Hydrodynamics of Ultracold Gases

Seminar Quark-Gluon Plasma and
Cold Atomic Physics

27.04.2009

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Outline of the Talk

- What are we able to do with Ultracold Quantum Gases? (experimental issues including traps, analysis of images)

- Hydrodynamic behaviour in expanding thermal clouds

- Collective Modes & Hydrodynamics: BEC regime and thermal clouds

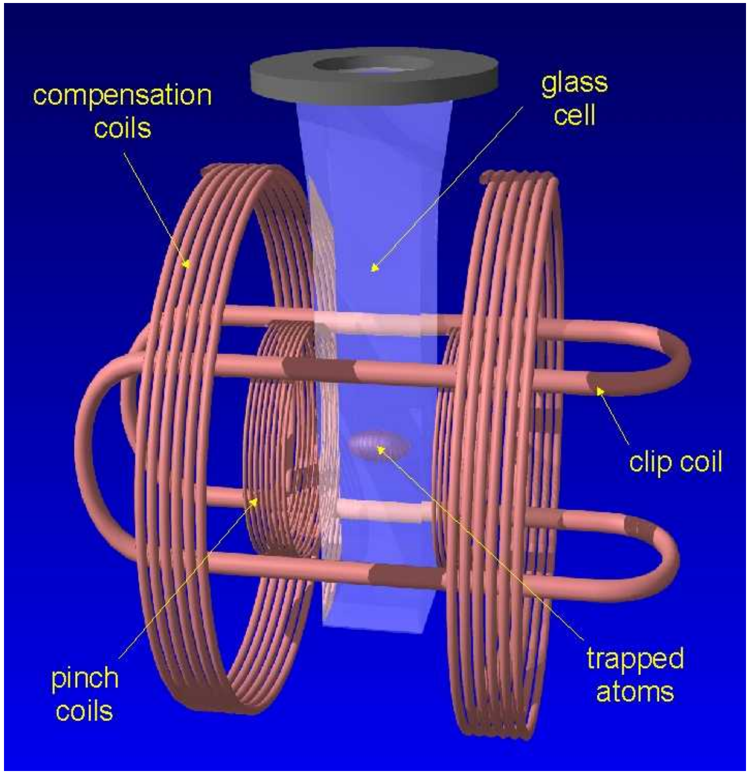
- Outlook

Common types of traps used in ultracold atom experiments

Magnetic: trap atoms in *B*-field minimum

Optical: Trap atoms in intensity maximum of red detuned light field (AC Stark shift) → allow for easy use of Feshbach resonances to change scattering length *a*

Trap geometry determines trap frequencies → Possibility of dynamical modification and hence excitation of collective modes etc.



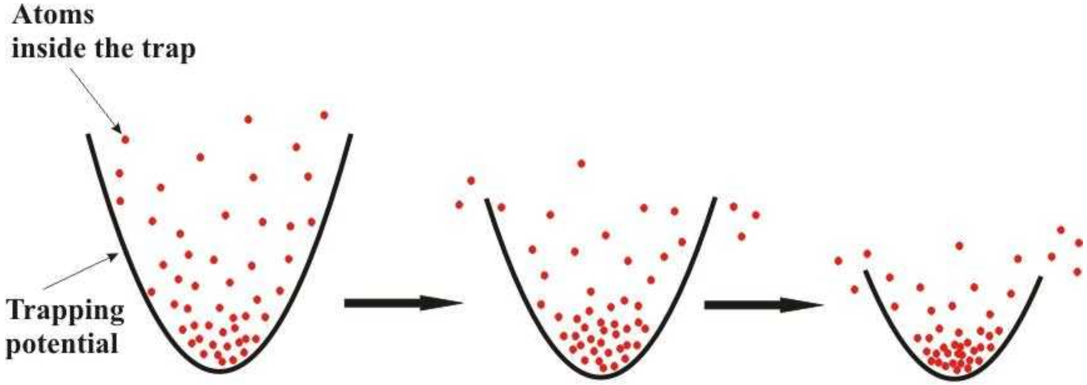
http://www.mpq.mpg.de/qdynamics/projects/bec/BECtrap.html

http://www.npl.co.uk/server.php?show=ConWebDoc.1739

Cold → Ultracold atoms

Ultracold temperatures commonly reached by evaporative cooling: Remove hottest atoms

- Magnetic traps: Use radiofrequency / microwave
- Optical dipole traps: lower trap depth



http://cold-atoms.physics.lsa.umich.edu/projects/bec/evaporation.html

Different types of ramps allow to adjust most important parameters, e.g. N_{atoms}

