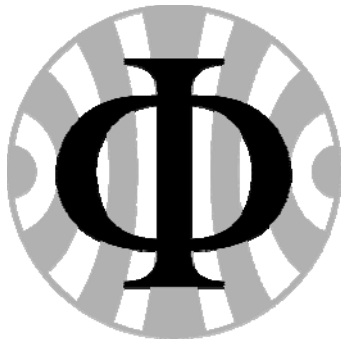


Evaporative Cooling

Marc Repp
AG Weidemüller



Quantum dynamics of atomic and molecular systems
Ruprecht-Karls-University Heidelberg
Physics Institute

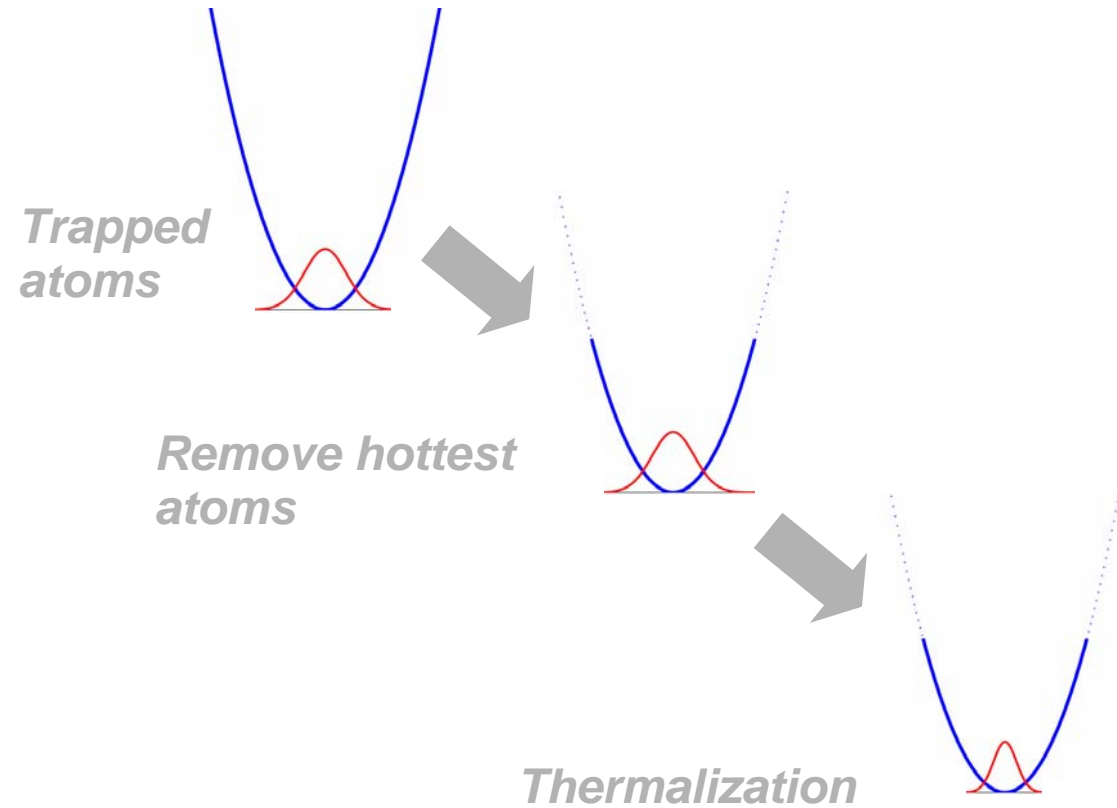
What is evaporative cooling?



In the Office:



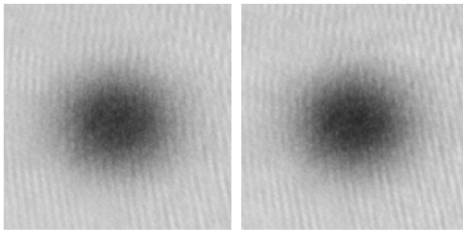
In the Lab:



Evaporation in the Lab

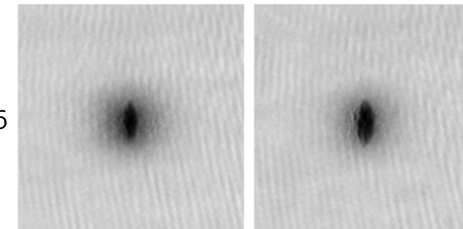


$\epsilon_t = 10.2\mu\text{K}$
 $T = 1.8\mu\text{K}$
 $N = 1.9 \times 10^7$



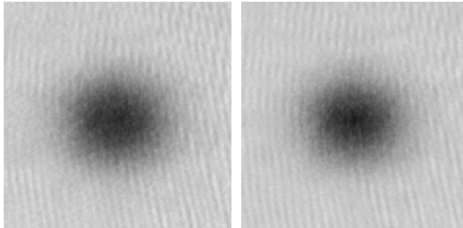
$\epsilon_t = 9.8\mu\text{K}$
 $T = 1.7\mu\text{K}$
 $N = 1.9 \times 10^7$

$\epsilon_t = 5.1\mu\text{K}$
 $T = 820\text{nK}$
 $N = 9.9 \times 10^6$



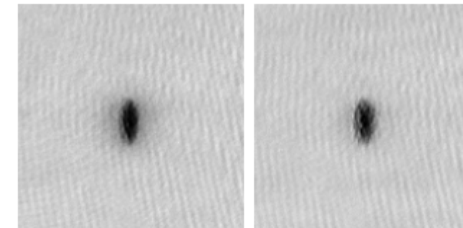
$\epsilon_t = 4.1\mu\text{K}$
 $T = 630\text{nK}$
 $N = 5.9 \times 10^6$

$\epsilon_t = 8.9\mu\text{K}$
 $T = 1.6\mu\text{K}$
 $N = 1.7 \times 10^7$



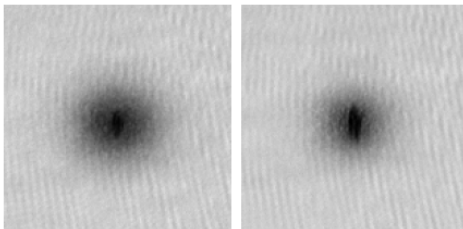
$\epsilon_t = 7.9\mu\text{K}$
 $T = 1.5\mu\text{K}$
 $N = 1.5 \times 10^7$

$\epsilon_t = 3.1\mu\text{K}$
 $T = 530\text{nK}$
 $N = 4 \times 10^6$



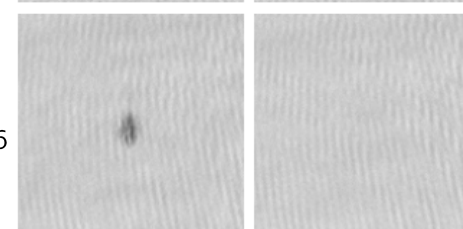
$\epsilon_t = 2.2\mu\text{K}$
 $T < 500\text{nK}$
 $N = 1.7 \times 10^6$

