

→ in **propagation**, these interactions are negligible because the GW amplitudes are small

G plug into EE:

$$\partial^2 h_{\mu\nu} - (\partial_\mu \partial^\sigma h_{\nu\sigma} + \partial_\nu \partial^\sigma h_{\mu\sigma}) + \partial_\mu \partial_\nu h$$

$$+ \eta_{\mu\nu} \partial^\sigma \partial^\sigma h_{\rho\sigma} - \eta_{\mu\nu} \partial^2 h = -16\pi G_N \overline{T}_{\mu\nu}$$

technically here $g = \eta$

this can be rewritten by introducing $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h$

$$\Rightarrow \partial^2 \bar{h}_{\mu\nu} - 2 \left[\underbrace{\frac{1}{2} (\delta_\mu^\alpha \partial_\nu + \delta_\nu^\alpha \partial_\mu)}_{\text{Symmetrisation: } \delta_\mu^\alpha \partial_\nu} - \frac{1}{2} \eta_{\mu\nu} \partial^\alpha \right] \partial_\beta \bar{h}^\beta_\alpha = -16\pi G T_{\mu\nu}$$

includes $\frac{1}{2}$
(=normalisation
to unit strength)

compare to QED:

$$\partial^2 A_\mu - \partial_\mu \partial^\nu A_\nu = J_\mu$$

wave equation, but has gauge redundancy
(= non-trivial kernel)

$$A_\mu \rightarrow \tilde{A}_\mu + \partial_\mu \lambda$$

$\Rightarrow$ have to gauge-fix, e.g. with
Lorentz gauge $\partial^\mu A_\mu = 0$

the *same* is true in gravity:

if $\bar{h}_{\mu\nu}^1$ is a solution, so is

$$\bar{h}_{\mu\nu}^2 = \bar{h}_{\mu\nu}^1 + \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu - \eta_{\mu\nu} \partial^\lambda \varepsilon_\lambda$$

for any ε

$\Rightarrow$ not all modes of $\bar{h}_{\mu\nu}$ are physical !

↳ the "gauge redundancy" in gravity is

diffeomorphism invariance:

→ physics does not depend
on your choice of
coordinates

Computation:

$$x^\alpha \rightarrow x'^\alpha = x^\alpha + \epsilon^\alpha$$

$$\Rightarrow g_{\mu\nu} \rightarrow \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x'^\beta}{\partial x^\nu} g'_{\alpha\beta}$$

$$\hookrightarrow \eta_{\mu\nu} + h_{\mu\nu} \rightarrow (\delta^\alpha_\mu + \partial_\mu \epsilon^\alpha) (\delta^\beta_\nu + \partial_\nu \epsilon^\beta) (\eta_{\alpha\beta} + h'_{\alpha\beta})$$

$$\hookrightarrow h_{\mu\nu} \rightarrow h'_{\mu\nu} + \underbrace{\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu}_{\text{increased diffeo. dr. fo.}} + \mathcal{O}(\epsilon^2)$$

- as in "normal" gauge theories, this is **not** a physical symmetry, but rather a **redundancy** in our description: $h_{\mu\nu}$ and $h_{\mu\nu}^{\prime}$ are **physically equivalent** ^{*}
 ↑
 conditions apply, you can have an entire course about this
- to get rid off this, we have to **gauge-fix**
 = impose condition on $\bar{h}_{\mu\nu}$
 can be used to simplify your life
 → remember Lorenz gauge in QED

e.g. [Strömteger
1703.05448]

popular choice: de Donder gauge

$$\partial^\mu \bar{h}_{\mu\nu} = 0$$

Sanity check: Is this a valid condition, i.e. can we always find an ϵ_μ such that $\partial^\mu \bar{h}_{\mu\nu} = 0$ without restricting physical configurations?

→ given some $h_{\mu\nu}$ which does not satisfy $\partial^\mu \bar{h}_{\mu\nu} = 0$,
search for $h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu$ so that

$$\partial^\mu \bar{h}'_{\mu\nu} = 0$$

$$\text{recall } \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h$$

$$= \partial^\mu \left[h'_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h' \right]$$

$$= \partial^\mu \left[\underbrace{h_{\mu\nu}} + \underbrace{\partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu}_{\text{cancel!}} - \frac{1}{2} \eta_{\mu\nu} (\underbrace{h} + 2 \underbrace{\partial^\alpha \varepsilon_\alpha}) \right]$$

$$0 = \partial^\mu \underbrace{\bar{h}_{\mu\nu}} + \partial^2 \varepsilon_\nu$$

→ this is a wave equation
for ε_ν and "always" admits
a solution

→ de Donder gauge is an incomplete

gauge fixing: we can still shift

ε_ν by solutions to the
homogeneous wave eq.

as in
QED!

this happens pretty universally in gauge theories,
and will eventually come to bite you

- back to linearised EE: in deDonder gauge,

$$\partial^2 \bar{h}_{\mu\nu} = -16\pi G_N T_{\mu\nu}$$

→ wave equation yay!

↳ we wanted to quantise the free field in vacuum

→ solve $\partial^2 h_{\mu\nu} = 0$

check why
 $\partial^2 \bar{h}_{\mu\nu} = 0 \Leftrightarrow \partial^2 h_{\mu\nu} = 0$

this has a general solution in terms of a superposition of plane waves

$$h_{\mu\nu}(x) = \int d^4q \, \tilde{h}_{\mu\nu}(q) e^{iq_\alpha x^\alpha} \quad \text{with } q^2 = 0$$

we will consider a single Fourier mode with momentum p_α ,

$$h_{\mu\nu}(x) = \pi_{\mu\nu} e^{ip_\alpha x^\alpha} + \pi_{\mu\nu}^* e^{-ip_\alpha x^\alpha}$$

$\uparrow$
polarisation tensor

similar to
polarisation vector
of the photon

note: $p^2 = 0$ to solve the wave equation

$\hookrightarrow$ gravitons are **massless** in GR (and in Minkowski!)