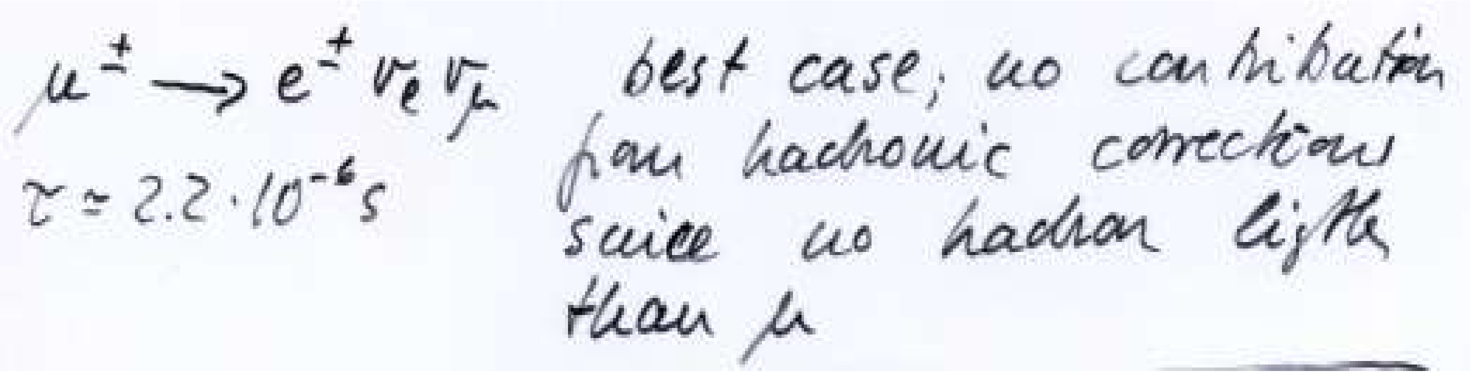
The Fermi Coupling constant G_F

plays key role in all precision tests of electroweak standard model together with α and mass of Z-boson best measured quantity in electroweak interaction -> input to all higher-order calculations (historically strongest constraint on top mass before it was directly measured now e.g. limit for Higgs mass)

< - > G_F is sensitive to physics at very high energy scales

experimentally, G_F is extracted from muon lifetime $\tau_\mu = 1/\Gamma_\mu$



$\mu^\pm \rightarrow e^\pm \nu_e \bar{\nu}_\mu$

$\tau \simeq 2.2 \cdot 10^{-6}\,s$

best case; no contribution from hadronic corrections since no hadron lighter than μ

purely leptonic process - > very clean experimentally and theoretically

• weak coupling constant

dimensionless coupling constant $\frac{g^2}{\hbar c}$

analog to $\alpha = \frac{e^2}{\hbar c}$

historically defined strength of weak interaction by Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

note: from propagator would expect just $G = g^2 / M_W^2$ but originally G was defined for pure Fermi (i.e. pure vector) transitions. to accomodate (V-A): $G \to G/\sqrt{2}$

also initially helicity operator $(1-\gamma_5)$ instead of $\frac{1}{2}(1-\gamma_5)$; applied for both vertices factor 4

and

g defined for coupling to W^1, W^2, W^3. Now for charged current we have $W^\pm = \frac{1}{\sqrt{2}}(W^1 \pm iW^2)$

gives factor $\frac{1}{2}$ using g^2

→ Fierz relation

➜ Strom-Strom-Kopplung der Fermi-Theorie

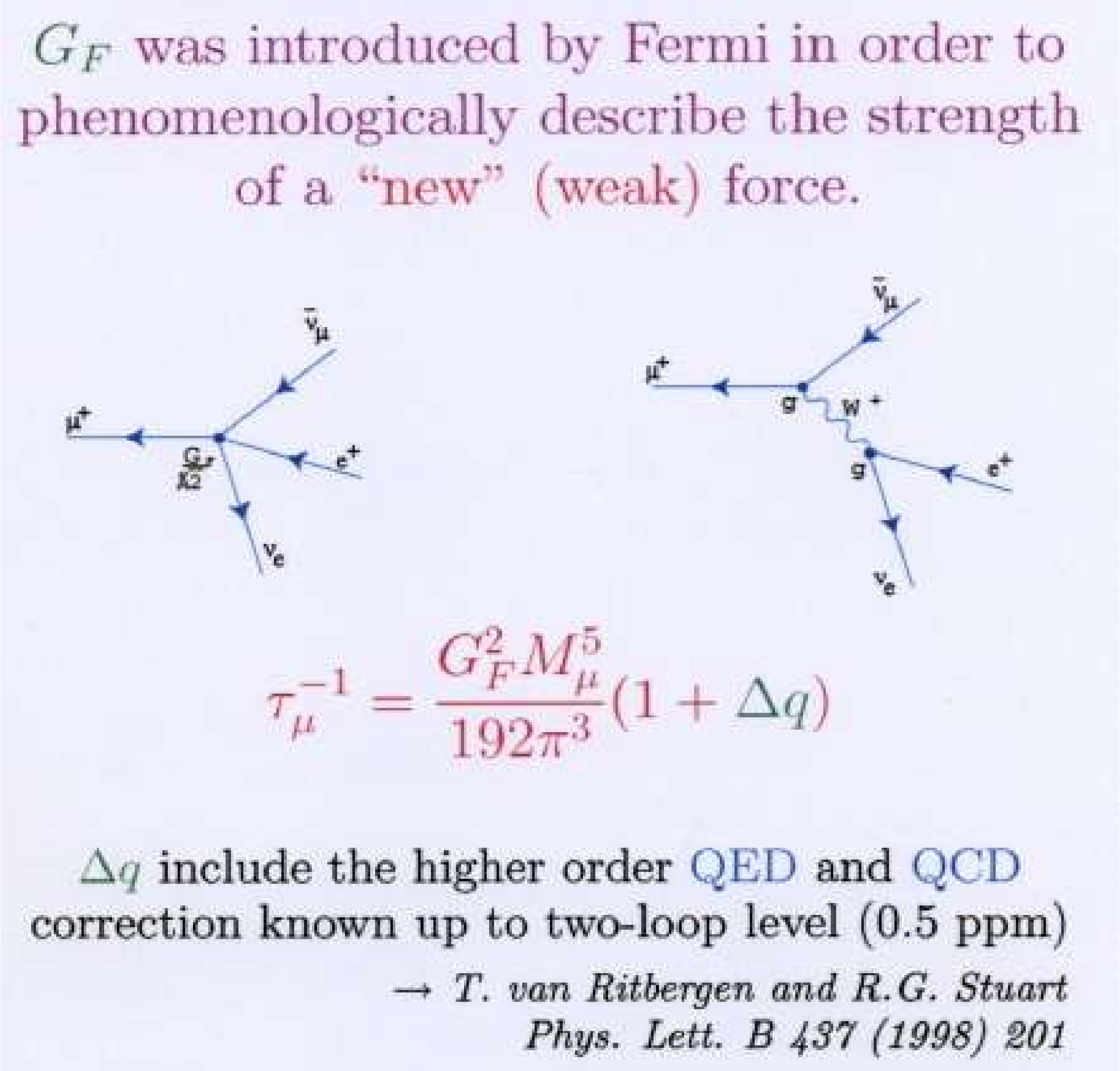
$$\mathcal{L}^{\mathsf{Fermi}} = \frac{G_F}{\sqrt{2}} \left(\bar{e}\gamma_\mu(1-\gamma_5)\nu\right) \left(\bar{\nu}\gamma^\mu(1-\gamma_5)e\right)$$