$\epsilon_t = 6.9\mu\text{K}$
 $T = 1.3\mu\text{K}$
 $N = 1.3 \times 10^7$



$\epsilon_t = 5.6\mu\text{K}$
 $T = 1.1\mu\text{K}$
 $N = 9 \times 10^6$

$\epsilon_t = 0.9\mu\text{K}$
 $T < 500\text{nK}$
 $N = 1.1 \times 10^6$



$\epsilon_t = 0\mu\text{K}$
 $T < 0\text{nK}$
 $N = 0 \times 10^6$



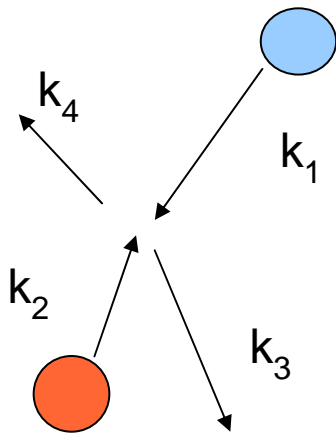
- **Thermalization of an ultracold gas**
- **Model of evaporation**
- **Efficiency of evaporative cooling**
 - Speed of evaporative cooling
 - Influence of loss processes
- **Conclusion**



- **Thermalization of an ultracold gas**
- Model of evaporation
- Efficiency of evaporative cooling
 - Speed of evaporative cooling
 - Influence of loss processes
- Conclusion



Elastic collisions in a dilute and cold gas



- **Dilute gas**
→ only binary collisions
- **Elastic collision of two particles**
→ Exchange of energy and momentum
- **S-wave cross-section and collision rate**

$$\sigma = \frac{8\pi a^2}{1+k^2 a^2}$$

$$\sigma = 8\pi a^2 \text{ for } k^2 a^2 \ll 1 \quad \text{Zero - energy limit}$$

$$\sigma = \frac{8\pi}{k^2} \text{ for } k^2 a^2 \gg 1 \quad \text{Unitarity limit}$$

$$\tau^{-1} = n\bar{v}_r\sigma$$

- **Thermal Equilibrium:**
→ Occupation number (for Bosons)

$$f(\vec{k}) = \left(\exp\left(\frac{E-\mu}{k_B T}\right) - 1 \right)^{-1}$$



Thermalization of an ultracold gas

Numerical simulation of the thermalization

- Starting with an unequilibrium

$$e.g. f(k) = f_0 \delta(k - k_0)$$

- Probability for bosons with wavevectors (k_1, k_2) to scatter to (k_3, k_4):

$$S(\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4) =$$

$$M^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \delta(E_1 + E_2 - E_3 - E_4)$$

$$\times f(\mathbf{k}_1) f(\mathbf{k}_2) [1 \pm f(\mathbf{k}_3)] [1 \pm f(\mathbf{k}_4)]$$

- Scattering rate into/ out of state with momentum k :

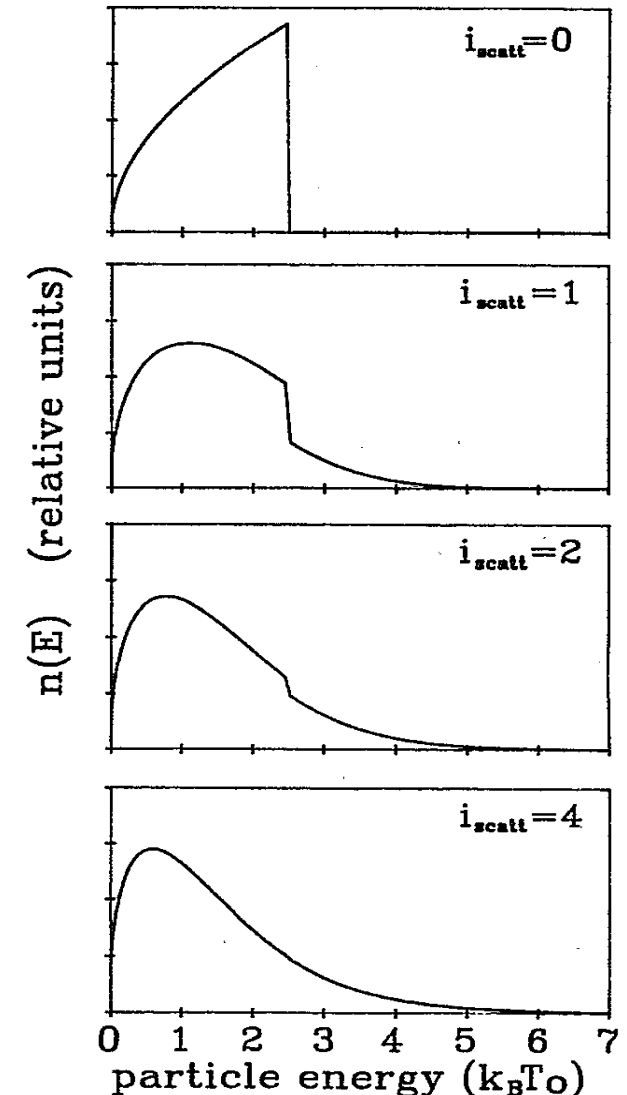
$$\Gamma_{in}(\mathbf{k}) = \int d^3k_1 d^3k_2 d^3k_3 S(\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k})$$

$$\Gamma_{out}(\mathbf{k}) = \int d^3k_1 d^3k_2 d^3k_3 S(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3)$$

- Equilibrium:

Only 4 collision events necessary

(harmonic quadrupole trap: 2.7 collisions)





Thermalization of an ultracold gas

Measurement of thermalization

Measurement principle

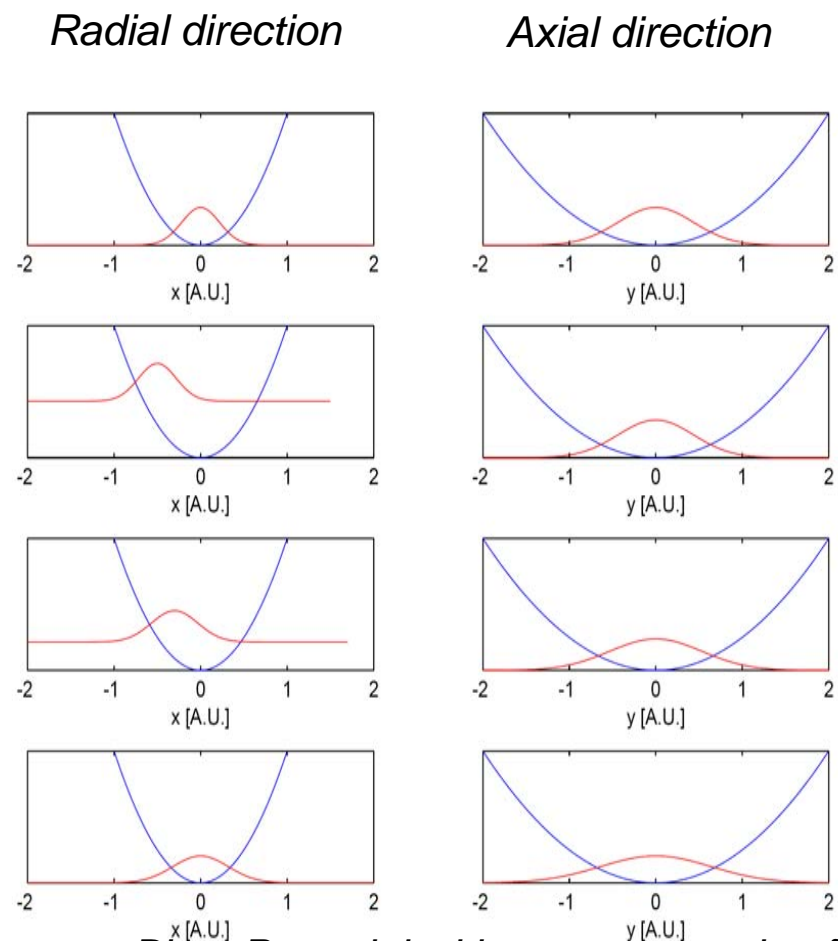
➤ Density of a thermalized sample in a anisotropic harmonic trap ($\sigma_x \neq \sigma_y$):

$$n = n_0 \exp \left[-\beta \left(\left(\frac{x}{2\sigma_x} \right)^2 + \left(\frac{y}{2\sigma_y} \right)^2 + \left(\frac{z}{2\sigma_z} \right)^2 \right) \right]$$

