# Perturbative QG as an EFT ← effective field theory

- order by order in perturbation theory, we can absorb the divergences of GR in finitely many counterterms

  ↳ as a recipe for a <u>fundamental</u> theory this fails

  ⇝ when curvature is "large", the theory is not predictive

↳ what happens at "small" curvature?

we have to clarify what this means since curvature has units

observation: n-loop terms come with $G_N^n$ relative to GR

$$\hookrightarrow \quad S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[ R + \alpha G_N R^2 + \beta G_N R_{\mu\nu} R^{\mu\nu} + \dots \right]$$

remember that $[G_N] = -2$

$\hookrightarrow$ $1/G_N$ defines the square of a mass, the Planck mass $M_{Pl}^2$

$$M_{Pl} = \sqrt{\frac{\hbar c}{G_N}} \approx 20\,\mu g \rightsquigarrow 10^{19}\,GeV$$

recall $m_{e^-} \simeq 511\,keV$

↳ the above expansion is in terms of curvature in Planck units

Q: when are higher order terms negligible?

for this, consides scattering of gravitons with energy $\sim E$

→ GR term contributes $E^2$

→ quadratic-in-curvature terms contribute $E^4/M_{Pl}^2$

relative factor: $E^2/M_{Pl}^2$

Conclusion: if $\alpha, \beta \sim \mathcal{O}(1)$, the two contributions above are negligible if $E^2 \ll M_{Pl}^2$

(Conversely, if $\alpha$ and $\beta$ are large, this reduces this bound)

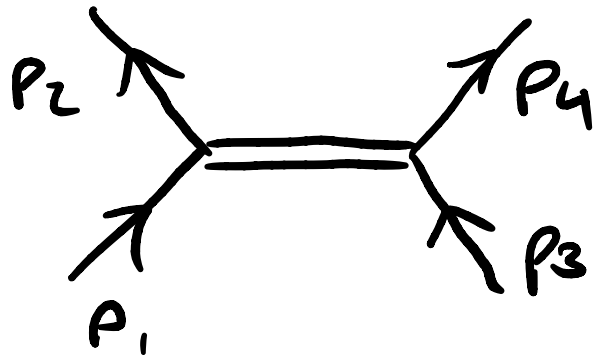
⇝ EFT of gravity to determine the leading quantum gravity corrections

## One-loop Newtonian potential

let us compute the gravitational attraction between two massive non-relativistic particles = "upgraded" Newtonian potential

a) tree level — this gives the classical Newtonian potential

consider the tree level diagram




~ scalar field that only interacts gravitationally

recall graviton propagator in de Donder gauge

once again we drop factors of "i"

$$P_{\mu\nu\sigma\tau} \propto \frac{1}{p^2}\left[\mathbb{1}_{\mu\nu\sigma\tau} - \frac{1}{2}\eta_{\mu\nu}\eta_{\sigma\tau}\right]$$

here: $p^2 = (p_1 - p_2)^2$

for the vertices, we assume a free massive scalar field action,

$$S_{matt} = \int d^4x \sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m^2\phi^2\right]$$

we have to expand this to linear order in $h_{\mu\nu}$ about $\eta_{\mu\nu}$ and take functional derivatives

recall $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G_N}\, h_{\mu\nu}$

$$\hookrightarrow \quad V^{\mu\nu}(p_1, p_2) \propto \sqrt{8\pi G_N}\left[ p_1^{(\mu} p_2^{\nu)} - \frac{1}{2}\eta^{\mu\nu}(p_1\cdot p_2 + m^2)\right]$$

$(\mu\nu)$: symmetrisation of indices

assuming two different mass scalar fields

$$\rightsquigarrow \quad V^{\mu\nu}(p_1,p_2)\, P_{\mu\nu\sigma\delta}(p_1-p_2)\, V^{\sigma\delta}(p_3,p_4) = -\frac{16\pi G_N m_1^2 m_2^2}{(p_1-p_2)^2}$$

recall that $p^2 = m^2$ for external lines

$$\approx -\frac{16\pi G_N m_1^2 m_2^2}{(\vec{p}_1-\vec{p}_2)^2}$$

$\approx$: non-relativistic limit

e.g. $p_{1,2\,\mu} = (\sqrt{m_\phi^2 + \vec{p}_{1,2}^{\,2}}, \vec{p}_{1,2})$

$\hookrightarrow (p_1 - p_2)^2 \approx (\vec{p}_1 - \vec{p}_2)^2 + \mathcal{O}\left(\frac{|\vec{p}|^4}{m_\phi^2}\right)$

