Landau-Ginzburg analysis of tensorial group field theories

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workshop "Random Geometry in Heidelberg" - 19.5.2022 based on:

L. Marchetti, DO, A. Pithis, J. Thürigen, JHEP 12 (2021) 201, e-Print: 2110.15336 [gr-qc]

L. Marchetti, DO, A. Pithis, J. Thürigen, to appear

(and A. Pithis, J. Thürigen, Phys.Rev.D 98 (2018) 12, 126006, e-Print: 1808.09765 [gr-qc])

(and indirectly on all work on RG analysis of TGFTs)

(and, in later part, D. Benedetti, J.Stat.Mech. 1501 (2015) P01002, e-Print:1403.6712 [cond-mat.stat-mech])

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(includes, tensor models, tensor field theories, group field theories, ...)

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$a, b, c \in (1 \ldots \infty)$ ~momenta on $T^3 = (S^1)^3$
UV cutoff N

$$S = \sum_{a,b,c} \left(\phi_{abc}(a^2 + b^2 + c^2)\phi_{abc} + \text{invariants}\right)$$

non-local arguments which are dynamical, simplicial interpretation for field, quanta and processes, theories of random discrete geometry

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here: TGFT models for QG from random geometry perspective (and, in the end, quantum geometric ones)

How to extract continuum gravitational physics?

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TGFT as (non-perturbative) completion of simplicial path integral/spin foam models for quantum gravity

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collective QG physics

- need distinctively field-theoretic approximations

result of collective quantum dynamics of fundamental discrete degrees of freedom

Extracting continuum spacetime & gravitational physics

✳ ideally, TGFT free energy itself (and its derivatives) or full TGFT quantum effective action should be used to compute continuum geometric observables and their quantum dynamics

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expect different phases

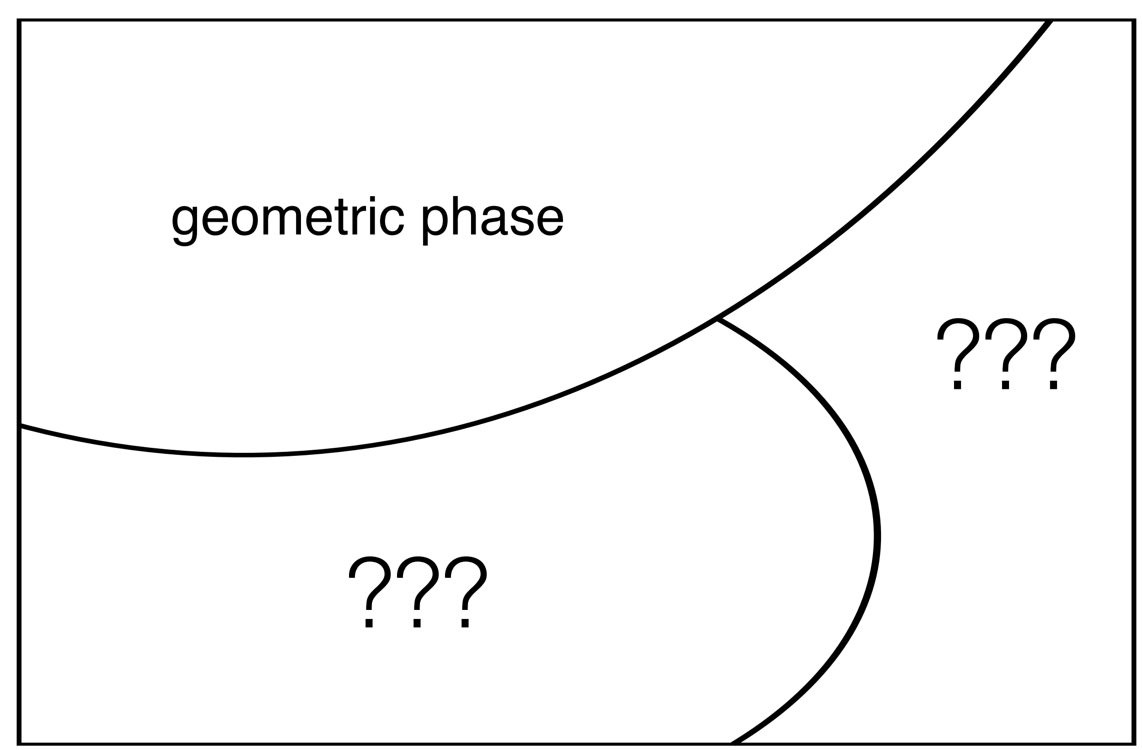
and phase transitions

as result of quantum dynamics

which ones are "geometric"

in which one does spacetime emerge?