➜ entspicht SM-Grenzfall $q^2 \to 0$

$$\mathcal{L}^{\mathsf{SM}} = \frac{g^2}{8} \left(\bar{e}\gamma_\mu(1-\gamma_5)\nu\right) \frac{1}{m_W^2 - q^2} \left(\bar{\nu}\gamma^\mu(1-\gamma_5)e\right)$$

➜ Zusammenhang

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{Q_e^2}{8m_W^2 \sin^2\theta_w} = \frac{\alpha\pi}{2m_W^2 \sin^2\theta_w}$$



G_F was introduced by Fermi in order to phenomenologically describe the strength of a "new" (weak) force.

$$\tau_\mu^{-1} = \frac{G_F^2 M_\mu^5}{192\pi^3}(1 + \Delta q)$$

Δq include the higher order QED and QCD correction known up to two-loop level (0.5 ppm)

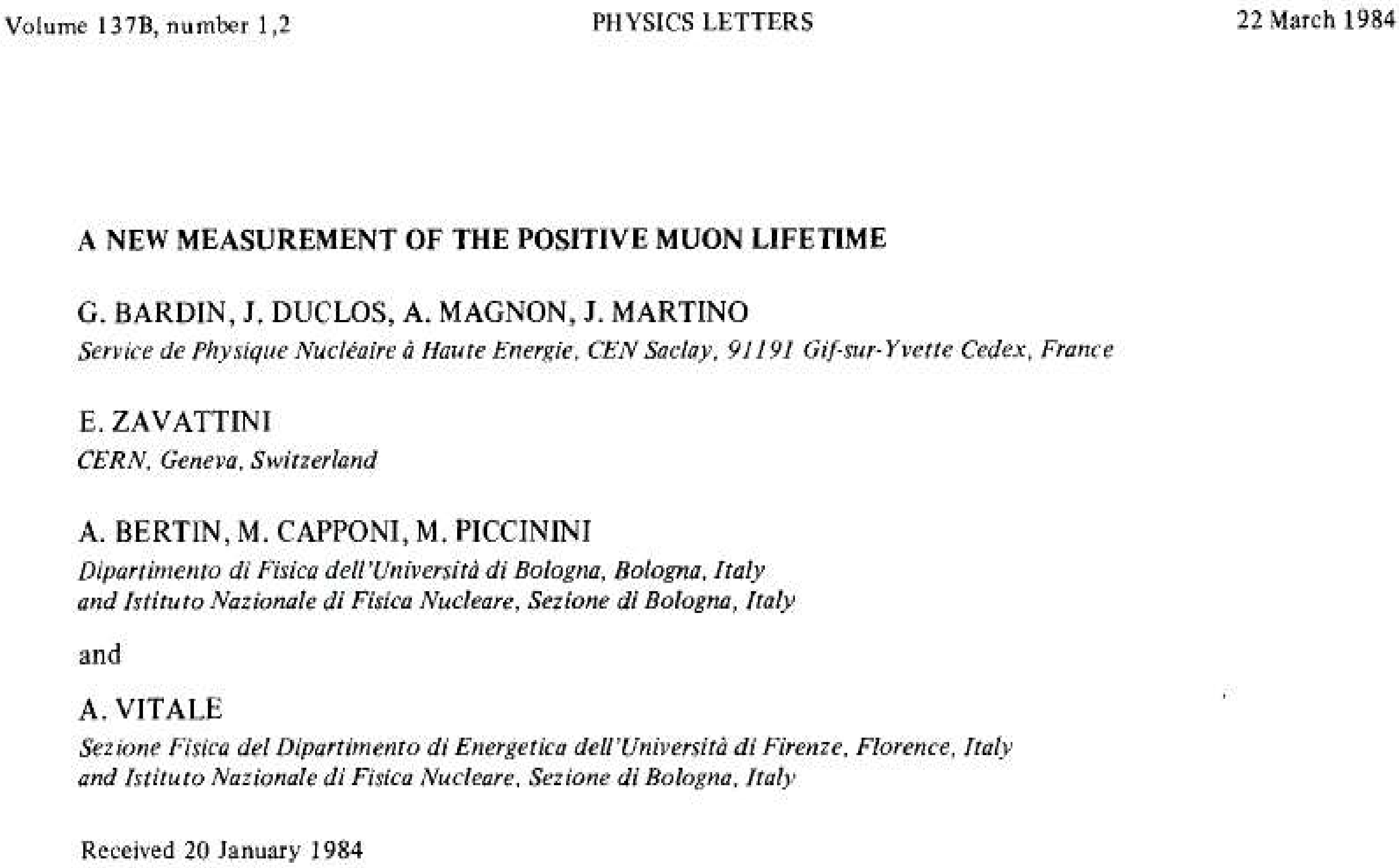
→ *T. van Ritbergen and R.G. Stuart*
Phys. Lett. B 437 (1998) 201
and Phys. Rev. Lett. 82 (1999) 488

1-loop QED contributions calculated already 40 years ago by Kinoshita, Sirlin, Berman.

presently data from 2 experiments with results published in 1984

generally use decay of secondary pion beam into muons and observe their decay into electrons + ...

1.