→ $T>T_{C/F}$, $T<T_{C/F}$ possible

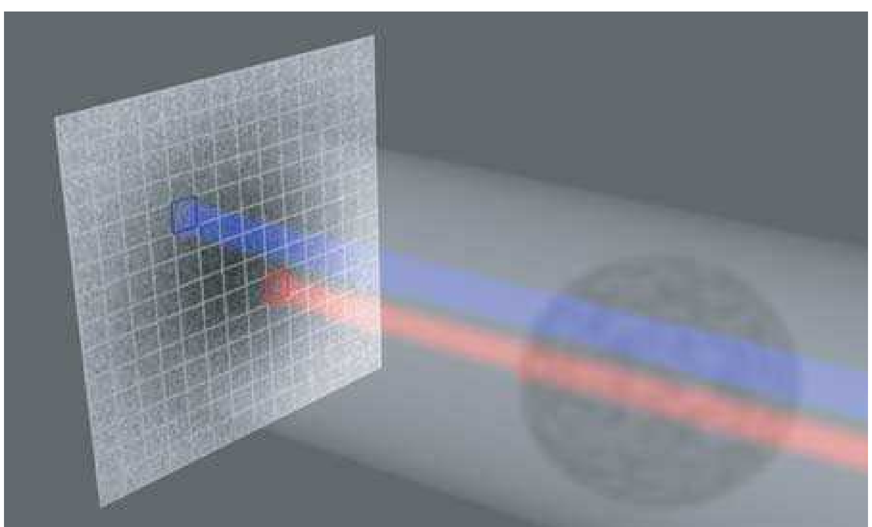
→ Important for testing theories in various parameter regimes

Image analysis

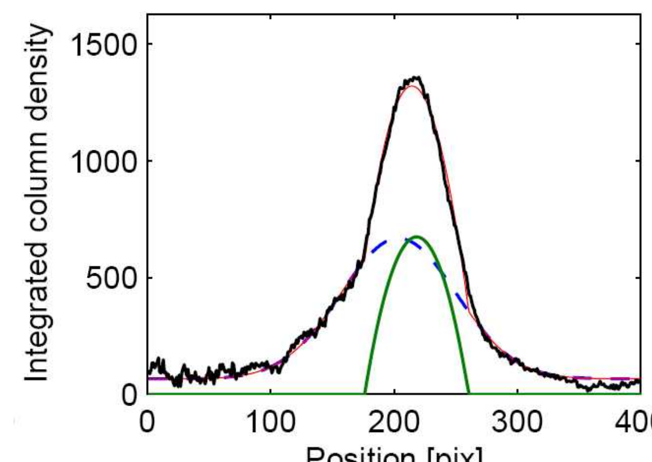
Most commonly absorption imaging to image the atoms used
→ Get 2 dimensional density profile of atom cloud
Can extract important quantities (typical values for BECs):

- Temperature (100nK ... 1μK)
- Total atom number (up to ~10^8 in condensate)
- Chemical potential (100nK ... 1μK) /
peak density (10^{14}cm^{-3}... 10^{15} cm^{-3})
- Condensate atom number / fraction (adjustable from 0 to 1)

Can even derive e.g. correlation functions



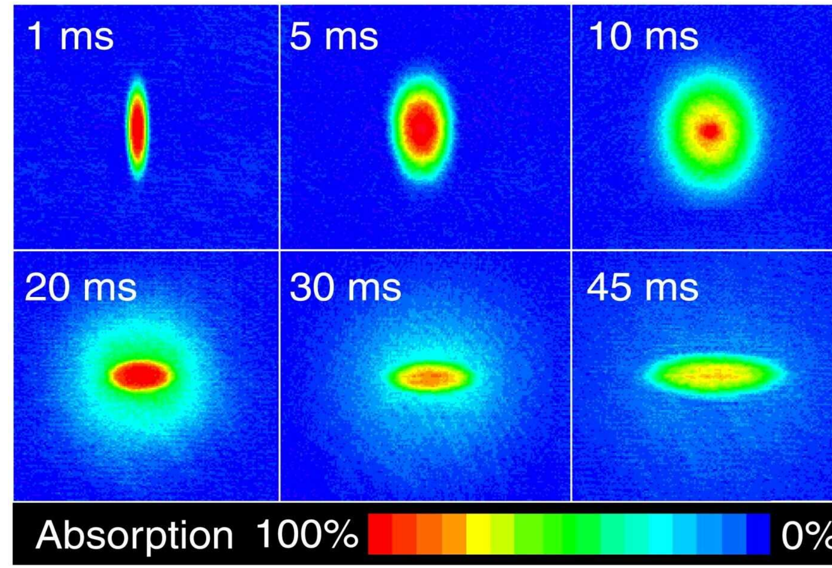
Foelling *et al.,* Nature 434, 481 (2005)