same from quantum geometric perspective



Koslowski, '07; DO, '07

A. Ashtekar, J. Lewandowski, '94 T. Koslowski, H. Sahlmann, '10 B. Dittrich, M. Geiller, '14; B. Bahr, B. Dittrich, M. Geiller, '16; S. Gielen, DO, L. Sindoni, '13 A. Kegeles, DO, C. Tomlin, '16

TGFT non-perturbative renormalisation

S. Carrozza, '16

FRG for (tensorial) GFT models (similar to matrix/tensor models but distinctively field-theoretic)

Eichhorn, Koslowski, '14

- Polchinski formulation based on SD equations Krajewski, Toriumi, '14
- general set-up for Wetterich-Morris formulation based on effective action
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- consequences of Ward identities from symmetries on RG flow

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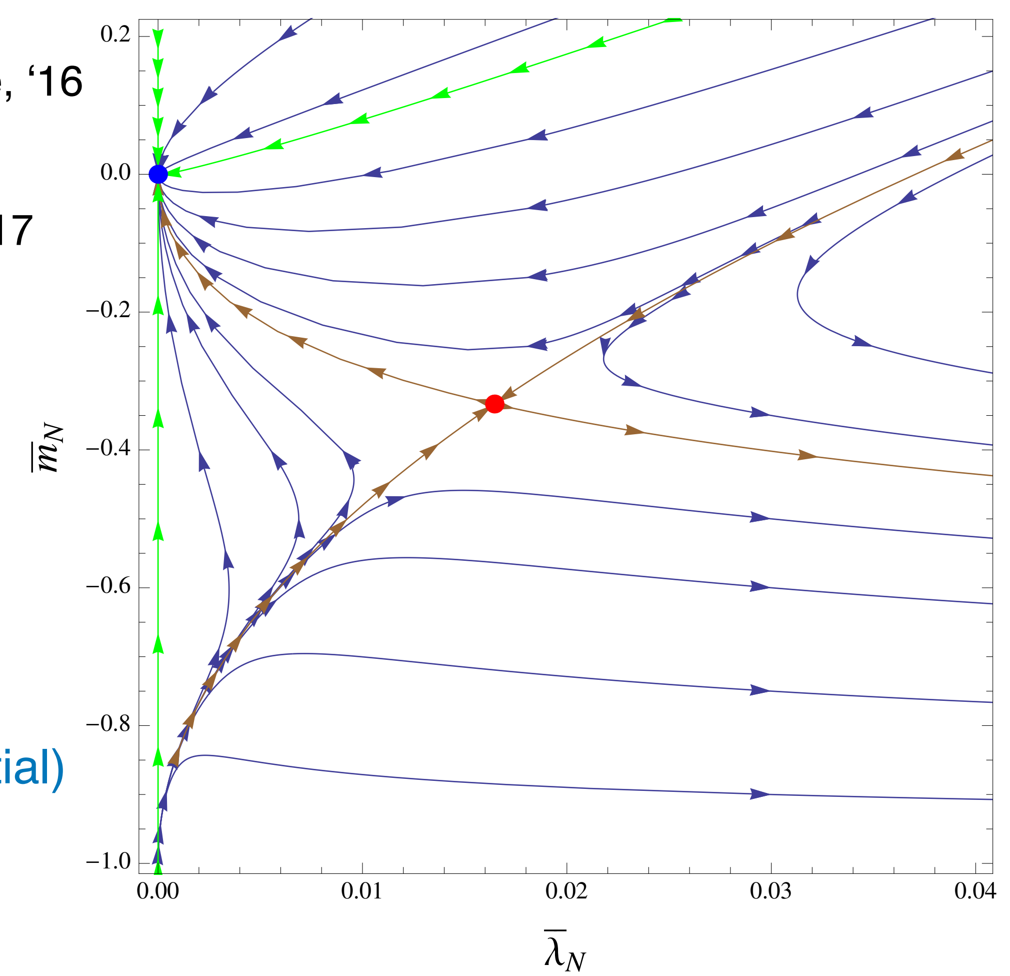