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A NEW MEASUREMENT OF THE POSITIVE MUON LIFETIME

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Received 20 January 1984

- 140 MeV pion beam from Saclay Linac, pulsed beam
- stop pion beam in S-target during 3 microsecond beam burst
- target designed such that most decay muons stop in target before decay
- measure time distribution of decay positrons detected in 6 telescopes of plastic scintillator covering 75% of total solid angle in 65 microsecond gate after start signal

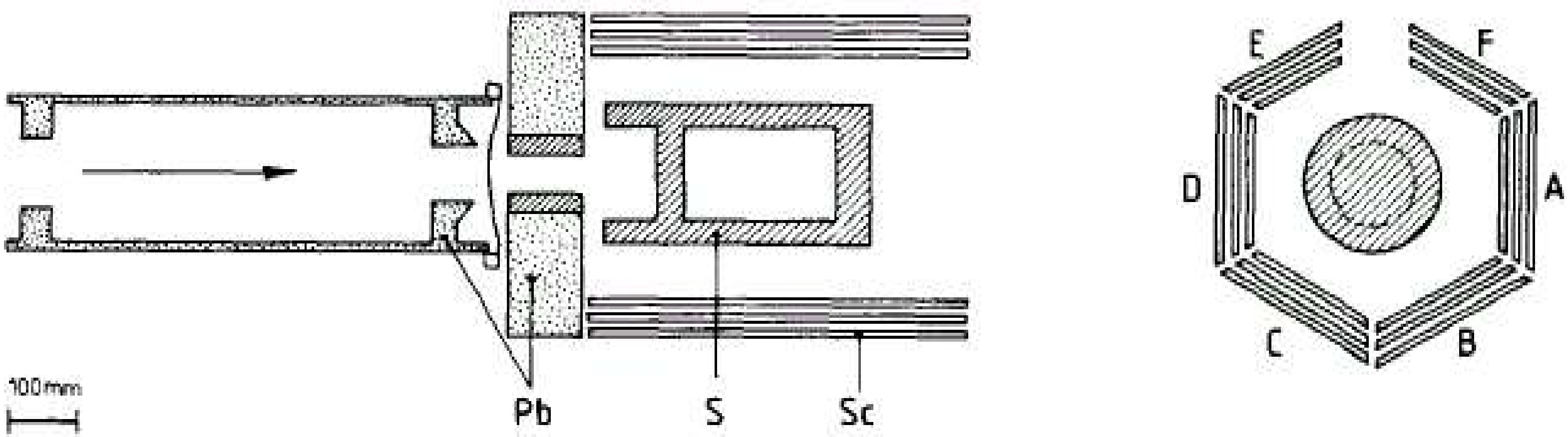


Fig. 1. Simplified scheme of the experimental set-up: Pb = lead collimator, S = sulphur target, Sc = plastic scintillator telescopes, placed at the positions A, B, C, D, E and F around the target.

difficulty: polarization of muons at decay;
stopped pions produce unpolarized muons, but beam contains fraction (5%) of polarized muons from decay in flight.
Sulfur depolarizes muons effectively, shape optimized for stopping high energy beam muons

why is polarization disturbing?
spin precession in magnetic field, rate of decay electrons depends on angle and is time modulated (used in g-2 experiment to measure muon magnetic moment)
experiment has to minimize residual magnetic field (mainly terrestrial), Helmholtz coils
careful measurement of polarization effects

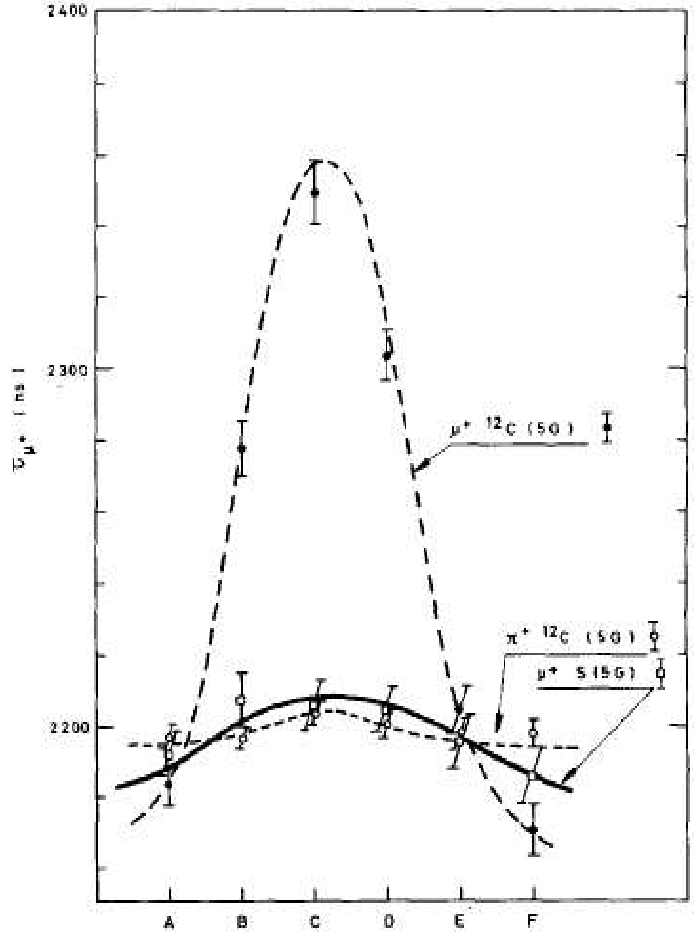
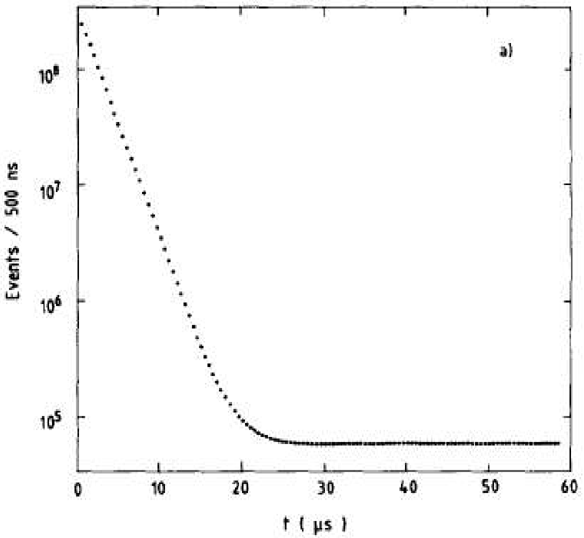
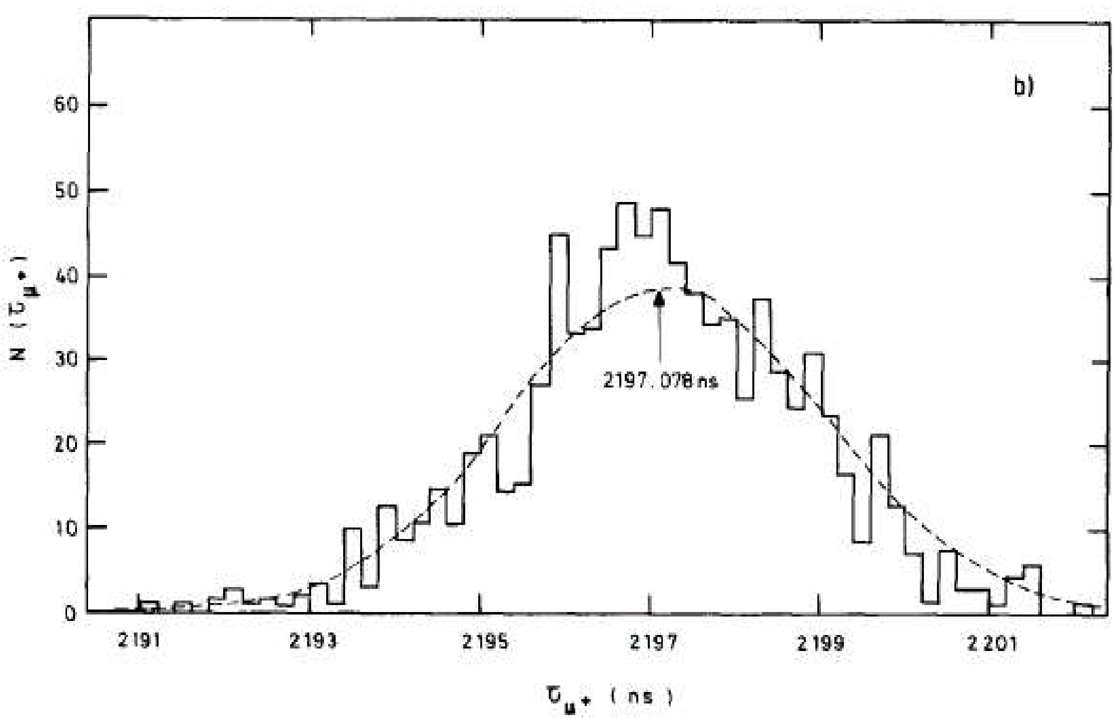