Hydrodynamic behaviour in expanding thermal clouds of ^{87}Rb

Shvarchuck *et al.*, PRA **68**, 063603 (2003)

Pure BEC in trap (Thomas-Fermi limit $E_{kin} << E_{int}$): Interaction driven inversion of aspect ratio in TOF.
Thermal gas without interactions (collisionless): Isotropic Expansion



http://cua.mit.edu/ketterle_group/Projects_1996/Ballistic_expansion/Ballistic_expansion.htm

Parameters of the experiment:
collision rate τ_c^{-1} = 6000 1/s, cigar-shaped trap with
$\omega_\rho = 2\pi*477$ Hz , $\omega_z = 2\pi*20.8$ Hz
→ Knudsen criterion for hydrodynamic behaviour

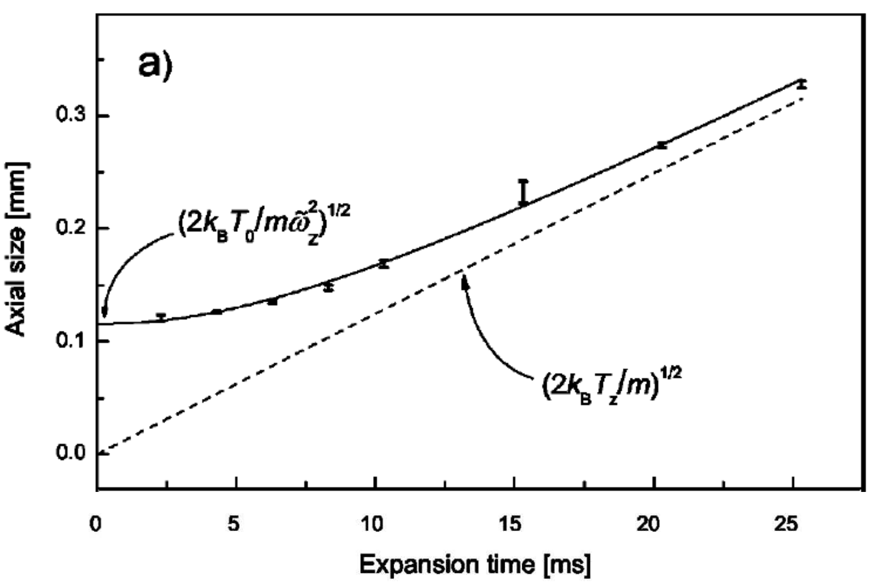
$$\frac{\lambda_0}{l_i} \simeq \omega_i \tau_c \ll 1$$

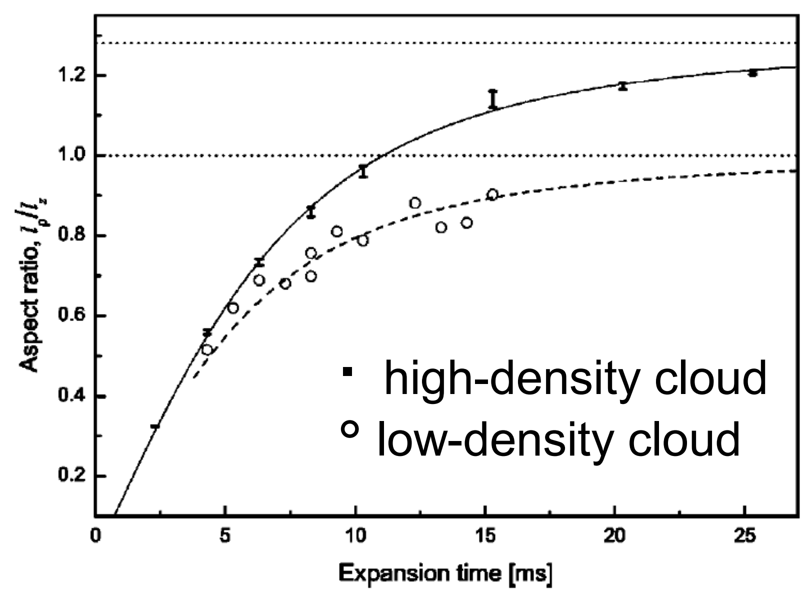
well satisfied in z direction, in ρ direction crossover regime.

