Exercises for

Advanced Quantum Theory

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Exercise 1

Let a_j, $a_j^\dagger$ be a complete set of bosonic annihilation and creation operators. And let $\mathcal{F}$ be the so called Fock space, the union of all Hilbert spaces with arbitrary number of particles.

1. $|\psi\rangle = \exp(\sum_i z_i a_i^\dagger)|\text{vac.}\rangle$ is an element of $\mathcal{F}$. What is $\langle\psi|\psi\rangle$?

2. Calculate $a_j \exp(\sum_i z_i a_i^\dagger)|\text{vac.}\rangle$.

3. Does $a_j^\dagger$ have eigenstates in $\mathcal{F}$?

4. Is it possible to construct eigenstates of the annihilation operator for Fermions? Which algebraic properties would the eigenvalues have if that was possible?

Exercise 2

Show (for fermions and bosons) that the operator that counts the number of pairs in the two states $|i\rangle$, $|j\rangle$ is given by

$$P_{ij} = a_i^\dagger a_j^\dagger a_j a_i$$

What is the operator that counts the number of n-tuples in the states $|i_1\rangle$, $|i_2\rangle, \ldots, |i_n\rangle$?

How would a Hamiltonian look like, that contains a kinetic energy and a purely local, repulsive interaction?