Scattering potential $\propto$ Fourier transform in $\vec{q} = \vec{p}_1 - \vec{p}_2$ "momentum transfer"

up to normalisation factors $= \frac{1}{4 m_1 m_2}$

$$\hookrightarrow V(r) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{-16\pi G_N m_1^2 m_2^2}{\vec{q}^{\,2}} \frac{1}{4 m_1 m_2} = -\frac{G_N m_1 m_2}{r}$$

sanity check satisfied

## b) one-loop

this computation has a long history, and it took many attempts (= wrong papers) to get the final result

Donoghue gr-qc/9310024, Bjerrum-Bohr/Donoghue/Holstein hep-th/0211072

⇝ redo above but with one loop : only here: restored $c$ and $\hbar$

$$V(r) = -\frac{G_N m_1 m_2}{r}\left[1 + 3\frac{G_N(m_1+m_2)}{r c^2} + \frac{41}{10\pi}\frac{G_N \hbar}{r^2 c^3} + \dots\right]$$

„GR!

this is interesting: both classical and quantum

correction from one-loop diagram

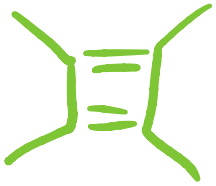
↳ fact: loop expansion $\neq \hbar$-expansion Holstein / Donoghue hep-th/0405239

→ one can calculate Post-Minkowskian (PM) corrections from loop diagrams

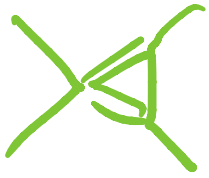
→ has opened up a new interesting line of research

In recent years: using QFT methods to improve GW calculations which are needed to interpret LIGO data

let us discuss the quantum tum

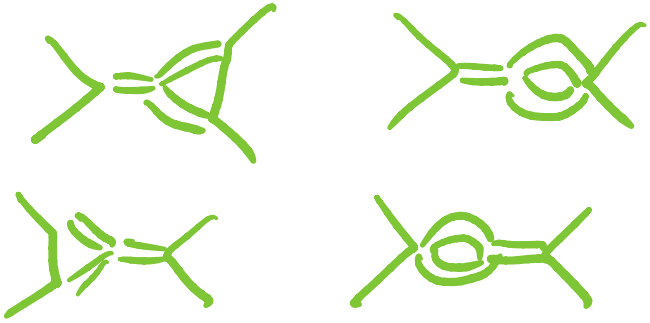
diagrams: box 

triangle 



crossed box 

double seagull 

vertex correction digrams 

vacuum polarisation 

grav. Faddeev-Popov ghost

diagrams all have divergences → does the counterterm enter the potential (including unknown prefactors)?

<u>NO</u>: a term $\frac{1}{r^3}$ comes from Fourier-transforming

$$\int \frac{d^3\vec{q}}{(2\pi)^3} \ln \frac{\vec{q}^2}{\mu^2} e^{i\vec{q}\cdot\vec{r}}$$

this is distinct from local counterterms $\sim R^2, R_{\mu\nu}R^{\mu\nu}$

these produce polynomials in $q^2$

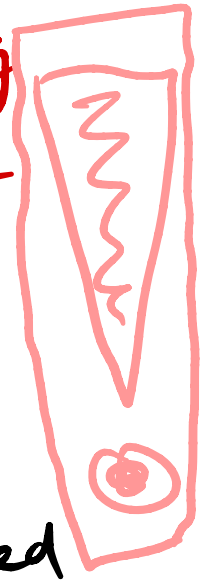
$\Rightarrow$ we **can** disentangle the IR (=long-range) effect from the UV (=short-distance) effect

↖ correction to $V(r)$, $r \to \infty$

↖ local corrections ("$r \to 0$")

↳ we **can** combine quantum effects with gravity and obtain unambiguous predictions within EFT

the **problem** is that the EFT breaks down at scales $r \approx \ell_{Pl}$ and loses predictivity $\Rightarrow$ we **still** need a UV completion



# QG crossroads

- perturbative QFT only works as an EFT
- experimental clues out of sight  recall $M_{Pl} \sim 10^{19}$ GeV

  → many approaches to QG, each with different assumptions/starting points/...

what could we do differently?

