Exercises for

Advanced Quantum Theory

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Exercise 1

Consider the Hamiltonian

$$H(t) = -\frac{\Delta}{2}\sigma_x + h(t)\sigma_z$$

with $h(t) = h_0\delta(t)$.

Calculate the exact unitary operator

$$U(t, t_i) = T\left[\exp\left(-i\int_{t_i}^{t} dt' H(t')\right)\right] \tag{1}$$

which describes the time evolution from an initial time $t_i < 0$ to time t in the form $U(t, t_i) = a(t, t_i) + b(t, t_i)\sigma_x + c(t, t_i)\sigma_y + d(t, t_i)\sigma_z$. Look at the script, Section 3.1, Eqs. (3.1) to (3.10) for the precise definition of the time ordered exponential in (1).

Note that for $t > 0$, you get a product of three exponentials. You can decompose the exponentials into a cos and sin. Those can then be multiplied using the algebra of the Pauli matrices.

Exercise 2

Do the same calculation as in exercise 1 for $h(t) = h_0$ for $t < 0$, . $h(t) = 0$ for $t > 0$.

Note that for $t > 0$ it is a product of two exponentials. Note that for the first factor (for $t < 0$) this cannot be dicrectly decomposed into a cos and sin because σ_x and σ_y do not commute. Here you must transform the Hamiltonian to a diagonal form using a unitary transformation $U = \exp(i\beta\sigma_y)$ with a suitable β, so that you end up with a product of four exponentials, each of which can be decomposed into cos and sin.