Exercises for

Advanced Quantum Theory

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Exercise 1

In the lecture you learned that the spin operator $S_i = \frac{1}{2}\sigma_i$, where σ_i are the Pauli matrices (1.11), yield a two-dimensional representation of the algebra $[L_j, L_k] = i\sum_l \epsilon_{j,k,l} L_l.$, $L_\pm = L_1 \pm iL_2$. Calculate S^2 explicitely and show that $S^2 = s(s+1)$ with $s = \frac{1}{2}$.

Now show that the matrices

$$\Sigma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Sigma_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\Sigma_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

obey the same algebra. They thus present another representation of $su(2)$. What is the corresponding spin s?

Exercise 2

Take the three-dimensional representation Σ_i of the $su(2)$ algebra from exercise 1. The general representation for the time reversal operator is $T = UK$ where K is complex conjugation and U is an arbitrary unitary operator. Use $T\Sigma T^{-1} = -\Sigma$ to explicitly construct U for the three-dimensional representation of the $su(2)$ algebra from exercise 1.