Hydrodynamic behaviour in expanding thermal clouds of ^{87}Rb

Shvarchuck *et al.*, PRA **68**, 063603 (2003)

Knudsen criterion: Consider classical particles
Expect cooling to happen in z direction (only particles with v||z can scatter), thus heating in ρ direction (collisionless)
Theoretical description using two-stage-model: 1st stage hydrodynamic (here heating/cooling occurs), 2nd collisionless
Experiment: T_0 =1.17μK, T_z=0.83μK, T_ρ =1.35μK



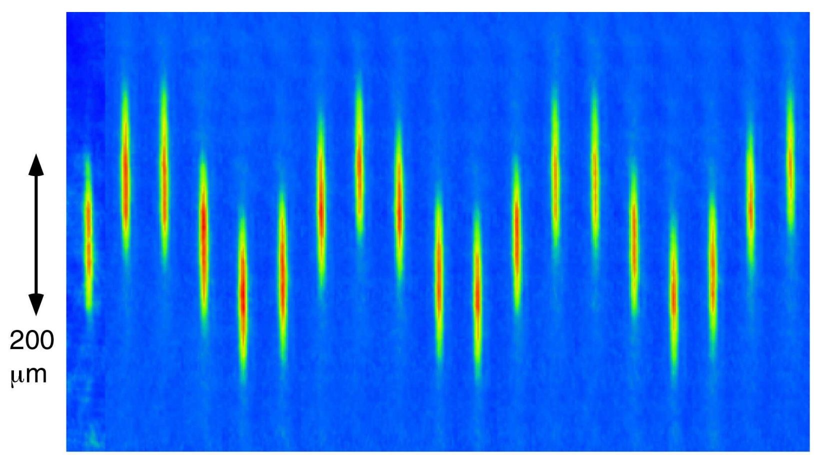


Collective Modes and Hydrodynamics

Collective mode = (low-lying) excitation of condensate
Experimentally relevant: BEC in trap (axially symmetric)
→ Write collective modes in terms of spherical harmonics
How can we excite them?
→ Change appropriate trap parameters

Simple example: Dipole mode l=1
What can we learn from the dipole mode?
→ Determine trap frequencies ω_i with high accuracy
But no interaction effects due to pure c.o.m. motion.
→ Same for BEC/thermal cloud

→ More complicated modes necessary to study physics, e.g. hydrodynamics!



http://cua.mit.edu/ketterle_group/Projects_1998/Coll_exc/Collective_excitations.htm

Collective excitations of a trapped Bose-Condensed Gas

S. Stringari, PRL **77**, 2360 (1996)

Starting from the Gross-Pitaevskii equation,

$$i\hbar\frac{\partial}{\partial t}\Phi(\mathbf{r},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(\mathbf{r}) + \frac{4\pi\hbar^2 a}{m}|\Phi(\mathbf{r},t)|^2\right)\Phi(\mathbf{r},t)$$

one can derive the hydrodynamic equations

$$\frac{\partial}{\partial t}\rho + \nabla(\mathbf{v}\rho) = 0 \qquad\qquad m\frac{\partial}{\partial t}\mathbf{v} + \nabla\left(\delta\mu + \frac{1}{2}m\mathbf{v}^2\right) = 0$$

with $\rho(\mathbf{r},t) = |\Phi(\mathbf{r},t)|^2$ $\qquad \delta\mu = V_{\text{ext}} + \frac{4\pi\hbar^2 a}{m}\rho - \frac{\hbar^2}{2m\sqrt{\rho}}\nabla^2\sqrt{\rho} - \mu$

$$\mathbf{v}(\mathbf{r},t) = [\Phi^*(\mathbf{r},t)\nabla\Phi(\mathbf{r},t) - \nabla\Phi^*(\mathbf{r},t)\Phi(\mathbf{r},t)]/2mi\rho(\mathbf{r},t).$$

