Issues with RGI

- usually no unique choice of physical scale

  → competing effects/multiscale physics not captured well

  → if there are dimensionful classical quantities, then different identifications are possible

e.g. $k \sim 1/r$

or

$k \sim \frac{1}{r}\left(\frac{G_N M}{r}\right)^n$

only need

$k\nearrow \leftrightarrow r\searrow$

$k\searrow \leftrightarrow r\nearrow$

- where to implement RGI?
  - → action?
  - → equations of motion?
  - → solution to eqs. of motion?

more difficult

- in gravity: – backreaction can become important

  interplay of classical singular metric and quantum fluctuations that are supposed to regularise

  – coordinate invariance

  scale identification incompatible with diffeomorphism invariance

many attempts in the literature to fix these problems

→ we discuss some ideas

① constraints from Bianchi identity

it's a consistency condition

recall Bianchi identity: $D^\mu G_{\mu\nu} = 0$

Einstein tensor $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$

geometric identity, has nothing to do with dynamics

→ perform RGI at the level of $\begin{cases} \text{action} & \alpha = 1 \\ \text{EoM} & \alpha = 0 \end{cases}$

$$G_{\mu\nu} = 8\pi G_N(k) T_{\mu\nu} - \Lambda(k) g_{\mu\nu} + \alpha \Delta t_{\mu\nu}$$

$$\Delta t_{\mu\nu} = G_N(k) \left[D_\mu D_\nu - g_{\mu\nu} D^2 \right] G_N(k)^{-1}$$

assume $D^\mu T_{\mu\nu} = 0$ and impose Bianchi identity:

a) $\alpha = 1$: compute $D^\mu G_{\mu\nu} = 0$ and insert EoM (solve for $T_{\mu\nu}$)

$$G_N'(k) R = 2\left(G_N'(k)\Lambda(k) - \Lambda'(k) G_N(k)\right)$$

$\rightarrow$ specific RG trajectory fixes $k = k(R)$

near FP: $G_N(k) \sim g_* k^2$, $\Lambda(k) \sim \lambda_* k^2$

$$\Rightarrow \quad k^2 \simeq R/4\lambda_*$$

b) $\alpha = 0$: compute $D^\mu G_{\mu\nu} = 0$ and insert EoM (solve for $R_{\mu\nu}$)

$$\left[8\pi G_N'(k) T_{\mu\nu} - \Lambda'(k) g_{\mu\nu}\right] D^\nu k(x) = 0$$

perfect fluid: $p = w\rho$, $T_\mu{}^\nu = \mathrm{diag}(-\rho, p, p, p)$

near FP, $k^4 \simeq \dfrac{8\pi g_*}{\lambda_*}\rho$

② Self-consistency and iterative RGI

Platania 1903.10411

- observation: we use $g^{cl}_{\mu\nu}$ to identify $k(x)$ and obtain $g^{q}_{\mu\nu}$

  → should use $g^{q}_{\mu\nu}$ to identify scale self-consistently

↪ iterative RGI:

$$g^{cl}_{\mu\nu} \xrightarrow[RGI]{} g^{q,1}_{\mu\nu} \xrightarrow[RGI]{} g^{q,2}_{\mu\nu} \to \dots \to g^{q}_{\mu\nu}$$

- for Schwarzschild as starting point, this leads to the Dymnikova BH

$$G_N(r) = G_N\left[1 - e^{-r^3/r_s e^2}\right]$$

$r_s = 2G_N M$

$e^2 = \frac{3\xi}{8\pi g_*} \ell_{Pl}^2$

③ Invariant RGI

Held 2105.11458

- observation: RGI at the level of EoM/solution is not coordinate-invariant

$\rightarrow g_{\mu\nu}$ transforms as a tensor

- idea: take curvature invariant and RG-improve there

- for Schwarzschild:

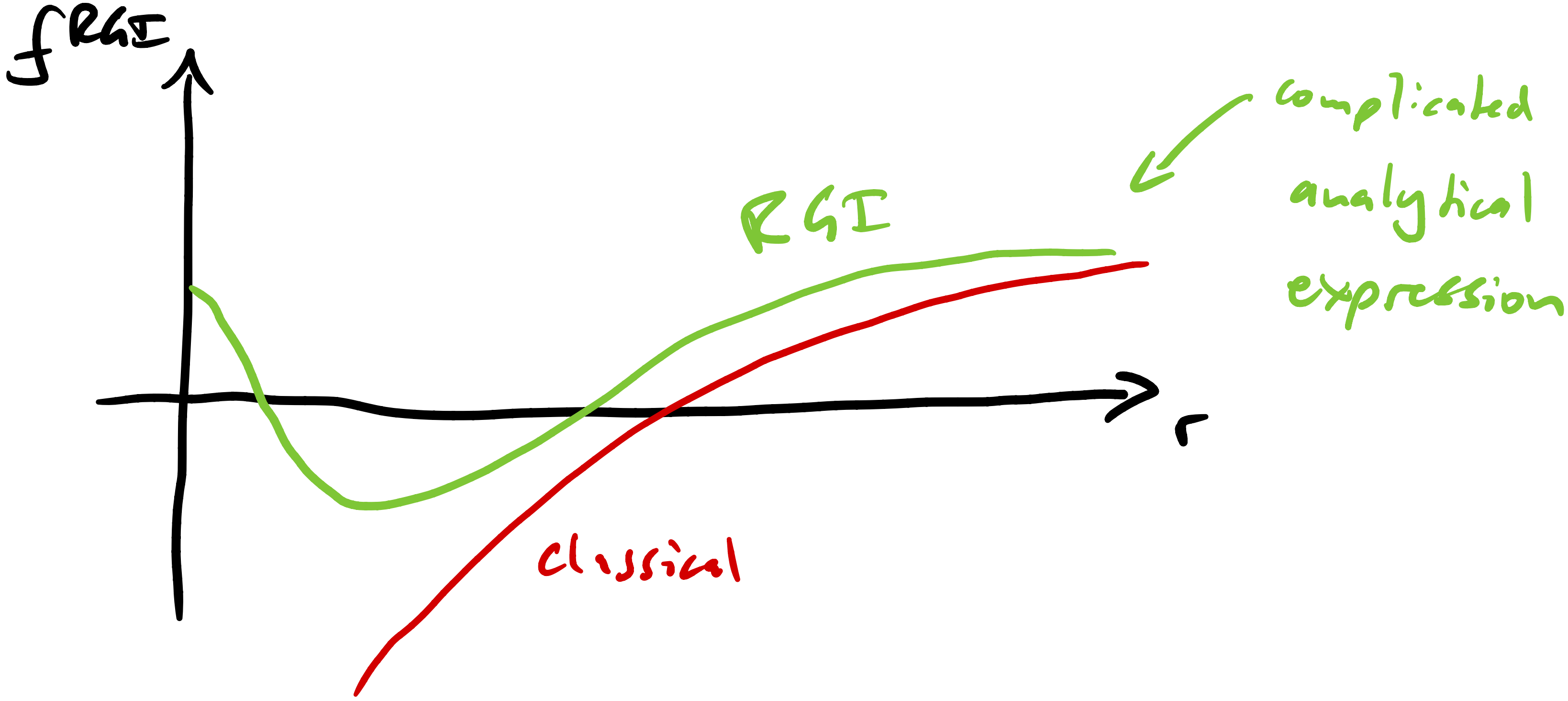
$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 48 \frac{G_N^2 M^2}{r^6}$$

compute in terms of

$$f^{RGI} = 1 - \frac{2 G_N(k(r)) M}{r}$$

RGI: $G_N \rightarrow G_N(k(r))$

$\Rightarrow$ differential equation for $k(r)$



obvious problem: what if there are several independent curvature invariants?