➤ Elongation of the cloud along one axis leads to oscillation along this axis

➤ Elastic collisions distribute the energy to all degrees of freedom

→ Thermalization when the aspect ratio reached his original value

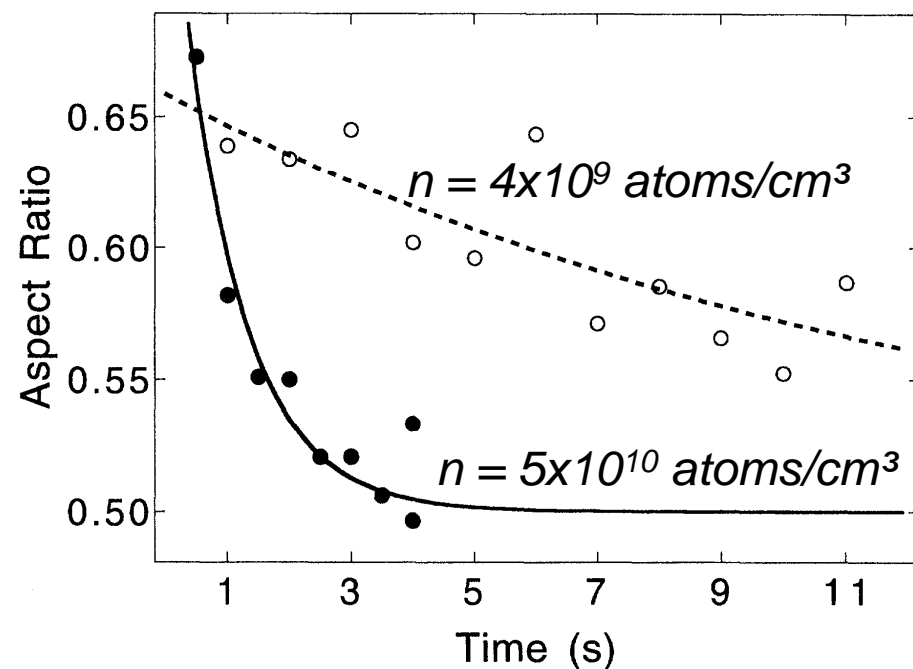


Blue: Potential with an aspect ratio of 2:1
Red: Density distributions along two axis



Thermalization of an ultracold gas

Thermalization of sodium atoms in a quadrupol trap with an aspect ratio of 2:1



Determinating the elastic cross section:

$$\tau = 1\text{s for } n = 5 \times 10^{10} \text{ atoms/cm}^3$$

$$\tau = 13\text{s for } n = 4 \times 10^9 \text{ atoms/cm}^3$$

$$n_{\text{eff}} \sigma v = 2.7 / \tau$$

$$\sigma = (6.0 \pm 3.0) \times 10^{-12} \text{cm}^2$$

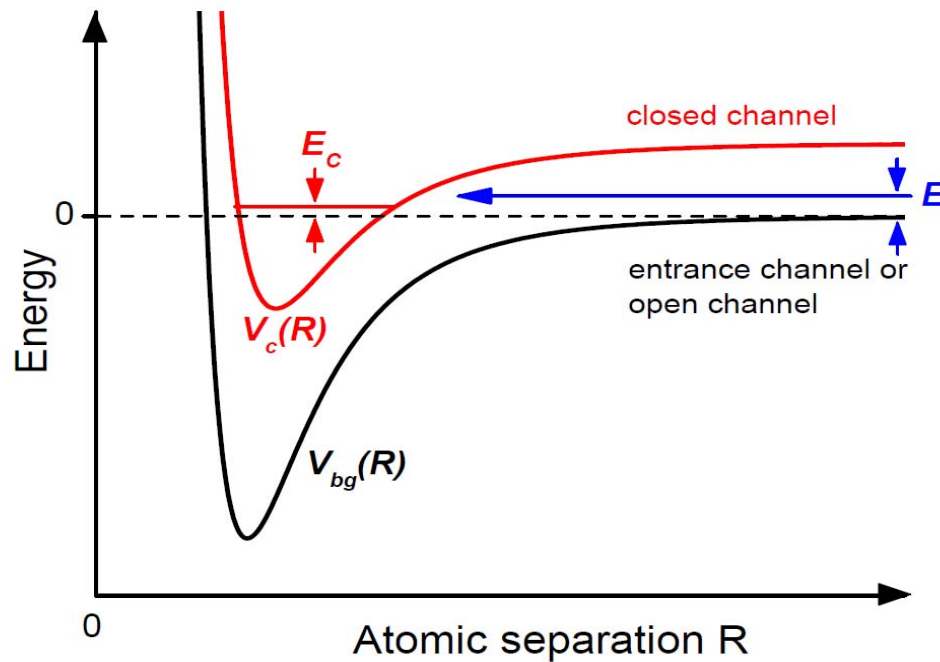
$$\sigma = 8\pi a^2$$

$$\Rightarrow a = \pm(92 \pm 25)a_0$$

Thermalization near a resonance

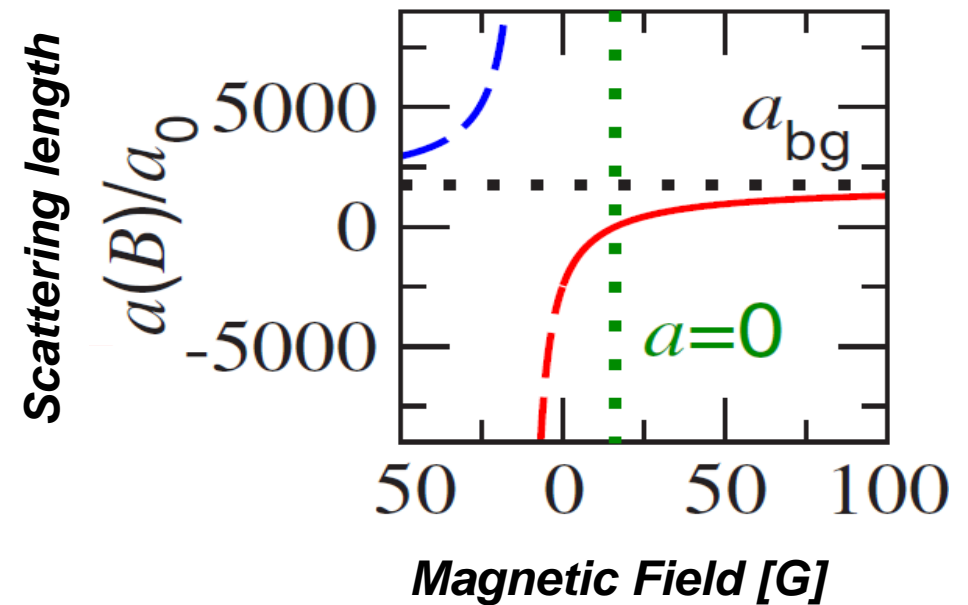


Feshbach -Resonances



C. Chin et al., arXiv 0812.1496v2

Zero-Energy resonance in ^{133}Cs in $F=3$ $m_F=+3$



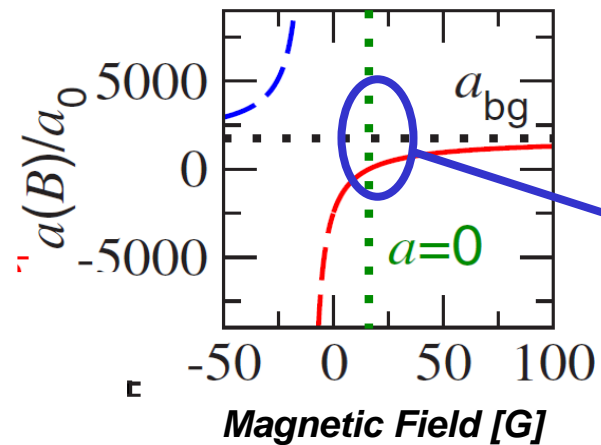
M. Lee et al., Phys. Rev. A **76**,12720 (2007)