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- asymptotic freedom/safety
- hints of broken/condensate phase? (non-trivial minimum of classical potential)

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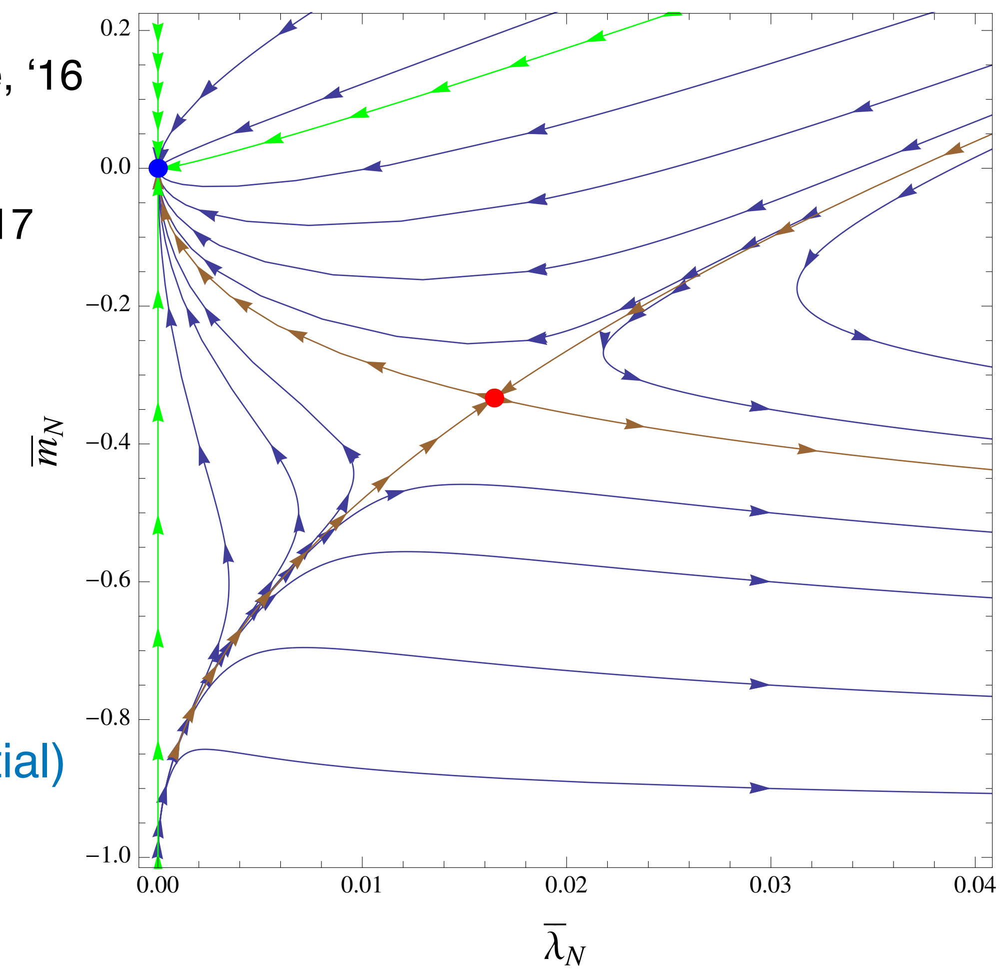