→ not obvious that consistent RGI procedure exists

Conceptual issues of RGI

Auber, Donoghue 1111.2875

- all of the above ideas hint towards that RGI has <u>conceptual</u> problems in gravity

- Q: Can RGI work at all in QG?

  in the sense of: there is a regime where RGI does provide the correct qualitative answers, similar to the Uehling potential

for this, we go back to perturbative QG and consider different $2 \to 2$ scattering processes involving gravity

$\Lambda = 0$

quick reminder: $2 \to 2$ scattering

→ 4 momenta $p_{1\mu}, p_{2\mu}, p_{3\mu}, p_{4\mu}$

Convention: all ingoing $p_1+p_2+p_3+p_4=0$

→ external lines are on-shell ⇒ $p_i^2 = m_i^2 = 0$

for gravitons

→ Mandelstam variables

$$s = (p_1+p_2)^2$$
$$t = (p_1+p_3)^2$$
$$u = (p_1+p_4)^2$$

with $s+t+u = \sum_{i=1}^{4} m_i^2$

→ the amplitude is a function of (s, t, u), or equivalently, in the centre-of-mass frame, (E^2, θ)

(energy transfer)^2

scattering angle in COM frame

⇒ if RGI makes sense, then there should be a replacement $G_N \to G_N(E^2)$ that, inserted into the tree-level result, gives the one-loop result

→ this should be universal, i.e., valid for all processes to maximise chances for success, we consider the differential cross section $\frac{d\sigma}{d\Omega}$ (measure for the effective area of a scattering target)

1) graviton-graviton scattering

helicities (don't worry about it)

$$\mathcal{A}^{++--}_{tree} \propto i G_N \frac{s^3}{tu}$$

2) grav. scatt. of massless scalars

$$\mathcal{A}_{tree} \propto i G_N \left[\frac{st}{u} + \frac{su}{t} + \frac{tu}{s} \right]$$

match RGI to one-loop $\frac{d\sigma}{d\Omega}$

$$G_N(E^2) \approx G_N \left[1 + c_g G_N E^2 \right]$$

grows with E

$c_g > 0$

$$G_N(E^2) \approx G_N \left[1 - c_s G_N E^2 \right]$$

decreases with E

$c_s > 0$

(similar issue if other helicity channels are considered)

→ suggests that there is **no** reasonable def. of RG running for G_N ???

what did we do then?

→ is RGI nonsense, or is gravity special?

to answer this, we have to go back to our starting point for renormalisation:

→ original observation: physics changes if a <u>physical</u> energy scale changes

→ what we compute with FRG: change when an IR cutoff changes "p running" vs. "k running"

$\rightsquigarrow$ physical energy scale is e.g. a momentum

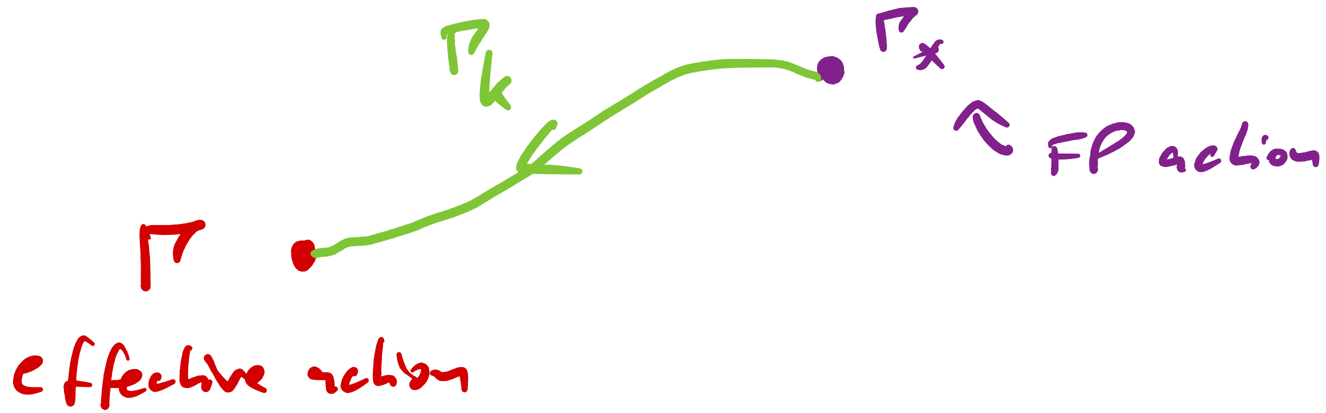
$\longleftrightarrow$ derivative in the action