$$a(B) = a_{bg} \left(1 - \frac{\Delta B}{B - B_0} \right)$$

Thermalization near a resonance

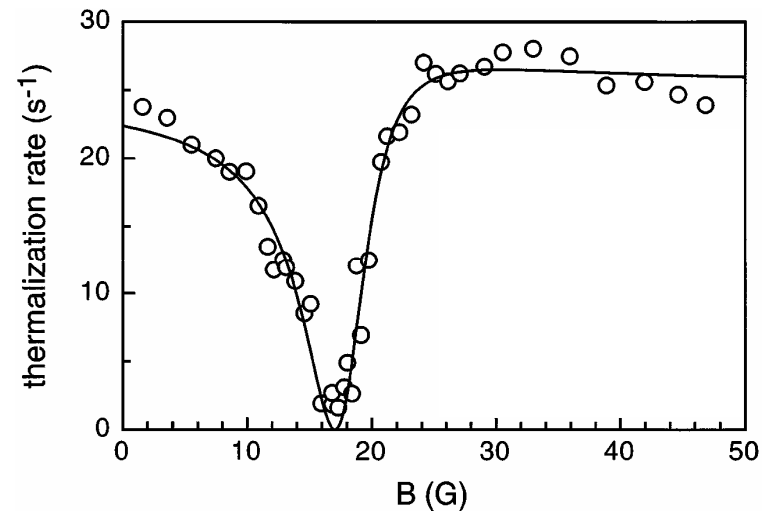


Thermalization at $a=0$



$$\sigma = 8\pi a^2$$
$$\Rightarrow \sigma = 0 \text{ for } a = 0$$

Thermalization rate for Cs near 17G



-> no thermalization at $a=0$

Thermalization near a resonance



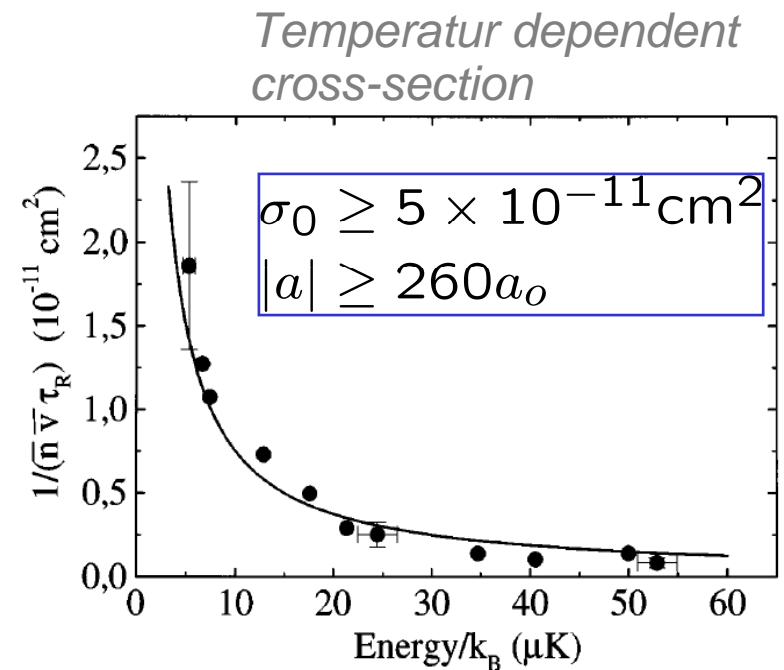
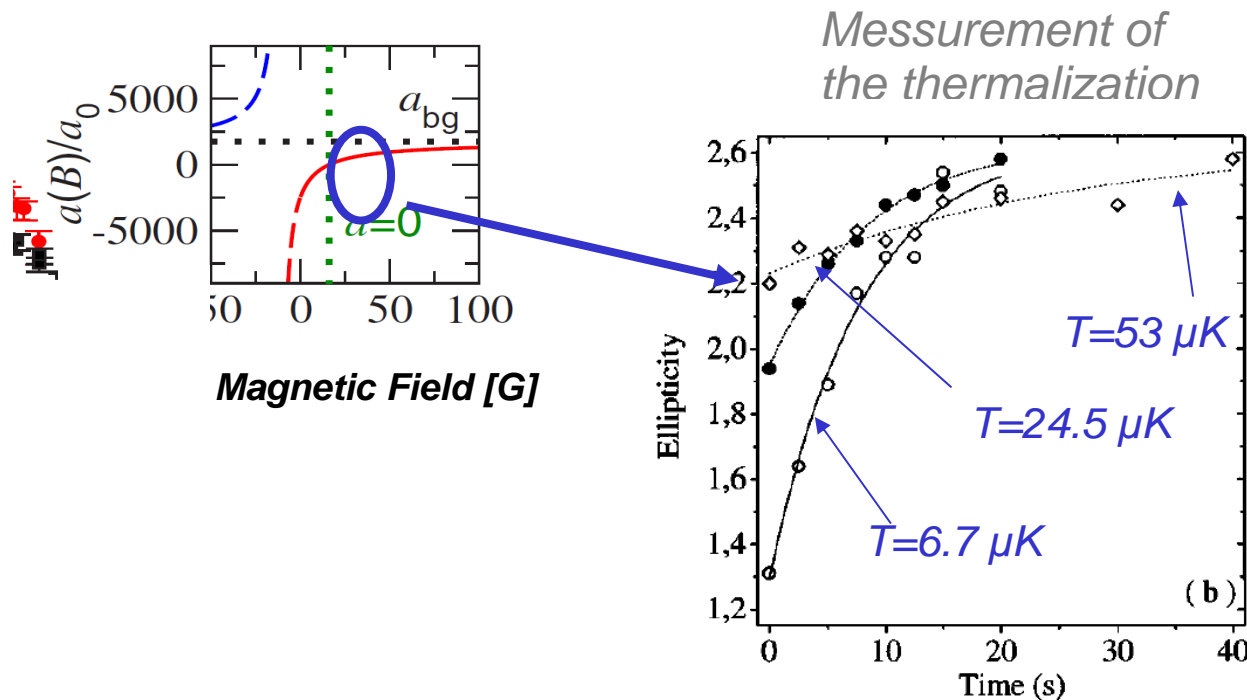
Thermalization in the unitarity limit

$$\sigma(k) = \frac{8\pi}{k^2}$$

$$k^{-1} = \lambda_{deB} = [4k_B T m / (\pi \hbar^2)]^{-1/2}$$

Collision rate now depends on the relative velocity: $\gamma_c = 128n\hbar^2/vM^2$

Monte-Carlo simulations: 10.7 collisions instead of 2.7 needed for thermalization