For an axially symmetric trap and strong interactions

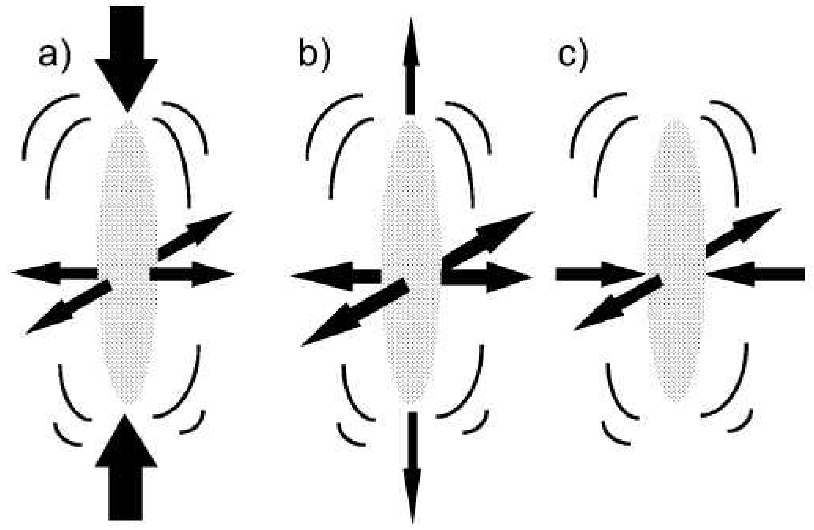
$$\omega^2(m = \pm\ell) = \ell\omega_\perp^2 \qquad\qquad \lambda = \omega_z/\omega_\perp$$

$$\omega^2(m = 0) = \omega_\perp^2\left(2 + \tfrac{3}{2}\lambda^2 \mp \tfrac{1}{2}\sqrt{9\lambda^4 - 16\lambda^2 + 16}\right)$$

Experimental Observation

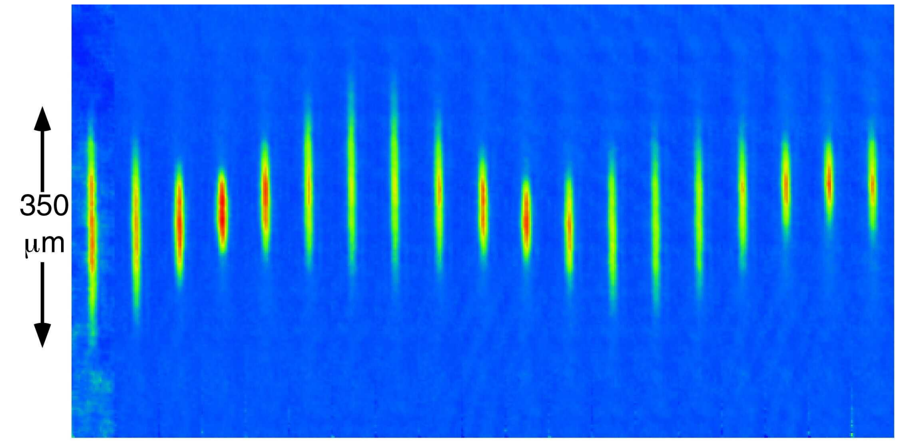
D.S.Jin *et al.*, PRL **77**,420 (1996) ; M.-O.Mewes *et al.*, PRL **77**, 988 (1996)

a) low-lying m=0 quadrupole mode
In TOP-Trap ($\lambda^2=8$) $\omega = 1.797\ \omega_\rho$
Cigar-shaped trap $\omega = (5/2)^{(1/2)}\ \omega_z$
b) fast m=0 quadrupole mode
Cigar-shaped trap $\omega = 2*\omega_\rho$
c) |m|=2 quadrupole mode $\omega = 2^{(1/2)}\ \omega_\rho$



http://cua.mit.edu/ketterle_group/Theses/thesis_DMSK.pdf

All predicted frequencies experimentally verified (e.g. a) as shown on the right)!



5 milliseconds per frame

http://cua.mit.edu/ketterle_group/Projects_1998/Coll_exc/Collective_excitations.htm

