Quantum gravity and the renormalisation group

45min + 15min break + 45min

lectures: Fr 11:15 – 13:00
Phil 12 kHS

tutorials: Mo 14:15 – 16:00
Phil 12 EG 059

central question: what is the quantum structure of spacetime?

how: Functional Renormalisation Group (FRG)

$\simeq$ "mathematical microscope for quantum field theories (QFTs)"

prerequisites:

- General Relativity (GR)
- QFT I (helpful: II)

you can try QFT I in parallel but it will be challenging

also helpful (but optional):

- QFT on curved spacetimes
- Standard Model / Particle Physics / ...
- Intro to FRG

<u>content</u>:

- perturbative quantisation of GR and breakdown of predictivity
- quadratic gravity and stability challenges
- asymptotic safety in QFTs
- FRG for gravity
- advanced topics (maybe: swampland, positivity bounds)

<u>useful material</u>:

- lecture notes of last year ← significant overlap
- Lectures in Quantum Gravity [SciPost Phys. Lect. Notes 98, YT recordings @Quantumgravity.nordita]

partial overlap, includes many more details + ST, quantum BHs, EFT, ...

two great books with many details

- Introduction to Covariant Quantum Gravity and Asymptotic Safety [Percacci 2017]
- Quantum Gravity and the FRG: The Road towards Asymptotic safety [Reuter, Saueressig 2019]

- Handbook of Quantum Gravity [2024]

the QG bible, "handbook" = 4300 pages

tutorials: Mo 14:15 - 16:00 with Fabian Wagner

→ where you really learn the material

→ weekly exercises + occasional presence exercises

no hand-in :)

exercises, lecture notes + other info will be posted

on the course webpage

→ thphys → people → research associates → BK

- ask **all your questions** – there are **no** stupid questions!
- provide **feedback** if something about the lecture is not ideal for you to learn the material

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(might change soon)

Why quantum gravity?

→ GR describes classical gravity ...

... but matter is quantum!

↳ Einstein's equations:

$$\underbrace{G_{\mu\nu}}_{\substack{\text{functions on} \\ \text{spacetime}}} = 8\pi G_N \underbrace{T_{\mu\nu}}_{\text{operators}}$$

$\nearrow R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$

note: we use natural units, i.e.
$c = 1$
$\hbar = 1$ (mostly)

→ mathematical inconsistency

→ physical inconsistency: RHS exhibits

uncertainty (e.g. position vs. momentum), but LHS has no uncertainty

$\Rightarrow$ what is the gravitational field of a "fuzzy" source?

Semiclassical gravity "let's close our eyes and pretend there is no issue here"

idea: maybe gravity couples to expectation value of stress-energy

$$T_{\mu\nu} \rightarrow \langle T_{\mu\nu} \rangle$$

which state?

finite? still tensor?

$$\hookrightarrow \qquad G_{\mu\nu} \overset{?}{=} 8\pi G_N \langle T_{\mu\nu} \rangle$$

corresponds to: matter described by QFT on curved spacetime + gravitational field sourced by $\langle \rangle$ of energy-momentum

Q: is this enough to consistently describe quantum matter on classical spacetime?

or: does it cook?

spoiler: maybe not

Example: grav. potential of a massive body that is in a superposition of states centred about two locations

$$|\psi(x,t)\rangle \simeq \frac{1}{\sqrt{2}}\left(\delta(\vec{x}-\vec{x}_A)+\delta(\vec{x}-\vec{x}_B)\right)$$

you can use wavepackets if you don't like the δ's

$\to$ expected position $\langle\vec{x}\rangle = \frac{\vec{x}_A+\vec{x}_B}{2}$

$\hookrightarrow$ gravitational field sourced at $\langle\vec{x}\rangle$

test mass attracted to $\langle\vec{x}\rangle$

isn't this strange here?

↪ Upon measurement, wave function collapses to either $\vec{x}_A$ or $\vec{x}_B$ → instantaneous change of gravitational field ?