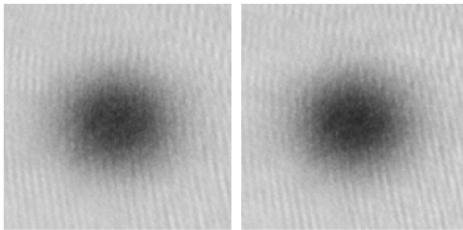


- Thermalization of an ultracold gas
- **Model of evaporation**
- Efficiency of evaporative cooling
 - Speed of evaporative cooling
 - Influence of loss processes
- Conclusion

Evaporation in the Lab

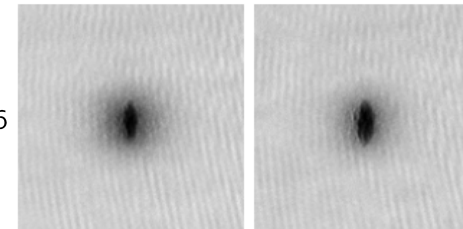


$\epsilon_t = 10.2\mu\text{K}$
 $T = 1.8\mu\text{K}$
 $N = 1.9 \times 10^7$



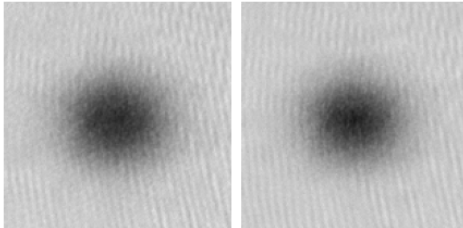
$\epsilon_t = 9.8\mu\text{K}$
 $T = 1.7\mu\text{K}$
 $N = 1.9 \times 10^7$

$\epsilon_t = 5.1\mu\text{K}$
 $T = 820\text{nK}$
 $N = 9.9 \times 10^6$



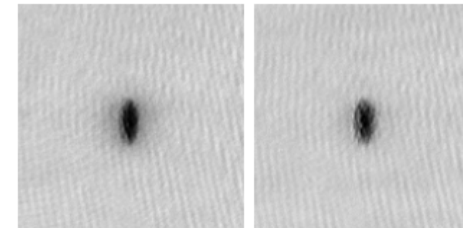
$\epsilon_t = 4.1\mu\text{K}$
 $T = 630\text{nK}$
 $N = 5.9 \times 10^6$

$\epsilon_t = 8.9\mu\text{K}$
 $T = 1.6\mu\text{K}$
 $N = 1.7 \times 10^7$



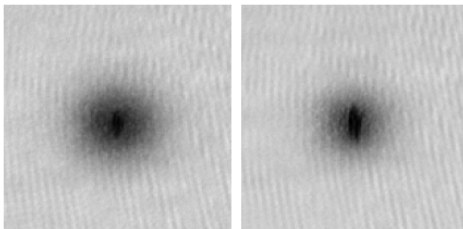
$\epsilon_t = 7.9\mu\text{K}$
 $T = 1.5\mu\text{K}$
 $N = 1.5 \times 10^7$

$\epsilon_t = 3.1\mu\text{K}$
 $T = 530\text{nK}$
 $N = 4 \times 10^6$



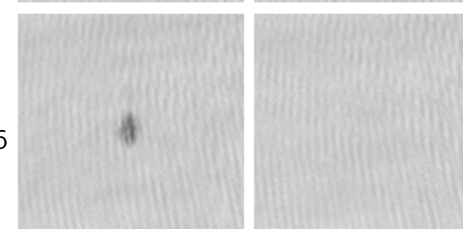
$\epsilon_t = 2.2\mu\text{K}$
 $T < 500\text{nK}$
 $N = 1.7 \times 10^6$

$\epsilon_t = 6.9\mu\text{K}$
 $T = 1.3\mu\text{K}$
 $N = 1.3 \times 10^7$



$\epsilon_t = 5.6\mu\text{K}$
 $T = 1.1\mu\text{K}$
 $N = 9 \times 10^6$

$\epsilon_t = 0.9\mu\text{K}$
 $T < 500\text{nK}$
 $N = 1.1 \times 10^6$

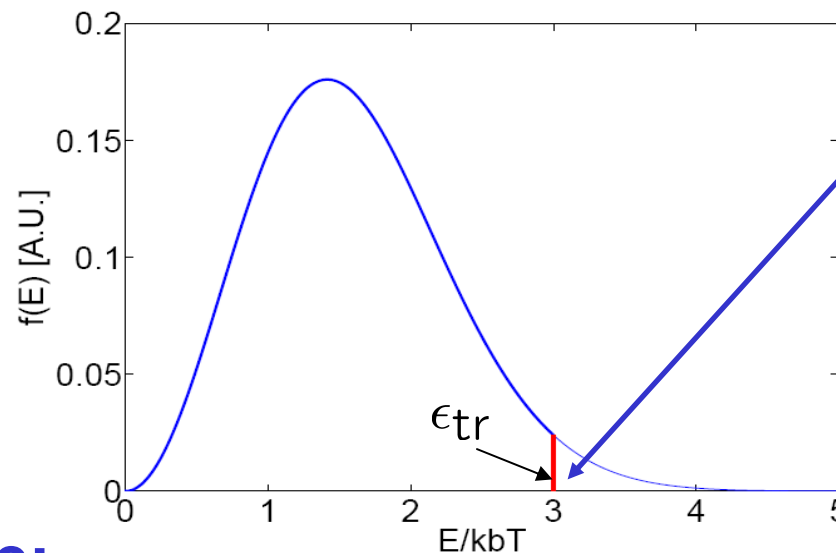


$\epsilon_t = 0\mu\text{K}$
 $T < 0\text{nK}$
 $N = 0 \times 10^6$



Model of evaporative cooling

Truncated Boltzman distribution



$$\eta = \frac{\epsilon_{tr}}{k_B T}$$

Truncation parameter

$$\eta = \frac{\epsilon_{tr}}{k_B T}$$

Evaporation rate:

➤ Removing all particles with an energy $E > \epsilon$ $\Gamma_{ev} = \frac{N_{untruncated}(E > \epsilon_{tr})}{\tau_{el}}$

➤ For large η : fraction of atoms with $E > \epsilon$ $\frac{N(E > \epsilon_{tr})}{N} = 2e^{-\eta} \sqrt{\eta/\pi}$

➤ Rate of evaporation: $\dot{N} = -N n_0 \sigma v \eta e^{-\eta} = \frac{-N}{\tau_{ev}}$

➤ Ratio of elastic collisions and time constant for evaporation: $\lambda = \frac{\sqrt{2}e^{\eta}}{\eta}$



Model of the evaporation

Temperature change:

Remove atoms with potential energy

$$\epsilon_{tr} = \eta k_B T$$

The system loses the energy

$$\dot{E} = (\eta + 1) \dot{N} k_B T$$

Total Energy of the system:

$$E = 3Nk_B T$$

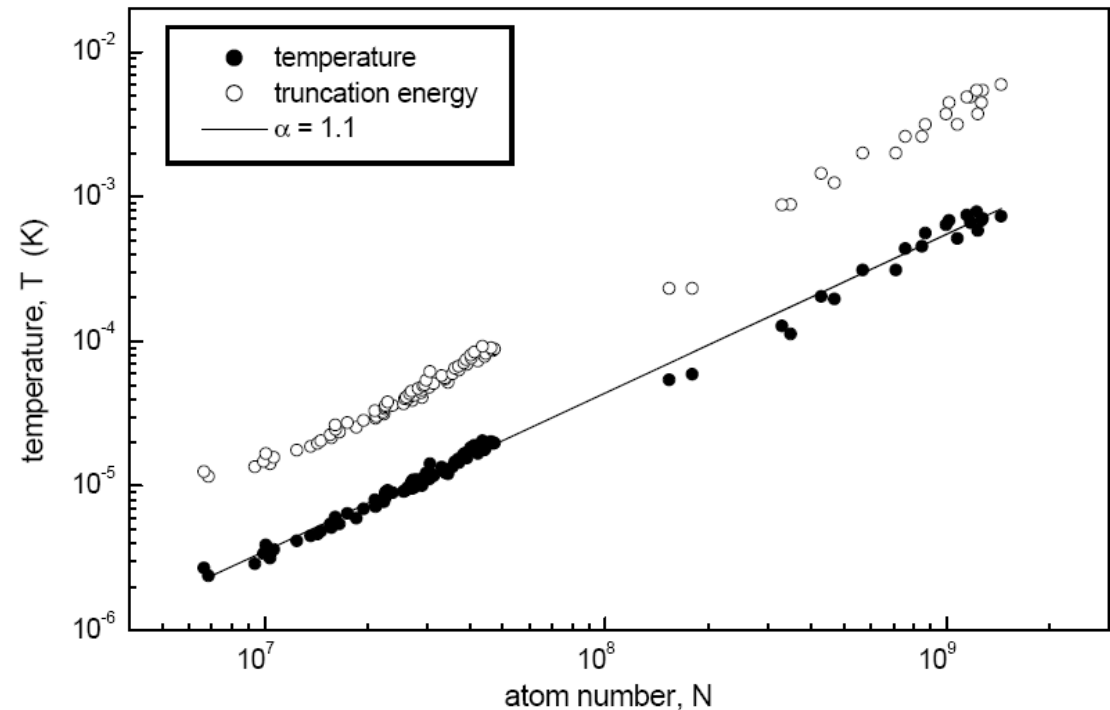
$$\Rightarrow \dot{E} = 3\dot{N}k_B T + 3Nk_B \dot{T}$$

$$\Rightarrow \frac{\dot{T}}{T} = \frac{1}{3}(\eta - 2) \frac{\dot{N}}{N}$$

$$\Rightarrow \frac{\dot{T}}{T_0} = \left(\frac{\dot{N}}{N_0} \right)^\alpha;$$

$$\alpha = \frac{1}{3}(\eta - 2)$$

Decrease of the temperature



K. Dieckmann. Thesis, University of Amsterdam (2001)

O. Luiten et al., Phys. Rev. A **53**, 381 (1996)



Model of the evaporation

Density and phasespace-density behavior:

$$N = n_0 V_e$$

$$V_e = \int e^{-U(r)/K_B T} d^3 r$$

$$\Rightarrow \frac{\dot{n}_0}{n_0} = \frac{\dot{N}}{N} - \frac{\dot{V}_e}{V_e}$$

$$\Rightarrow \frac{\dot{n}_0}{n_0} = \frac{\dot{N}}{N} - \frac{3\dot{T}}{2T}$$

$$\Rightarrow \frac{\dot{n}_0}{n_0} = \frac{1}{2}(4 - \eta) \frac{\dot{N}}{N}$$

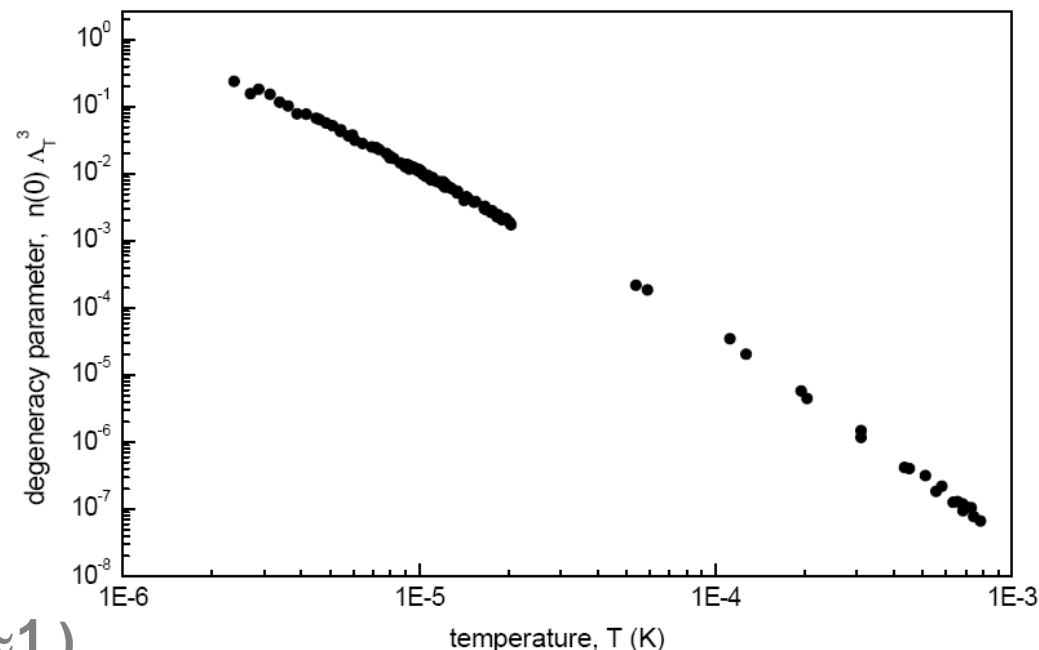
Phasespace density D (for BEC: $D \approx 1$)

$$D = n_0 \lambda^3$$

$$\lambda = \sqrt{2\pi^2 \hbar / (2mk_B T)}$$

$$\Rightarrow \frac{\dot{D}}{D} = (3 - \eta) \frac{\dot{N}}{N}$$

Increase of the phasespace-density



K. Dieckmann. Thesis, University of Amsterdam (2001)



- Thermalization of an ultracold gas
- Model of evaporation
- **Efficiency of evaporative cooling**
 - **Speed of evaporative cooling**
 - **Influence of loss processes**
- Conclusion