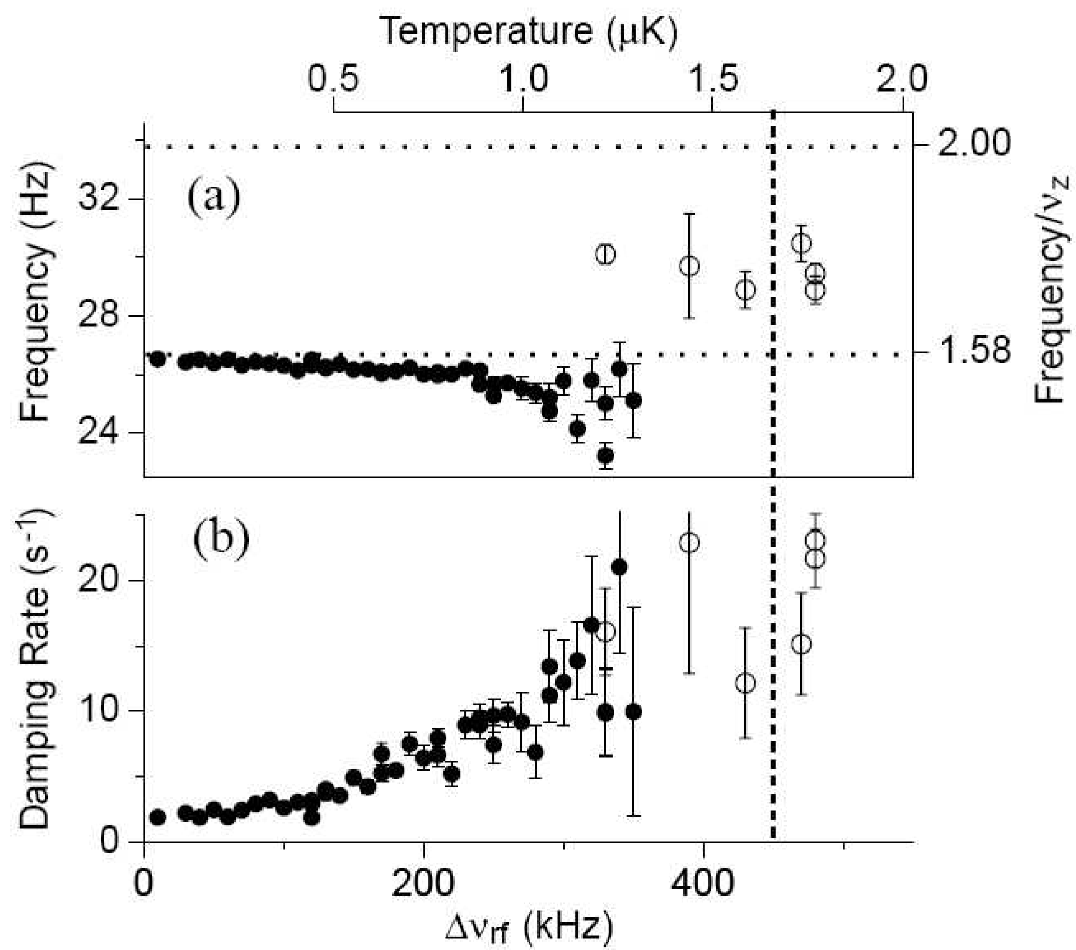
More advanced collective oscillations

D.Stamper-Kurn *et al.*, PRL **81**, 500 (1998)

So far: Only T=0 physics.
What happens at finite T?

- thermal cloud appears
- change in frequency
- finite damping

→ Does thermal cloud show hydrodynamic behaviour? Further theoretical and experimental analysis needed!



Hydrodynamic Modes in a Trapped Bose Gas above the Bose-Einstein Transition

A. Griffin *et al.*, PRL **78** (1997), 1838; G.M.Kavoulakis *et al.*, PRA **57**, 2938 (1998)

Hydrodynamic treatment of collective modes in thermal gas:
Modes with |m|=l, |m|=l-1have same frequency as in BEC regime
To find differences, have to compare frequencies of low-lying quadrupole mode: For a cigar shaped trap,
$\omega = (5/2)^{(1/2)}\ \omega_z$ in BEC regime
$\omega = (12/5)^{(1/2)}\ \omega_z$ for thermal hydrodynamic gas
Calculation of damping done with ansatz given in Landau Lifshitz

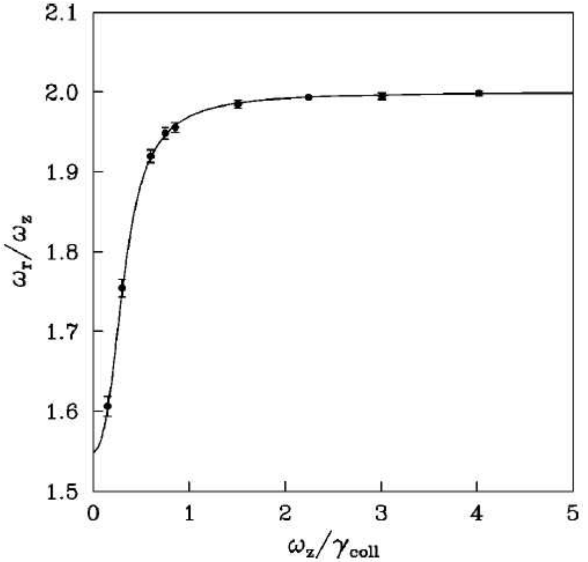
$$\dot{E}_{\mathrm{mech}} = -\int \frac{\kappa}{T}|\nabla T|^2 d\mathbf{r} - \int \zeta(\boldsymbol{\nabla}\cdot\mathbf{v})^2 d\mathbf{r} - \int \frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{i,k}\boldsymbol{\nabla}\cdot\mathbf{v}\right)^2 d\mathbf{r}$$