Fig. 2. Display of the observed values of τ_{μ^+} as a function of the telescope position (see for reference fig. 1) around the target. The experimental points correspond to μ^+ or π^+ beams, stopping in carbon (^{12}C) or sulphur (S) target, and to a 5 G magnetic field.



time distribution measured in a telescope
total time about 5 days, several runs per day, typically
a few million decays per run measured

distribution of resulting
muon lifetimes from all
runs and telescopes



typically run at very low beam rate such that only 0.1 electrons per gate
otherwise need to correct for fact that finite time resolution of electronics may not detect 2 decays both
control: high and low rate measurements and correction toward zero rate

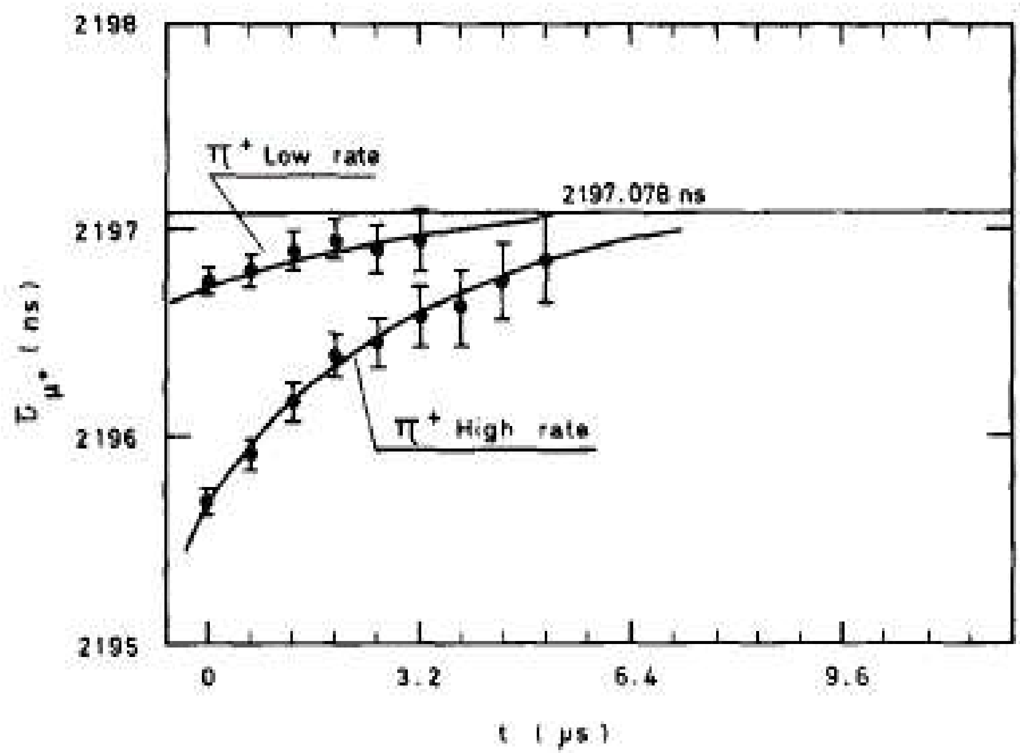


Fig. 4. Rate dependence of the observed values of τ_{μ^+}, as obtained by analysing the data starting from different initial times t within the measuring gate.