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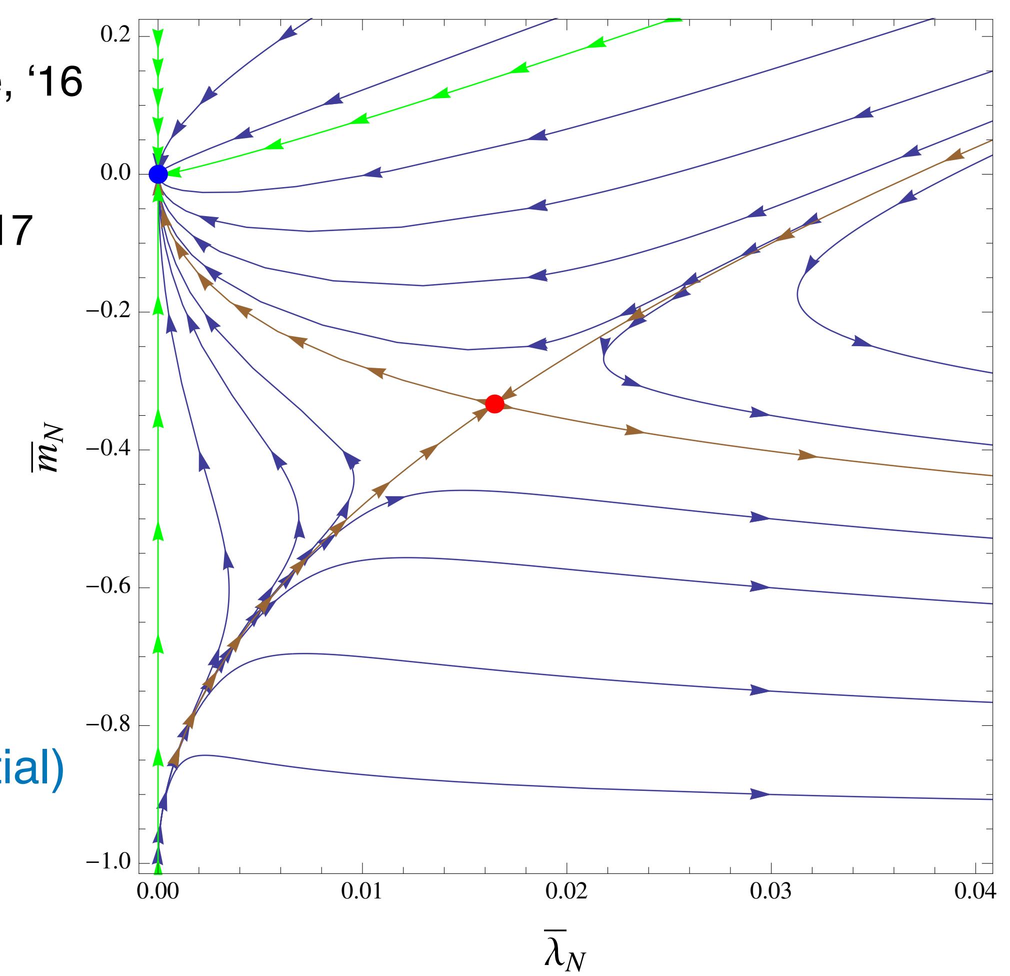

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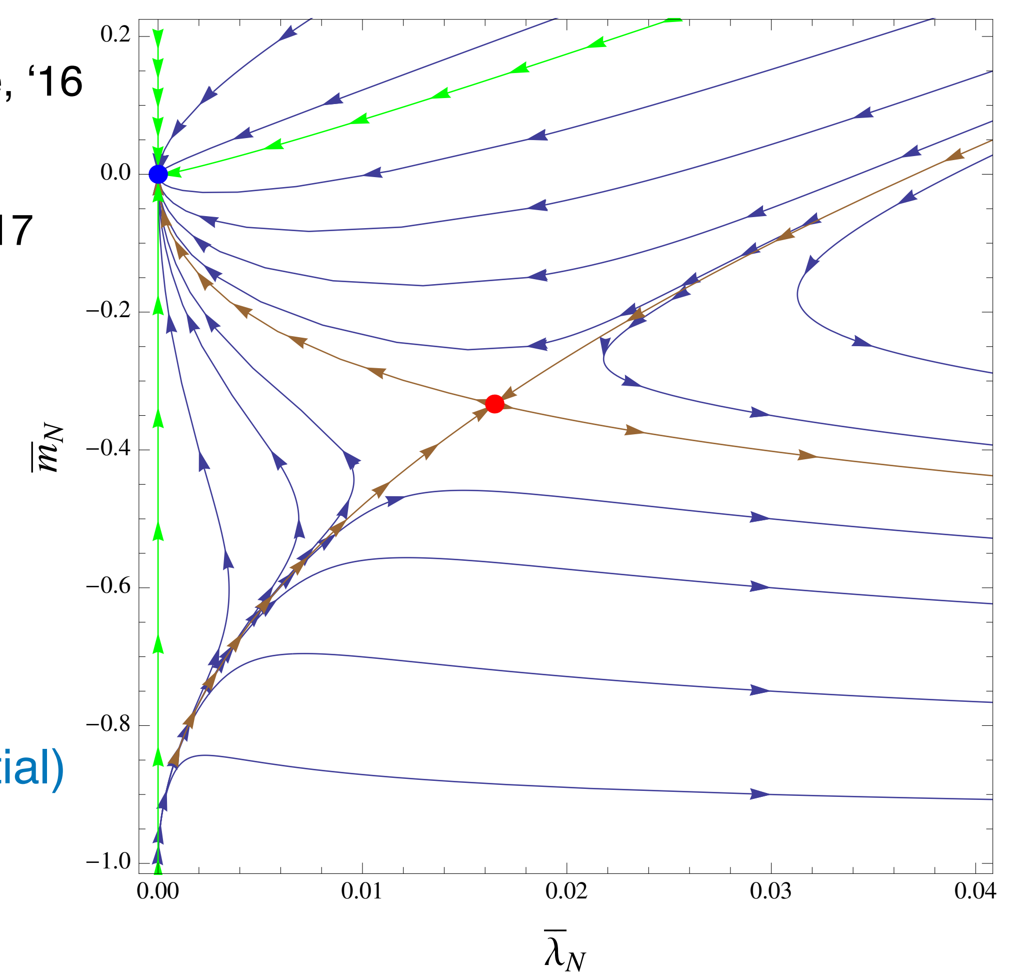