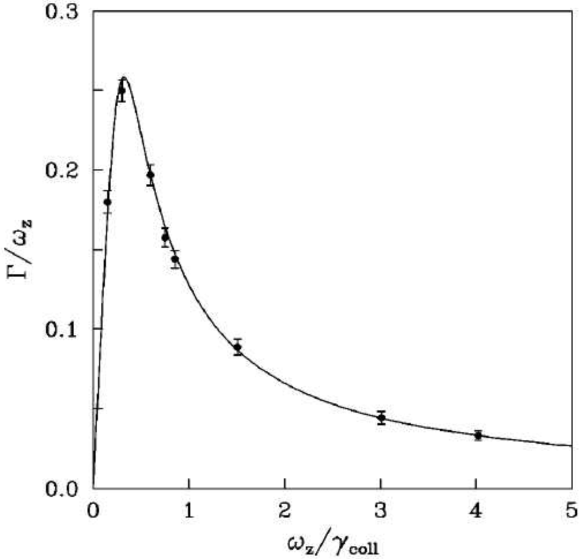
Bulk viscosity ζ vanishes, no temperature gradients
→ damping only due to shear viscosity η

Collective Oscillations of a classical gas confined in harmonic traps

D. Guery-Odelin *et al.*, PRA **60**, 4851(1999)

So far only purely hydrodynamic regime; damping rates turn out not to describe experimental data correctly
→ use Boltzmann equation to describe frequencies and damping in collisionless as well as hydrodynamic regime
With Gaussian ansatz for the distribution function f(**r**,**v**,t) get the following result for frequency and damping (solid line: Gaussian ansatz, points: numerical simulation)