PHYSICAL REVIEW D

PARTICLES AND FIELDS

THIRD SERIES, VOLUME 29, NUMBER 3 **1 FEBRUARY 1984**

Mean life of the positive muon

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W. A. Orance,†† W. C. Phillips, R. T. Siegel, W. F. Vulcan, R. E. W
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(Received 9 September 1983)

TRIUMF/Canada 500 MeV proton cyclotron
generate secondary pions of 150-170 MeV
stop in water Cherenkov counter
measure time distribution of positrons
from decay pi+ -> mu+ -> e+

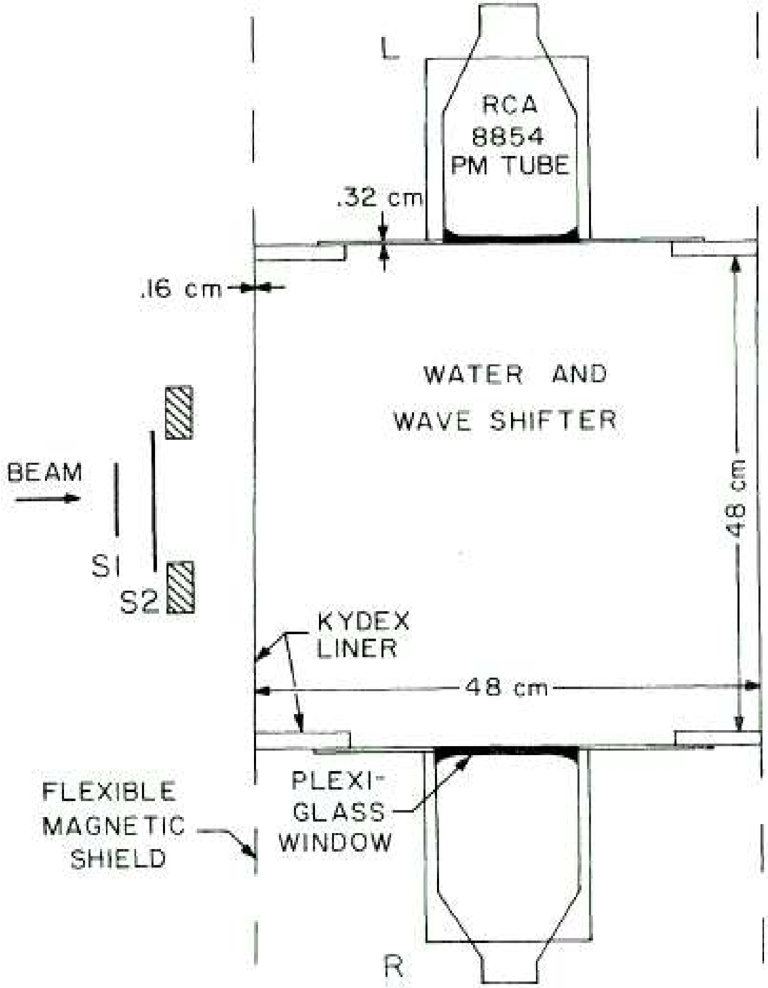
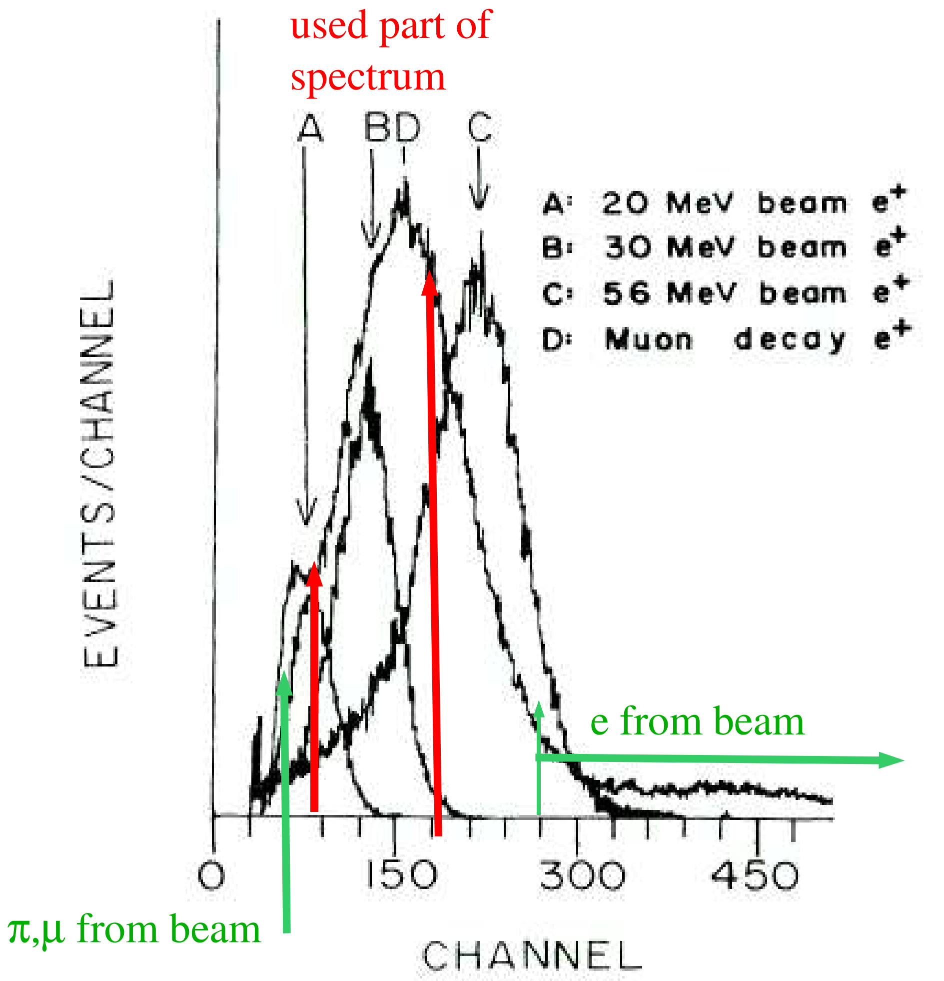


FIG. 1. Target and detector arrangement, top view. Entering beam particles were detected by scintillation counters *S*1 and *S*2. In the water, muon decay positrons caused Cherenkov radiation. This radiation was wavelength shifted by dissolved Na-*G* amino acid and then detected by the phototubes *R* and *L*.

still some muon and electron contamination from beam
but muons make signals of low pulse height
beam electrons of high pulseheigt
select a window

again difficulty: e+ from beam muons polarized, spin precession even in very low B-field


used part of spectrum

e from beam

π,μ from beam

FIG. 2. Energy calibration of the water Cherenkov counter. Curves A, B, and C were obtained with monoenergetic e^+ beams. Curve D was obtained with decay e^+ from stopped muons, and includes a high-energy contribution from direct beam e^+.