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- TGFTs can flow naturally out of melonic sector
- non-melonic TGFTs can be renormalizable
- truncation (incl. non-melonic, disconnected diagrams) matters

TGFT renormalization - matter is going to matter

example: rank-r TGFT with cyclic melonic interactions

A. Pithis, J. Thurigen, '20

$$\phi : \mathrm{U}(1)^r \to \mathbb{C}$$

$$\varphi(\boldsymbol{g}) := \langle \phi(\boldsymbol{g}) \rangle = \frac{\delta W[\bar{J}, J]}{\delta \bar{J}(\boldsymbol{g})}$$

$$\Gamma_k[\bar{\varphi}, \varphi] = (\varphi, (-Z_k \Delta + \mu_k)\, \varphi) + \sum_{c=1}^{r} \mathrm{Tr}_G V_k(\bar{\varphi} \cdot_{\hat{c}} \varphi)$$

$$(\bar{\varphi} \cdot_{\hat{c}} \varphi)(g_c, h_c) := \int \prod_{b \neq c} \mathrm{d}g_b \bar{\varphi}(g_1 ... g_c ... g_r) \varphi(g_1 ... h_c ... g_r)$$

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the RG flow equations are found to be:

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- at large-N, equivalence with O(N) models in d = r-1 (up to anomalous dimension!) - non-Gaussian FP for r = 4,5

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cyclic-melonic interactions dominate the RG flow of more general TGFT models

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in TGFT models with matter, matter adds non-compact directions $\longrightarrow$ expect different results!

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- can we say anything about phase transitions at such approximate level?
- can we estimate when mean field approximation is to be trusted?

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- deduce critical dimension below which mean field theory (and above analysis) fails (fluctuations too large)

in this talk:

consider "extended" TGFT models, with non-local (group) directions and local (flat) directions, both dynamical

"combination of TGFTs and TFT"

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- note: bringing together different branches of the TGFT family! (Y. Wang, V. Nador, DO, X. Pang, A. Tanasa, in progress)

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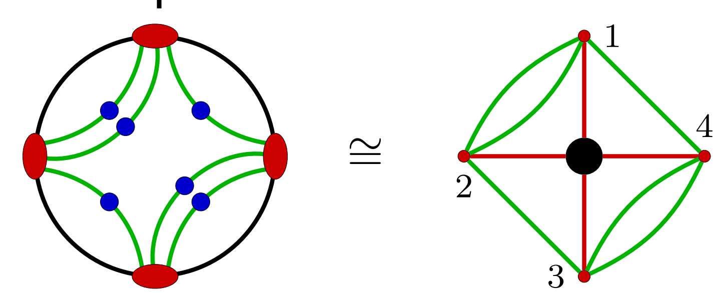