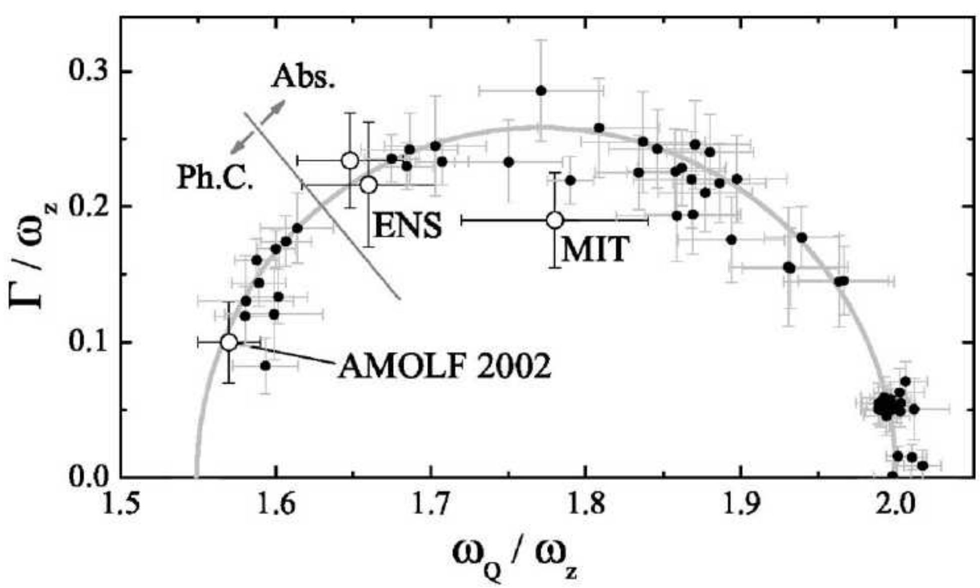


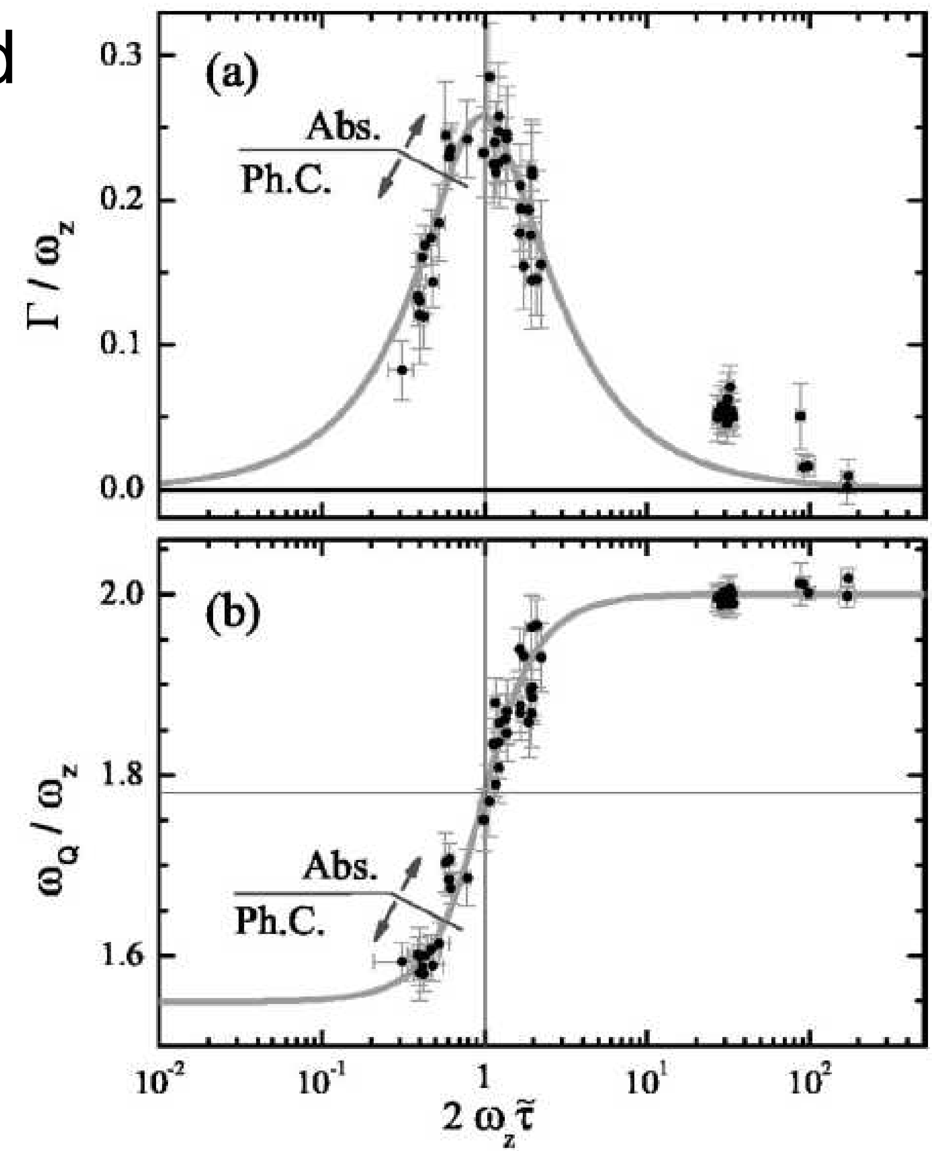
Shape oscillations in nondegenerate Bose gases: Transition from the collisionless to the hydrodynamic regime

Ch. Buggle *et al.*, PRA **72**, 043610 (2005)

Make experiment in cigar-shaped trap with varying densities (achieved by laser depletion followed by plain evaporation)

$$\omega^2 = \omega_{\mathrm{cl}}^2 + \frac{\omega_{\mathrm{hd}}^2 - \omega_{\mathrm{cl}}^2}{1 - i\omega\tilde{\tau}}$$





Conclusion and Outlook

- Introduction to traps and image analysis for ultracold quantum gases
- Hydrodynamic behaviour in expanding thermal clouds of ^{87}Rb → Aspect ratio as indicator
- Collective modes and hydrodynamics:
  - BEC regime
  - Thermal clouds: Transition from collisionless to hydrodynamic regime well described by Boltzmann equation

- No comprehensive experimental study of damping of collective modes of condensate at finite temperature
- Collective oscillations of Fermi Gases (BEC-BCS crossover) → Precision measurements done in group of R. Grimm (e.g. PRL **98**, 040401 (2007))