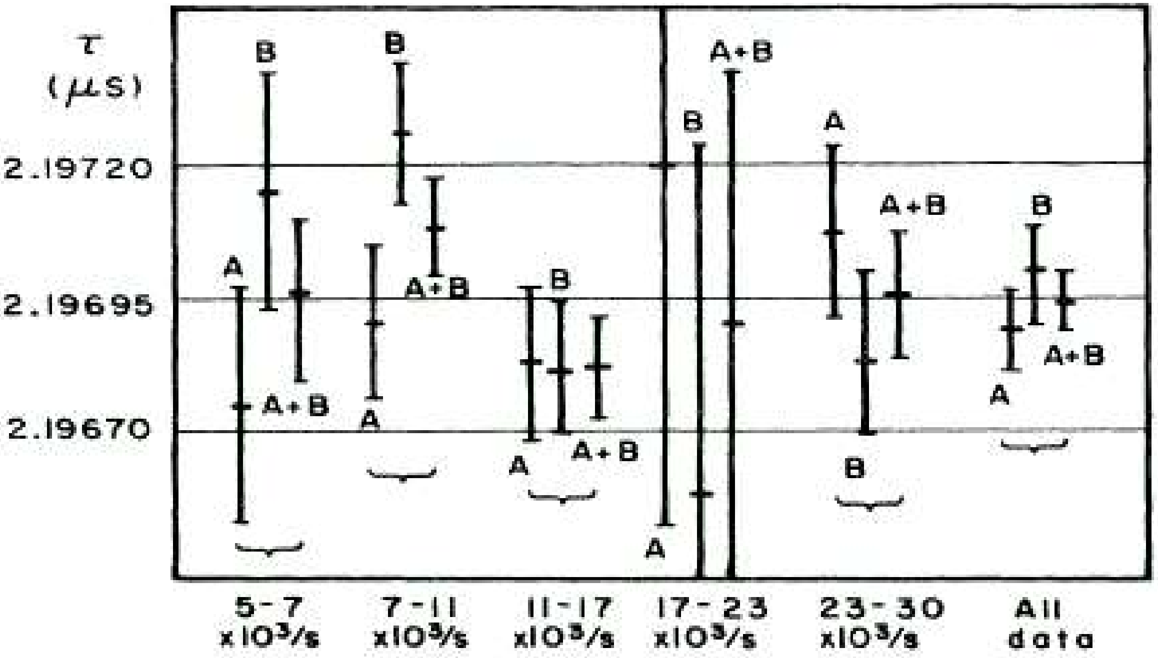


FIG. 4. Values of τ and their standard deviations determined from data obtained with beam intensities in five different ranges, and from all of the data. For each, there are shown separately the results from timing channel A, from timing channel B, and from the combination A + B.

PDG average of all results so far :

$\tau_\mu = 2.19703 \pm 0.00004\,\mu s$

with $m_W = 80.9$ GeV

$m_\mu = 105.65916 \pm 0.00030$ MeV

$m_e = 0.5110034 \pm 0.0000014$ MeV

and $O(\alpha)$ electroweak corrections (contain in computing $\alpha(m_\mu)$

$\rightarrow$ $\underline{G_F = 1.16637 \pm 0.00002 \cdot 10^{-5}\ \mathrm{GeV}^{-2}}$

17 ppm

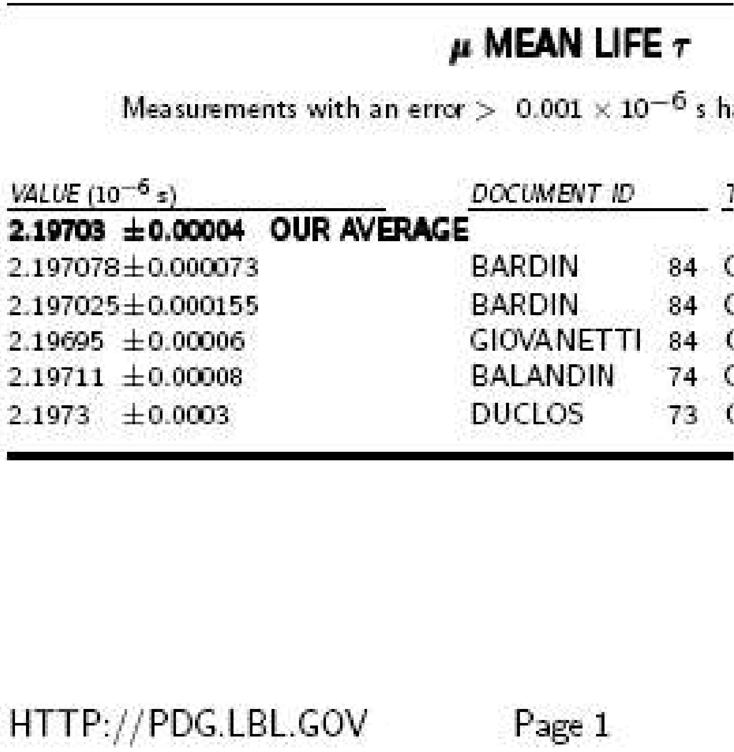
W. J. Marciano, A. Sirlin, Phys. Rev. Lett. 61 (1988) 1815

complete 2-loop QED contribution to τ_μ

T. van Ritbergen, R. Stuart, Phys. Rev. Lett. 82 (1999) 488

... $\rightarrow$ $1.16637 \pm 0.00001 \cdot 10^{-5}\ \mathrm{GeV}^{-2}$

+ hadronic & τ-loop corrections



μ MEAN LIFE τ

Measurements with an error > 0.001 × 10^{-6} s h

VALUE (10^{-6} s)	*DOCUMENT ID*	
2.19703 ±0.00004 OUR AVERAGE		
2.197078±0.000073	BARDIN	84
2.197025±0.000155	BARDIN	84
2.19695 ±0.00006	GIOVANETTI	84
2.19711 ±0.00008	BALANDIN	74
2.1973 ±0.0003	DUCLOS	73

HTTP://PDG.LBL.GOV Page 1

Where the uncertainty on G_F comes from?

$$\frac{\delta G_F}{G_F} = -\frac{5}{2}\frac{\delta m_\mu}{m_\mu} - \frac{1}{2}\frac{\delta \tau_\mu}{\tau_\mu} + \text{th.}\left(+4\frac{m_\nu^2}{m_\mu^2}\right)$$

$\frac{5}{2}\frac{\delta m_\mu}{m_\mu} = 0.38$ ppm *PDG2000*

theory $= 0.50$ ppm *Ritbergen and Stuart*

$4\frac{m_\nu^2}{m_\mu^2} = 10$ ppm *PDG2000*

(if you assume neutrinos to be massive)

$\frac{\delta \tau_\mu}{\tau_\mu} = 18$ ppm *PDG2000*

Dominant contributions from:

Balandin et al. *(1974)*
Bardin et al. *(1984)*
Giovanetti et al. *(1984)*

New efforts to improve muon lifetime (since it completely dominates error in G_F)

2 efforts at PSI (Switzerland)