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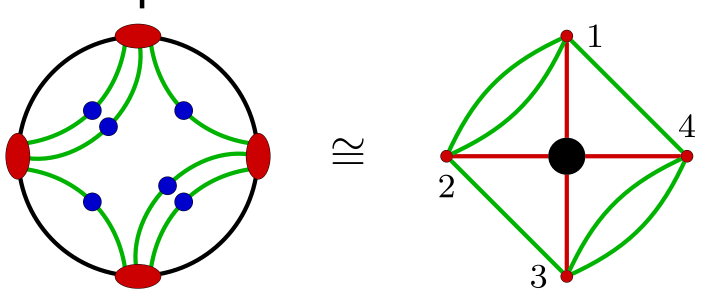

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  variation removes one field (corresp. to one vertex) for any interaction vertex (corresp. to one graph)

- project onto constant field solutions $\Phi(\boldsymbol{\phi}, \boldsymbol{g}) = \Phi_0$

$$\mu\Phi_0 + \sum_\gamma \lambda_\gamma V_\gamma a_G^{r\frac{V_\gamma-2}{2}} \Phi_0^{V_\gamma-1} = \left(\mu + \sum_\gamma \lambda_\gamma V_\gamma a_G^{r\frac{V_\gamma-2}{2}} \Phi_0^{V_\gamma-2}\right)\Phi_0 = 0$$

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i-th root of unity

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  - get 1st order eqn for fluctuation: $\mathcal{K}\delta\Phi + \sum_\gamma \lambda_\gamma \sum_{v,v'\in\mathcal{V}_\gamma} \mathrm{Tr}_{\gamma\backslash v}(\Phi_0, \delta\Phi_{v'}) = 0$

    the quadratic form in 2nd term is Hessian of interaction term of action (similar to FRG eqn)

$$(\mathcal{K} + F[\Phi_0])\delta\Phi(\boldsymbol{\phi},\boldsymbol{g}) = 0$$

$$F[\Phi](\boldsymbol{\phi},\boldsymbol{g};\boldsymbol{\phi}',\boldsymbol{h}) := \frac{\delta S_{\mathrm{IA}}[\Phi]}{\delta\Phi(\boldsymbol{\phi},\boldsymbol{g})\delta\Phi(\boldsymbol{\phi}',\boldsymbol{h})} = \delta(\boldsymbol{\phi}-\boldsymbol{\phi}')\sum_\gamma \lambda_\gamma \sum_{v,v'\in\mathcal{V}_\gamma} \mathrm{Tr}_{\gamma\backslash v\backslash v'}(\Phi)$$ at $\Phi(\boldsymbol{\phi},\boldsymbol{g}) = \Phi_0$

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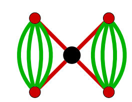
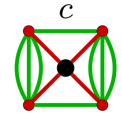
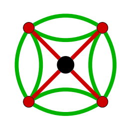
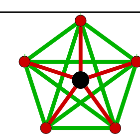
examples:

double-trace melon		$\hat{\mathcal{X}}_{\sqcup}(\boldsymbol{j}) = 4\left(2\prod_{c=1}^4 \delta_{j_c,0}+1\right)$
quartic melonic	c 	$\hat{\mathcal{X}}_{c}(\boldsymbol{j}) = 4\left(\prod_c \delta_{j_c,0}+\prod_{b\neq c}\delta_{j_b,0}+\delta_{j_c,0}\right)$
quartic necklace		$\hat{\mathcal{X}}(\boldsymbol{j}) = 4\left(\prod_c \delta_{j_c,0}+\delta_{j_1,0}\delta_{j_2,0}+\delta_{j_3,0}\delta_{j_4,0}\right)$
simplicial		$\hat{\mathcal{X}}(\boldsymbol{j}) = 5\sum_{i=0}^4\prod_{k\neq i}\delta_{j_{(ik)},0}$ (edges labeled by adjacent vertices i,k)

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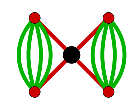
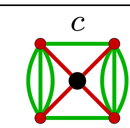
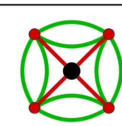
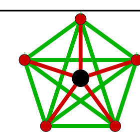
double-trace melon		$\hat{\mathcal{X}}(\boldsymbol{j}) = 4\left(2\prod_{c=1}^4 \delta_{j_c,0}+1\right)$
quartic melonic		$\hat{\mathcal{X}}(\boldsymbol{j}) = 4\left(\prod_c \delta_{j_c,0}+\prod_{b\neq c}\delta_{j_b,0}+\delta_{j_c,0}\right)$
quartic necklace		$\hat{\mathcal{X}}(\boldsymbol{j}) = 4\left(\prod_c \delta_{j_c,0}+\delta_{j_1,0}\delta_{j_2,0}+\delta_{j_3,0}\delta_{j_4,0}\right)$
simplicial		$\hat{\mathcal{X}}(\boldsymbol{j}) = 5\sum_{i=0}^4\prod_{k\neq i}\delta_{j_{(ik)},0}$ (edges labeled by adjacent vertices i,k)