$\rightsquigarrow$ k is a fiducial momentum scale

in some cases, k-running mimics p-running, but there are exceptions

$\rightarrow$ when talking about RG, one has to specify which is meant!

subtlety that leads to a lot of confusion recall what the main idea of the FRG was:



this is where the physics is

→ all quantum fluctuations included

= "path integral performed"

→ once we have Γ, we can forget that we used the FRG to compute it

$\Rightarrow$ pruning ~ momentum dependence in Γ

no pruning !!!

$\rightarrow$ in gravity, $\Gamma \simeq \int d^4x \sqrt{g} \left[\frac{2\Lambda - R}{16\pi G_N} + R \underline{f_R(\Box)} R + R_{\mu\nu} \underline{f_{Ric}(\Box)} R^{\mu\nu} + \dots \right]$

pruning here

$\Rightarrow$ G_N (and Λ) do not run with physical momenta!

Suppose they would, then e.g.

Q: what about

$\sqrt{R} \frac{1}{G_N(\Box)} \sqrt{R}$?

$$\int d^4x \sqrt{g} \frac{1}{G_N(\Box)} R$$

$\Rightarrow$ all potential non-trivial $\Box$-dependence are surface terms

$\Rightarrow$ k running = k-dependence of Γ_k

$$\hookrightarrow \quad \Gamma_k = \int d^4x \sqrt{g} \left[\frac{2\Lambda_k - R}{16\pi G_{N,k}} + R\, f_{R,k}(\Box) R + R_{\mu\nu}\, f_{Ric,k}(\Box) R^{\mu\nu} + \dots \right]$$

k running everywhere !

subject to constraints :

$k \to 0$: $\quad G_{N,k} \to G_N, \quad \Lambda_k \to \Lambda, \quad f_{R,k} \to f_R, \dots$

$k \to \infty$: $\quad g_k \to g_*, \quad \lambda_k \to \lambda_*, \quad f_{R,k} \to f_{R*}, \dots$

dimensionless in units of k

learning:

- G_N and Λ have krunning but no prunning $\Rightarrow$ RGI questionable

this explains the problem with the amplitude

- $f_R, f_{Ric}, \ldots$ have both krunning and prunning

idea for RGI of $f_{R,k}$:

$f_{R,k}(0) \longleftrightarrow f_R(p^2)$ "$k \to p$"

$\Rightarrow$ RGI potentially more promising

but still issue with Wick rotation: $p^2 \in \mathbb{R}$ but $k \in \mathbb{R}_+$

$\leadsto$ for the amplitude, relevant one-loop contributions come from R^2, $R_{\mu\nu} R^{\mu\nu}$; these have a different tensor structure than the Einstein-Hilbert term

$\Rightarrow$ cannot both be captured by a single coupling

Comments to put this into perspective:

- computing running is hard
  - $\rightarrow$ many computations rely on expansions in powers of $\Box$ or RGI where it makes sense
- pheno: people do what they can, analysing complicated actions with form factors :)

often unfeasible

i.e. for G_N & Λ

→ simple RGI often only way to make any statement at all

→ nevertheless: "AS-inspired" instead of "predictions of AS"

- large part of the literature focusses on establishing the FP in the first place

→ running important for this

- answering some of the most pressing open questions **need** to consider pruning, e.g. is the theory unitary & causal?

e.g. check bounds on scattering amplitudes

→ also strictly needs Lorentzian computations

of which we have some now

Some of these developments came out of constructive criticism from outside the community

Donoghue 1911.02967

⇒ having an open mind is important for progress, scientific or otherwise