MuLan Experiment *PSI Experiment R–99-07*

pulsed muon beam and fast segmented timing detector
180 triangular scint. tiles on sphere

FAST

R-99-06,

Fibre Active Scintillator Target

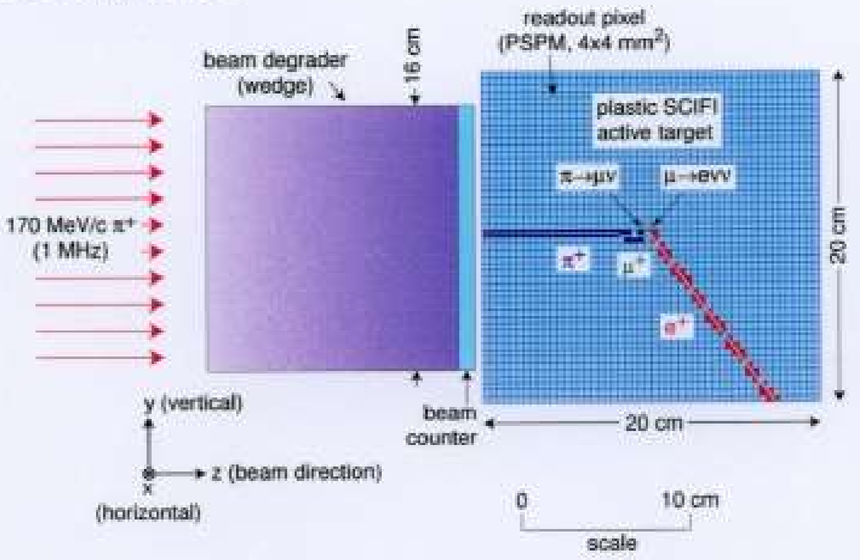
both proposed in 1999, aim to take 10^{12} decays to measure τ over 9 lifetimes in 1-2 months final data taking

both aim for 1 ppm accuracy in muon lifetime (vs. 18 presently)
data taking going on, no final results released yet
error presently down to half of the previous, but still way to go

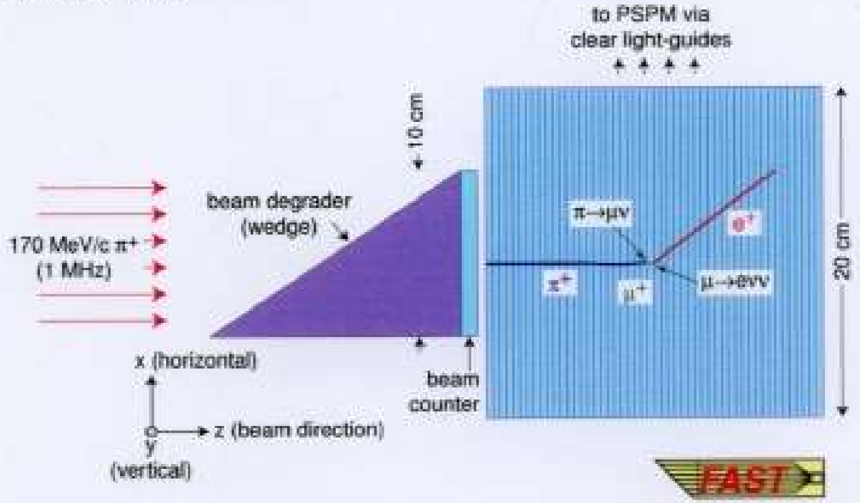
Concept:

A scintillating fiber detector, on a DC pion beam, able to handle up to 30 $\pi \rightarrow \mu \rightarrow$ e decay chains in a 30 μs window:

a) yz view (elevation)



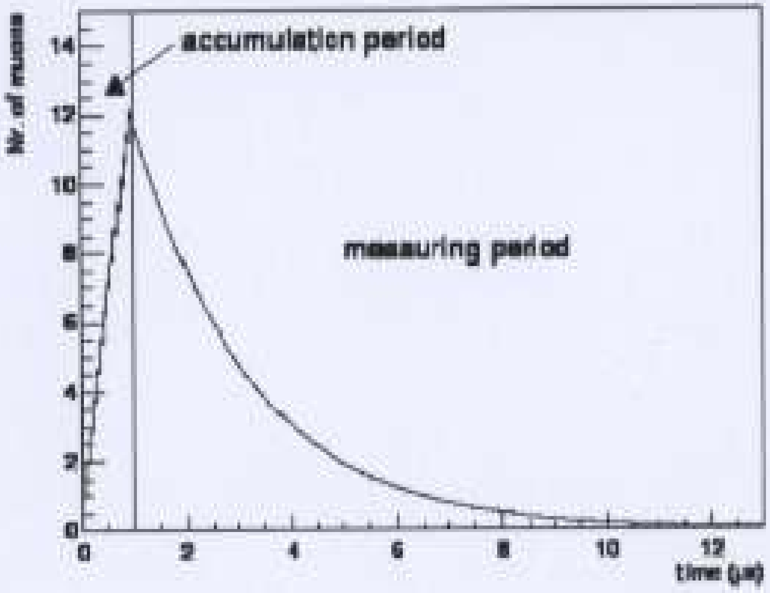
b) xz view (plan)