① pert. QFT is fine, but the gravitational starting point is wrong

  ⇝ at one loop, we get $R^2$ and $R_{\mu\nu}R^{\mu\nu}$,

what if we add them from the start?

Quadratic gravity

② pert. QFT is wrong, but QFT is fine

⇒ we need a non-pert. treatment

Asymptotic Safety

Loop Quantum Gravity

③ QFT breaks down

String theory

Causal set theory

# Quadratic gravity

if you can't beat them, join them

instead of quantising GR perturbatively, let us consider

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\left[\alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right]$$

as the starting point

we will discuss:

1. is adding these terms compatible with observations?
2. what is their effect on black holes + cosmology?
3. is this theory renormalisable?
4. why is this not considered to be <u>the</u> QG?

## ① Compatible with observations?

- in experimentally accessible situations, curvature is small

↳ $R/M_{Pl}^2 \ll 1$ even for black holes

e.g. Schwarzschild BH of mass M:

Kretschmann scalar $K = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$

$$K_{SBH} = \frac{48 G_N^2 M^2}{r^6}$$

↳

at horizon: $(r = r_H = 2G_N M)$

recall $G_N = \frac{1}{M_{Pl}^2}$

$$\left.\frac{K_{SBH}}{M_{Pl}^4}\right|_{r=r_H} = \frac{48}{M_{Pl}^4} \frac{G_N^2 M^2}{64 G_N^6 M^6}$$

$$= \frac{3}{4} \frac{M_{Pl}^4}{M^4}$$ tiny

- strongest bounds on $\alpha, \beta$ come from constraints on modifications of Newton's law

$$\rightsquigarrow \quad \phi(r) = -\frac{G_N m}{r}\left[1 + \frac{1}{3}e^{-m_0 r} - \frac{4}{3}e^{-m_2 r}\right]$$

$$m_0 = \sqrt{32\pi G_N (3\alpha - \beta)}$$

$$m_2 = \sqrt{16\pi G_N \beta}$$

short range terms

$\hookrightarrow$ $|\alpha|, |\beta| < 10^{60}$ → weak constraints

note: we can see the appearance of two massive modes from the Yukawa terms $\sim e^{-mr}$

→ will play an important role later

## ② Phenomenology

there are (at least) two settings where modifications of GR could be expected to play a major role

a) early universe cosmology:

- $R^2$-term gives a minimal model for inflation known as Starobinsky model

- generally good agreement with measurements of cosmic microwave background for $\beta = 0$, $\alpha \sim 10^5$

  → but: recently under pressure !

- attractive because there is only a single parameter ($\alpha$)

b) black holes:

- every vacuum solution of GR is also a vacuum solution to Quadratic gravity ↗ sheet 2

⇒ Kerr BHs are solutions

but there are others!

