Geometry & Topology in Physics

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Tuesday, 11:15-13:00, Pw 19 / SR [LSF]

Wednesday, 11:15-13:00, every 2nd week, kHS Pw 12 [LSF]

- Content
- Literature
- Exercises & bonus material
- Script

Prerequisites: Theoretical Physics I-IV, basic knowledge of QFT

Content of lecture series

The lecture course provides an introduction to geometrical and topological effects in physics, applications range from quantum mechanics to quantum field theory.

Outline in key words

- Symmetries & topological excitations:
  Scalar field theory, solitons, fermions & index theorems, dilute gas expansion

- Gauge theories & homotopy:
  Abelian Higgs model, Bogomol'nyi bound, homotopy, topological invariants & classification

- Non-Abelian gauge theories:
  Setting, instantons, zero modes, collective coordinates & moduli space, fermionic zero modes & Atiyah-Singer index theorem, topology & confinement
• Gauge anomalies:
  Cocycles, Wess-Zumino consistency condition, consistent & covariant anomalies

• Seiberg-Witten:
  Supersymmetry, chiral multiplets, N=2 Susy Yang-Mills, superconformal Ward-Id's, dualities

• Topology & dynamics:
  Dynamics, vortices, scaling

Literature

Coleman
Aspects of Symmetry
Cambridge University Press

Göckeler & Schücker
Differential Geometry, gauge theories, and gravity
Cambridge University Press

Nakahara
Geometry, Topology and Physics
Hilger

Nash & Sen
Topology and Geometry For Physicists
Academic

Rajaraman
Solitons and Instantons
North-Holland

Wu-Ki Tung
Group Theory in Physics
World Scientific

Zinn-Justin
Quantum Field Theory and Critical Phenomena
Oxford

Lecture notes

Bruckmann
Topological objects in QCD
Lecture notes, Schladming winter school 2007

Lenz
Topological concepts in gauge theories
Lecture notes

't Hooft, Bruckmann
Monopoles, Instantons and Confinement
Lecture notes, Saalburg summer school 1999
Motivation

EoM: $S[\phi]$ action, $S[\phi] = \int d^4x L(\phi)$

Field Theory: $\phi = \phi(x, t)$

mechanics: $\phi = \hat{\phi}(t)$

Physics: $\frac{\delta S}{\delta \phi} \bigg|_{\hat{\phi}} = 0$

Set of $\hat{\phi}$ possible physics evolution uniquely determined by initial condi.
boundary conditions.

Quantum Physics:

EoM: $\Gamma[\phi]$ effective action

$\frac{\delta \Gamma}{\delta \phi} \bigg|_{\bar{\phi}} = 0$

$\bar{\phi}$ mean field/classical field $\langle \phi \rangle$
\[
\frac{\partial \Gamma}{\partial \phi^n} \bigg|_{\phi} = \langle \phi^n \rangle
\]

How do compute?

- First (simplest) approach:

  Parturbation theory

  (i) theory is basically free / classical

  (ii) quantum fluctuations are perturbative

Example of real scalar theory in 1+1 dimensions

\[
S[\phi] = -\frac{1}{2} \int d^2 x \, \partial_\mu \phi \partial^\mu \phi + \int d^2 x \, V(\phi)
\]

\[V(\phi) = \begin{cases}
V(0) + \frac{1}{2} V''(0) \phi^2 + \frac{4}{4!} V^{(4)}(0) \phi^4 + \cdots \quad &\text{with } V(\phi) = V(-\phi)
\end{cases}
\]

\[
\phi = 0 : \quad V(\phi) = V(0) + \frac{1}{2} V''(0) \phi^2 + \frac{4}{4!} V^{(4)}(0) \phi^4 + \cdots
\]

\[
\phi \rightarrow \phi \phi^{1/2}
\]
Problems:

(1) convergence (at most asymptotic series (with radius of convergence 0))

(2) strong coupling (evidently)

(3) is the topic of this lecture

Non-perturbative

Examples:

\[(\phi(t, \phi(0), \phi(t = t', \phi(t_0))) = \phi)\]

smoothly into \((\phi(t_0), \phi(t = t_0))\) could be chosen.

Energy solutions could be chosen.
Quantization within saddle point expansion about all distinct vacua/classical solutions \( e^{i \frac{1}{T} \int d^4 x} \) \( + \sum e^{i m} \) \( (\phi) \)

(i) Classification of solutions

mandatory construction of solutions

(ii) Classical solutions carry interesting non-perturbative physics

(iii) Disclaimer: beware of naive belief of topological arguments (key word: Platonism behind the curtain argument)

in short: Topology is global, (most) physics is local

\( \Rightarrow \) topological densities important
Applications

- quantum mechanics (tunneling, geometric phases)
- solid state physics (vortices, tunneling)
  cosmology, top. field theories, SW, gauge fixing, global),
- string theory (dualities, fluxes, instantons, algebraic geometry)