μLAN: `http://www.npl.uiuc.edu/exp/mulan/muLanMain.html`

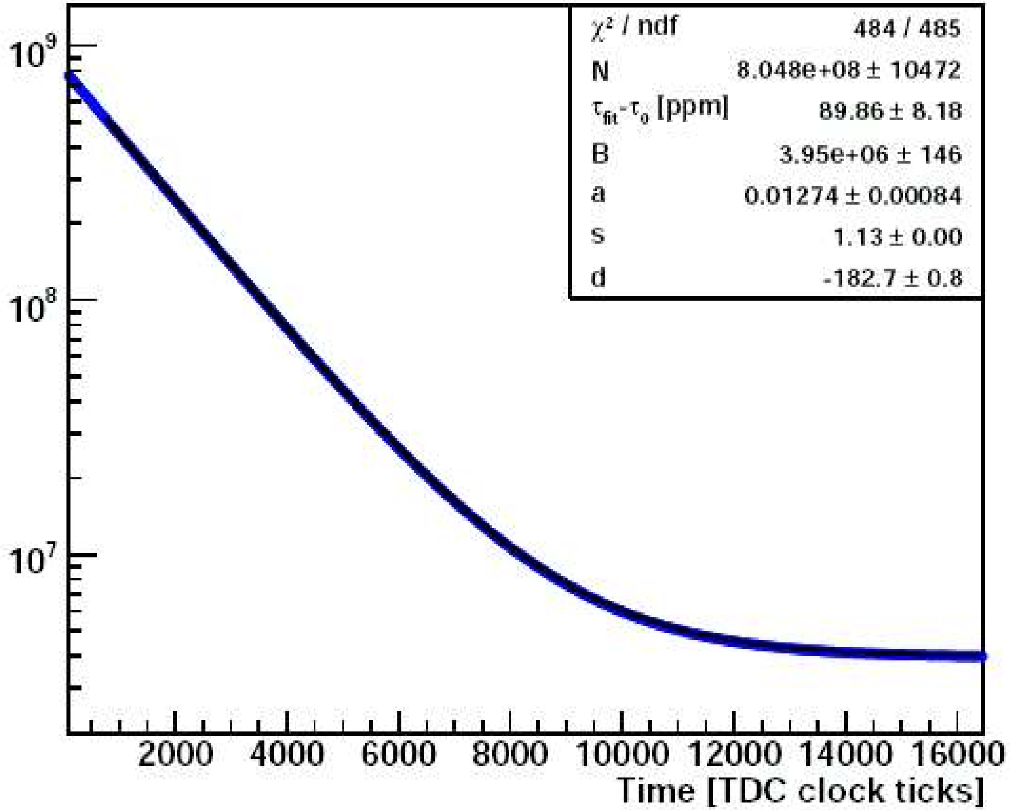
A structured muon beam (pulsed approach) is used to generate a radioactive muon source in a depolarazing sulfur target.

180 triangular timing tiles are distributed uniformly within the 20 SuperTriangles of an icosahedral geometry centered on the target.

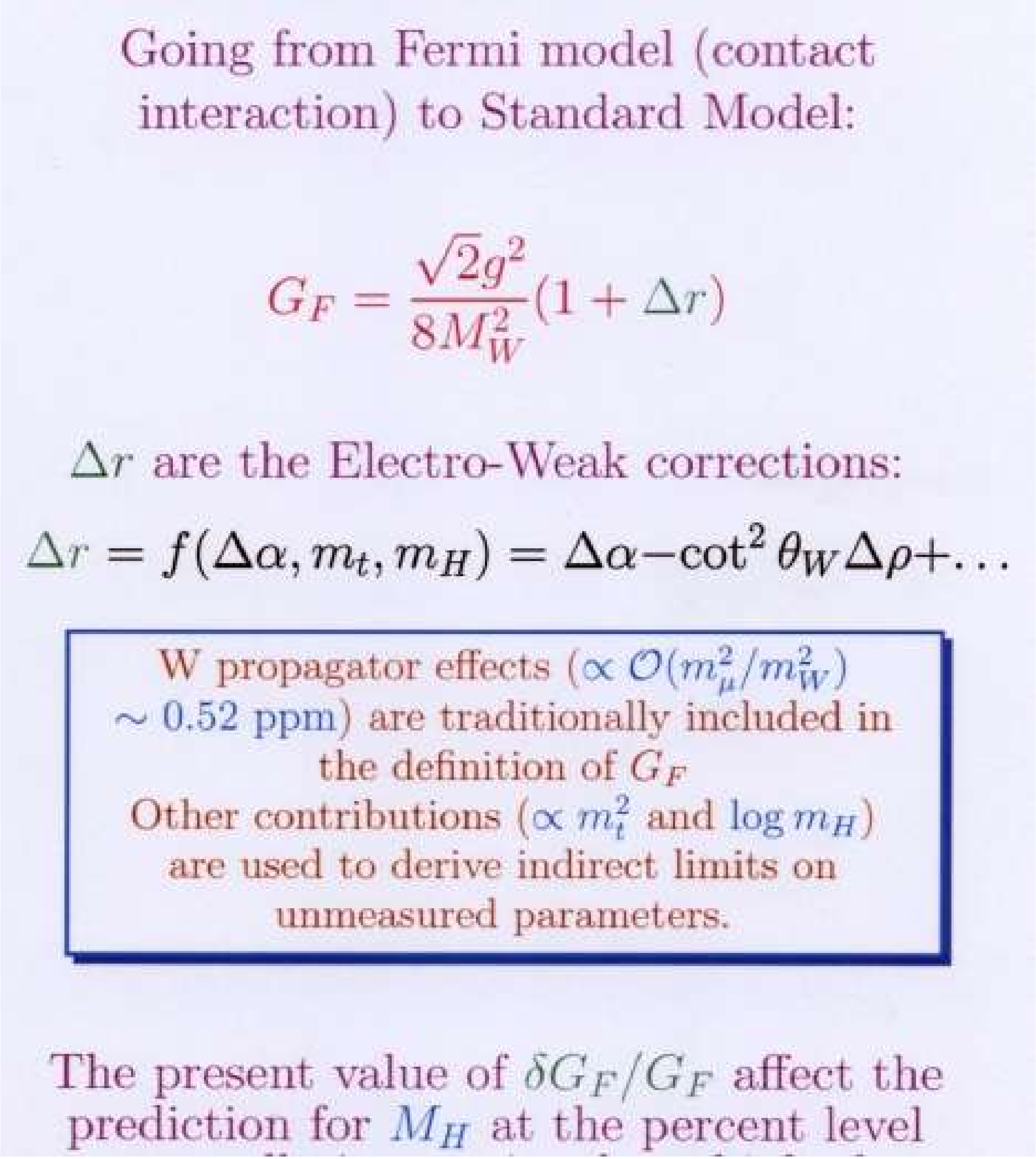


Given the different approaches (pion DC vs pulsed muon beam) FAST and μLAN will have complementary systematics sources: good cross-check!

2004 Analysis



Full statistics lifetime histogram with fit. The fit parameter $\tau_{\mathrm{fit}} - \tau_0$ shows the statistical precision of this data set to be 8.2 ppm.



Going from Fermi model (contact interaction) to Standard Model:

$$G_F = \frac{\sqrt{2}g^2}{8M_W^2}(1+\Delta r)$$

Δr are the Electro-Weak corrections:

$$\Delta r = f(\Delta\alpha, m_t, m_H) = \Delta\alpha - \cot^2\theta_W \Delta\rho + \ldots$$

W propagator effects ($\propto \mathcal{O}(m_\mu^2/m_W^2)$ ~ 0.52 ppm) are traditionally included in the definition of G_F
Other contributions ($\propto m_t^2$ and $\log m_H$) are used to derive indirect limits on unmeasured parameters.

The present value of $\delta G_F/G_F$ affect the prediction for M_H at the percent level