- this thought experiment suggests that semiclassical gravity breaks down in some situations

  (Page + Geilker 1981 were the first to attempt an actual experiment – challenging to obtain QM superposition of an object that is massive enough to exert measurable grav. field)

  See sheet 2

- in analogy to QED, we expect the gravitational field of an object in a superposition is also a superposition of fields

Classical curvature singularities

- even GR signals its own breakdown, no QFT needed

  → curvature singularities] black holes, big bang

  → incomplete geodesics]

Example: Schwarzschild BH

we use the correct signature :)

$$ds^2 = -\left(1-\frac{2G_N M}{r}\right)dt^2 + \left(1-\frac{2G_N M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

is a vacuum solution of GR, with $R_{\mu\nu} = 0$

however: $R_{\mu\nu\rho\sigma} \neq 0$, and in particular

$$K = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 48 \frac{G_N^2 M^2}{r^6}$$

Kretschmann scalar

→ infinite curvature at $r=0$

⇒ infinite tidal forces

similarly, the proper time τ_0 of a massive particle to reach the centre when starting at $r=r_0$ is

$$\tau_0 = \pi \sqrt{\frac{r_0^3}{8M}} < \infty$$

→ geodesics terminate in finite time

→ more complete theory need not be quantum per se, but it is a minimal assumption

everything else is quantum, why not gravity?

Perturbative QG

- try what worked for the Standard Model

  ↳ quantise free theory (gravitational waves)

  ↳ decompose into gravitons (similar to photons in QED)

  then add interaction perturbatively

→ learn why we are still working on QG despite many decades of research

Recap of gravitational waves

recall Einstein's equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$

highly non-linear in $g_{\mu\nu}$

- GW = solutions to linearized equations

→ use background field method:

we split the metric $g_{\mu\nu}$ into an arbitrary background metric $\bar{g}_{\mu\nu}$ and a perturbation $h_{\mu\nu}$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

variations exist, for us this is enough

→ you can define background curvatures, cov. derivatives etc.

→ indices are raised & lowered with $\bar{g}$

→ for GW, we consider $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$ (flat background)

→ as long as $|h_{\mu\nu}| \ll 1$ (= grav. field created by perturbation is small, i.e. self-interactions can be neglected) linearisation should work well

↳ linearise Christoffel symbol & curvature tensors: sheet 1

means equality up to terms outside of our interest (defined in context)

$$\Gamma^{\mu}{}_{\alpha\beta} \simeq \frac{1}{2}\eta^{\mu\nu}\left(\partial_\alpha h_{\nu\beta} + \partial_\beta h_{\nu\alpha} - \partial_\nu h_{\alpha\beta}\right)$$

$$R_{\mu\nu} \simeq \frac{1}{2}\left[\partial_\mu \partial^\alpha h_{\nu\alpha} + \partial_\nu \partial^\alpha h_{\mu\alpha} - \partial^2 h_{\mu\nu} - \partial_\mu \partial_\nu \underline{h}\right]$$

↳ $h = h^\alpha{}_\alpha$

$$R \simeq \partial^\mu \partial^\nu h_{\mu\nu} - \partial^2 h$$

note: expanding to higher order gives <u>interaction terms</u> of $h_{\mu\nu}$

→ important in strong-field regime, e.g. when GWs are created via black hole mergers

→ in propagation, these interactions are negligible because the GW amplitudes are small

↳ plug into EE:

$$\partial^2 h_{\mu\nu} - (\partial_\mu \partial^\rho h_{\nu\rho} + \partial_\nu \partial^\rho h_{\mu\rho}) + \partial_\mu \partial_\nu h$$
$$+ \eta_{\mu\nu} \partial^\rho \partial^\sigma h_{\rho\sigma} - \eta_{\mu\nu} \partial^2 h = -16\pi G_N \overline{T}_{\mu\nu}$$

technically here $g = \eta$

this can be rewritten by introducing $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$

$$\Rightarrow \quad \partial^2 \bar{h}_{\mu\nu} - 2\Big[\underbrace{\tfrac{1}{2}(\delta_\mu{}^\alpha \partial_\nu + \delta_\nu{}^\alpha \partial_\mu)}_{} - \tfrac{1}{2}\eta_{\mu\nu}\partial^\alpha\Big]\partial^\beta \bar{h}_{\alpha\beta} = -16\pi G_N T_{\mu\nu}$$

Symmetrisation: $\delta_{(\mu}{}^\alpha \partial_{\nu)}$ ← includes $1/2$ (= normalisation to unit strength)

compare to QED:

$$\partial^2 A_\mu - \partial_\mu \partial^\nu A_\nu = J_\mu$$

wave equation, but has gauge redundancy (= non-trivial kernel)

$$A_\mu \to \tilde{A}_\mu + \partial_\mu \lambda$$

$\Rightarrow$ have to gauge-fix, e.g. with
Lorenz gauge $\partial^\mu A_\mu = 0$

the same is true in gravity:

if $\bar{h}^1_{\mu\nu}$ is a solution, so is

$$\bar{h}^2_{\mu\nu} = \bar{h}^1_{\mu\nu} + \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu - \eta_{\mu\nu} \partial^\alpha \varepsilon_\alpha$$

for any ε

$\Rightarrow$ not all modes of $\bar{h}_{\mu\nu}$ are <u>physical</u>!