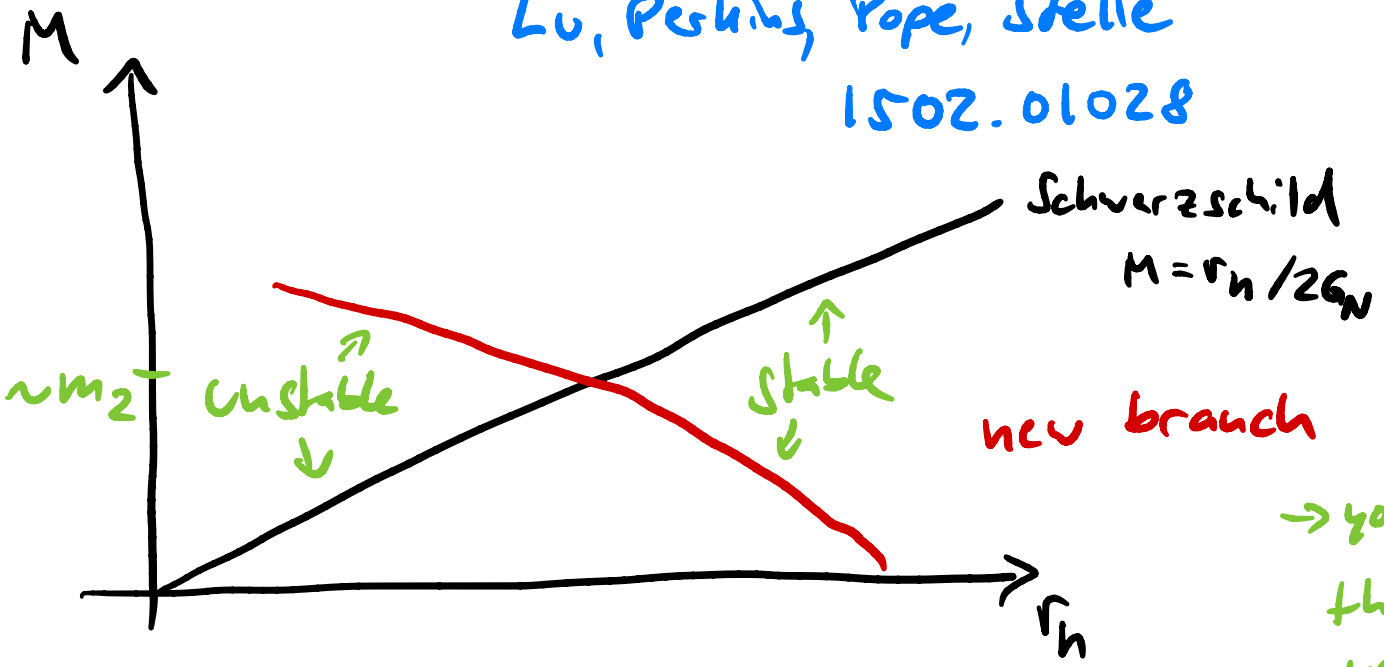
(only known numerically)

Lu, Pershing, Pope, Stelle

1502.01028

⇒ no uniqueness theorem as for GR in $d=4$

(in $d>4$ there can be other solutions like black strings)




→ you can also study the thermodynamics of these new solutions

⇝ light enough BHs might be described by this new branch

⇝ more branches exist, e.g. also wormholes and nonsingular holes

↳ lots of buzz in recent years to investigate solutions to classical gravity theories beyond GR

## ③ Renormalisability

compute superficial degree of divergence of Quadratic Gravity:

at large momenta, the $R^2$-terms dominate

$\rightsquigarrow$ propagator $\propto p^{-4}$ recall $R \sim \partial\partial h \sim p^2 h$

vertices $\propto p^4$

in $d$ dimensions, a diagram with $P$ propagators, $V$ vertices and $L$ loops scales like

$$D = dL - 4P + 4V = (d-4)L + 4 \quad = 4 \text{ in } d=4$$

$L = P - V + 1$

this means that divergences scale like $p^4$ at worst,

independent of the loop order

→ we have $p^4$ terms in the action

→ we can absorb all divergences !

⇒ Quadratic Gravity is perturbatively renormalisable !

so this is it ? can we go home and call it a day ?

↳ Is perturbative renormalisability enough to ensure that a theory is fundamental, i.e. valid at all scales?

⇝ NO.

⇝ renormalisability only ensures that a finite number of counterterms is needed to absorb all divergences

⇝ no implication by itself on how couplings change under the renormalisation group (RG) flow

↳ can still have divergences in the finitely many retained couplings → Landau pole

We will introduce the RG flow more properly later;

for now the idea is the following:

- we can probe the theory at different scales

  e.g.: <u>experimentally</u> by performing scattering at different energies

  <u>theoretically</u> by introducing a cutoff and shifting it around

these are related, more on this later (Pandora's box ahead)

$\rightsquigarrow$ values of couplings change

- we call the "probing scale" $\mu$ for now

  units of energy = momentum

and we introduce the $\beta$-function

$$\beta_g = \mu \partial_\mu g(\mu) = \partial_{\ln \mu}$$

for the coupling $g(\mu)$