Loss Processes

Collisions in an ultracold sample

“Good collisions”: Elastic collisions

→ Thermalization

$$\Gamma_{el} = n\sigma v$$

$$\Gamma_{el} \sim n \times T^{1/2}$$

“Bad collisions”: Inelastic collisions

→ Loss of atoms

- Inelastic one-body collisions
 - Collisions with background gas

$$\Gamma_{Bg} \sim p$$

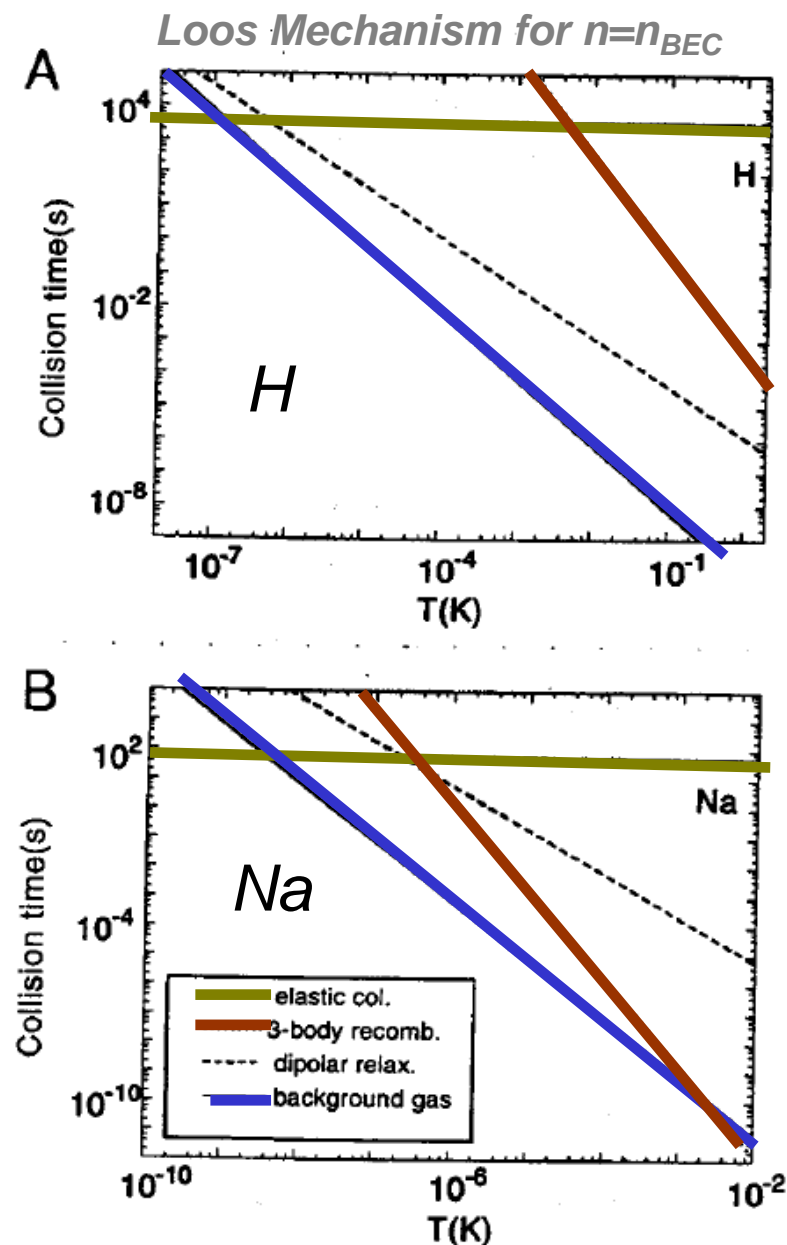
- Inelastic two-body collisions
 - Dipolar and spin relaxation

$$\Gamma_{2body} \sim n$$

- Inelastic three-body collisions

- Three-body recombination

$$\Gamma_{3body} \sim n^2$$





Change of the elastic collision rate:

$$\begin{aligned}\frac{d(n\sigma v)}{dt} / n\sigma v &= \frac{\dot{n}}{n} + \frac{\dot{v}}{v} - \frac{1}{\tau_{loss}} \\ &= \frac{\dot{n}}{n} + \frac{1}{2} \frac{\dot{T}}{T} - \frac{1}{\tau_{loss}}\end{aligned}$$

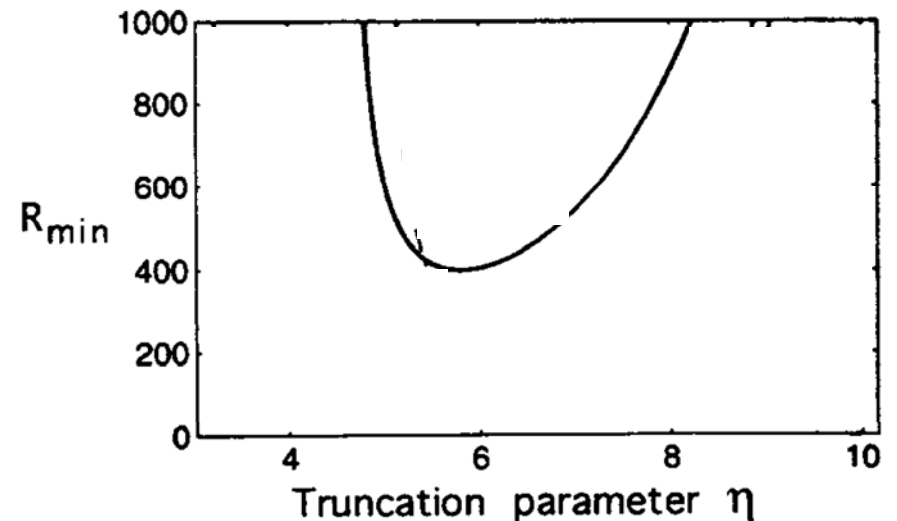
$$\begin{aligned}\frac{\dot{T}}{T_0} &= \alpha \frac{\dot{N}}{N_0} \\ \frac{\dot{n}}{n} &= \frac{\dot{N}}{N} - \frac{3}{2} \frac{\dot{T}}{T} \Rightarrow\end{aligned}$$

$$\frac{d(n\sigma v)}{dt} / n\sigma v = \left(\frac{\alpha-1}{\tau_{ev}} - \frac{1}{\tau_{loss}} \right)$$

→ Elastic collision rate increases

for $\frac{\tau_{el}}{\tau_{loss}} \geq R_{min}$ („Runaway Evaporation“)

Limit for Runaway evaporation
In a harmonic trap



Efficiency of evaporative cooling

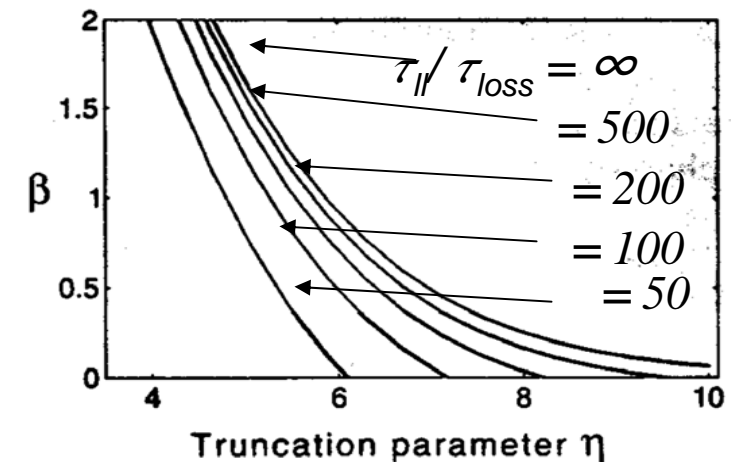


Maximizing phase-space density

Increase of phase space density
(per 100 elastic collisions)

$$\beta = 100\tau_{el} \frac{d}{dt} (\log_{10} D) = \frac{100}{\ln 10} \left(\frac{\alpha-1}{\lambda} - \frac{1}{\tau_{loss}/\tau_{el}} \right)$$

Calculated Phasespace density increase



Calculated efficiency parameter

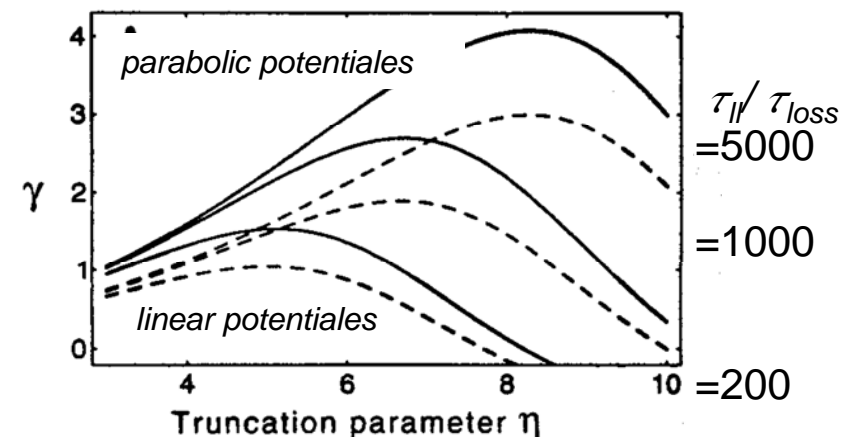
Efficiency of evaporative cooling:

Efficiency of one step:

$$\gamma = -\frac{d(\ln D)}{d(\ln N)} = \frac{\alpha}{1+\lambda/R} - 1$$

Efficiency of the whole process:

$$\gamma_{tot} = \frac{\ln(D_{final}/D_{init})}{\ln(N_{final}/N_{init})}$$





Efficiency of evaporative cooling

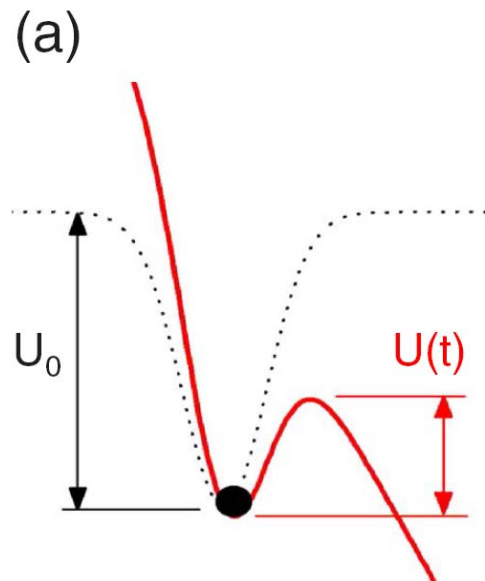
Evaporation in an optical dipole trap

Problem:

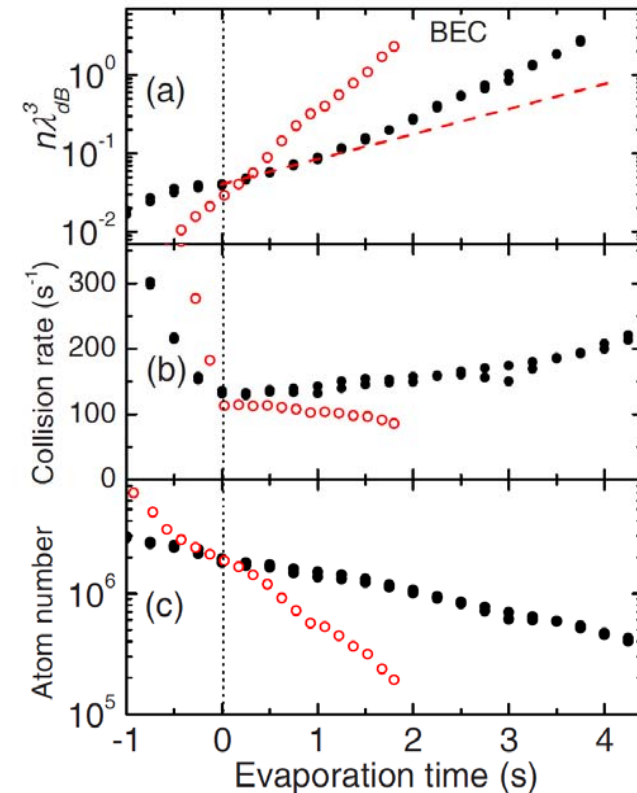
Lowering the trap depth also decreases the collision rate

Solution:

Evaporation via a magnetic gradient

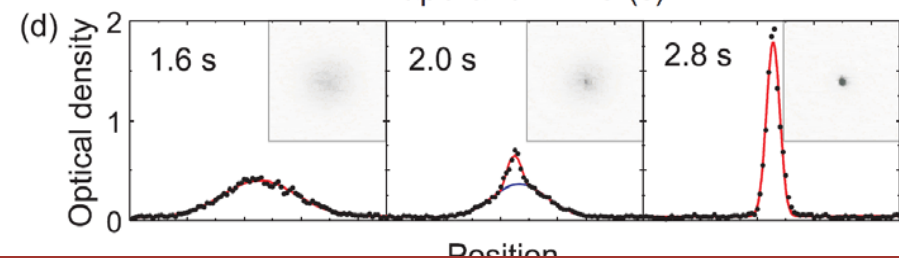


Evaporation of Cs atoms in an optical trap at different truncation parameters



Red:
 $\eta = 4.6$
 $\gamma = 1.9$

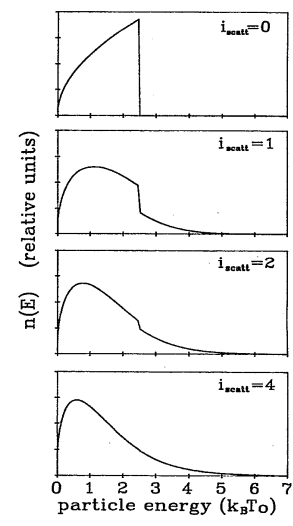
Black:
 $\eta = 6.5$
 $\gamma = 3.4$





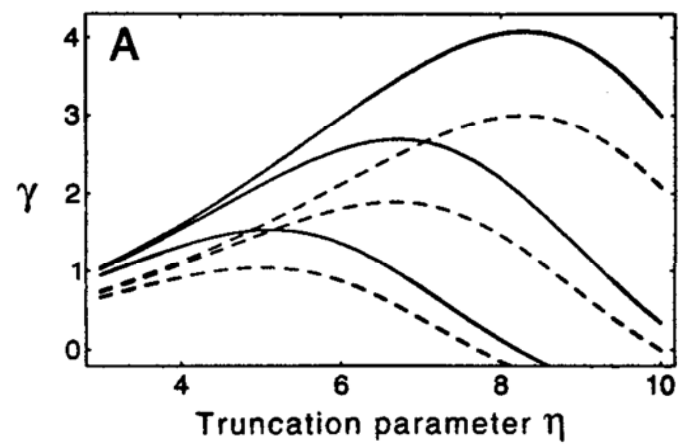
Conclusion

● Thermalization of an ultracold gas



● Model of evaporation

- Efficiency of evaporative cooling
- Speed of evaporative cooling
 - Influence of loss processes





Literature

- *W. Ketterle, N. J. van Druten,*
Evaporative cooling of atoms,
Advances in Atomic, Molecular, and Optical Physics **37**, 181 (1996)
- *Luiten, O. J., Reynolds, M. W., Walraven, J. T. M.,*
Kinetic theory of the evaporative cooling of a trapped gas,
Phys. Rev. A **53**, 381 (1996)
- *D. W. Snoke, J. P. Wolfe,*
Population dynamics of a Bose gas near saturation,
Phys. Rev. B **39**, 4030 (1989)
- *K. B. Davis, M.-O. Mewes, M. A. Joffe, M. R. Andrews, W. Ketterle,*
Evaporative Cooling of Sodium Atoms,
Phys. Rev. Lett. **74**, 5202 (1995)
- *Chen-Lung Hung, Xibo Zhang, Nathan Gemelke, Cheng Chin,*
Accelerating evaporative cooling of atoms into Bose-Einstein condensation in optical traps,
Phys. Rev. A **78**, 011601 (2008)
- *M. Arndt, M. Ben Dahan, D. Guery-Odelin, M. W. Reynolds, J. Dalibard,*
Observation of a Zero-Energy Resonance in Cs-Cs Collisions,
Phys. Rev. Lett. **79**, 625 (1997)
- *Vladan Vuletic, Andrew J. Kerman, Cheng Chin, Steven Chu ,*
Observation of Low-Field Feshbach Resonances in Collisions of Cesium Atoms,
Phys. Rev. Lett. **82**, 1406 (1999)
- *Kai Dieckmann*

Bose-Einstein Condensation with High Atom Number in a Deep Magnetic Trap

University of Amsterdam, 2 March 2001 ;Thesis advisor: Prof. Dr. J.T.M. Walraven

from www.staff.science.uva.nl/~walraven/walraven/Theses.htm