↳ the "gauge redundancy" in gravity is

diffeomorphism invariance:

→ physics does not depend on your choice of coordinates

Computation:

$$x^\alpha \longrightarrow x'^\alpha = x^\alpha + \varepsilon^\alpha$$

$$\Rightarrow \quad g_{\mu\nu} \longrightarrow \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x'^\beta}{\partial x^\nu} g'_{\alpha\beta}$$

$$\hookrightarrow \quad \eta_{\mu\nu} + h_{\mu\nu} \longrightarrow \left(\delta^\alpha{}_\mu + \partial_\mu \varepsilon^\alpha\right)\left(\delta^\beta{}_\nu + \partial_\nu \varepsilon^\beta\right)\left(\eta_{\alpha\beta} + h'_{\alpha\beta}\right)$$

$$\hookrightarrow \quad h_{\mu\nu} \longrightarrow h'_{\mu\nu} + \underline{\partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu} + \mathcal{O}(\varepsilon^2)$$

linearised diffeo. trafo.

- as in "normal" gauge theories, this is not a physical symmetry, but rather a redundancy in our description: $h_{\mu\nu}$ and $h'_{\mu\nu}$ are physically equivalent*

conditions apply, you can have an entire course about this

e.g. [Strominger 1703.05448]

- to get rid off this, we have to

gauge-fix

= impose condition on $\bar{h}_{\mu\nu}$

can be used to simplify your life

→ remember Lorenz gauge in QED

popular choice: de Donder gauge

$$\partial^\mu \bar{h}_{\mu\nu} = 0$$

Sanity check: Is this a valid condition, i.e. can we always find an ε_μ such that $\partial^\mu \bar{h}_{\mu\nu} = 0$ without restricting physical configurations?

→ given some $h_{\mu\nu}$ which does not satisfy $\partial^\mu \bar{h}_{\mu\nu} = 0$, search for $h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu$ so that

$$\partial^\mu \bar{h}'_{\mu\nu} = 0$$

recall $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$

$$= \partial^\mu \left[h'_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h' \right]$$

$$= \partial^\mu \Big[h_{\mu\nu} + \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu - \tfrac{1}{2}\eta_{\mu\nu}\left(h + 2\partial^\alpha \varepsilon_\alpha\right)\Big]$$

cancel!

$$0 = \partial^\mu \bar{h}_{\mu\nu} + \partial^2 \varepsilon_\nu$$

→ this is a wave equation for ε_ν and "always" admits a solution

→ de Donder gauge is an incomplete gauge fixing: we can still shift ε_ν by solutions to the homogeneous wave eq.

as in QED!

this happens pretty universally in gauge theories, and will eventually come to bite you

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- back to linearised EE: in deDonder gauge,

$$\partial^2 \bar{h}_{\mu\nu} = -16\pi G_N T_{\mu\nu}$$

$\rightarrow$ wave equation yay!

$\hookrightarrow$ we wanted to quantise the free field in vacuum

$\rightarrow$ solve $\partial^2 h_{\mu\nu} = 0$

check why

$$\partial^2 \bar{h}_{\mu\nu} = 0 \iff \partial^2 h_{\mu\nu} = 0$$

this has a general solution in terms of a superposition of plane waves

$$h_{\mu\nu}(x) = \int d^4q \; \tilde{h}_{\mu\nu}(q)\, e^{i q_\alpha x^\alpha} \qquad \text{with } q^2 = 0$$

we will consider a single Fourier mode with momentum p_α,

$$h_{\mu\nu}(x) = \Pi_{\mu\nu}\, e^{i p_\alpha x^\alpha} + \Pi^*_{\mu\nu}\, e^{-i p_\alpha x^\alpha}$$

$\uparrow$

polarisation tensor

similar to polarisation vector of the photon

note: $p^2=0$ to solve the wave equation

$\hookrightarrow$ gravitons are massless in GR (and in Minkowski :)