- then the 2-point correlation function can be computed as:

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with non-local interactions producing an effective mass: $b_{\boldsymbol{j}} := \mu\left(1-\sum_\gamma \tilde{\lambda}_\gamma \hat{\mathcal{X}}_\gamma(\boldsymbol{j})\right)$

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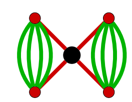
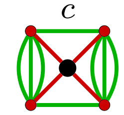
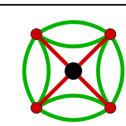
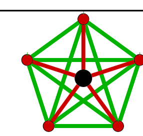
double-trace melon		$\hat{\mathcal{X}}_{\sqcup}(\boldsymbol{j}) = 4\left(2\prod_{c=1}^{4}\delta_{j_c,0}+1\right)$
quartic melonic		$\hat{\mathcal{X}}_{c}(\boldsymbol{j}) = 4\left(\prod_c \delta_{j_c,0}+\prod_{b\neq c}\delta_{j_b,0}+\delta_{j_c,0}\right)$
quartic necklace		$\hat{\mathcal{X}}(\boldsymbol{j}) = 4\left(\prod_c \delta_{j_c,0}+\delta_{j_1,0}\delta_{j_2,0}+\delta_{j_3,0}\delta_{j_4,0}\right)$
simplicial		$\hat{\mathcal{X}}(\boldsymbol{j}) = 5\sum_{i=0}^{4}\prod_{k\neq i}\delta_{j_{(ik)},0}$ (edges labeled by adjacent vertices i,k)

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- with closure condition (on group variables)

$$\Phi(\boldsymbol{\phi},g_1,\dots,g_r) = \Phi(\boldsymbol{\phi},g_1h,\dots,g_rh) \quad \forall h\in G \longrightarrow \hat{C}(\boldsymbol{k},\boldsymbol{j}) = \frac{\int \mathrm{d}h \bigotimes_{c=1}^{r} D^{j_c}(h)}{\alpha(\boldsymbol{j})\sum_i k_i^2+\frac{1}{a_G^2}\sum_c \mathrm{Cas}_{j_c}+b_{\boldsymbol{j}}}$$

Correlation length in TGFTs

note: from now on, restrict to abelian groups $G^r \cong \mathrm{U}(1)^{d_{\mathrm{nl}}}$ $\quad d_{\mathrm{nl}} \equiv r d_G$ $\quad a_G \equiv a^{d_G} \equiv (2\pi\tilde{a})^{d_G}$

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TGFT field domain is not spacetime, thus correlation length indicates "internal" scale

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- up to 2nd order in "momenta" (and using isotropy):

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- in vicinity of phase transition, then: finite non-local contribution to correlation length, negligible to (diverging) local contirbution - no phase transition without local directions

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- correlation length is then (after subtracting divergent factor)
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  - for positive effective mass $\quad \xi_{\mathrm{nl}}^2 = \sum_{s=s_0}^{r} \frac{d_{\mathrm{G}}(r-s_0)}{d_{\mathrm{nl}}} \sum_{(c_1,\ldots,c_s)} \frac{b_{\boldsymbol{0}}}{b_{c_1,\ldots,c_s}^2}$

$d_{\mathrm{G}}s_0$ = minimal number of delta functions in interactions

diverges at phase transition, just like local case

(differences only quantitative)

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same as for 0-dim QFT and local QFT on compact domain

infinite volume limit: $Q \underset{a\to\infty}{\sim} \dfrac{-4f_{s_0}\lambda_\gamma}{\mu^2}\left(\dfrac{\xi}{a}\right)^{d_{\mathrm{G}}(s_0-r)} \underset{\xi\to\infty}{\sim} -4f_{s_0}\lambda_\gamma \dfrac{\xi^{4-d_{\mathrm{G}}(r-s_0)}}{a^{d_{\mathrm{G}}(s_0-r)}}$

critical rank: $r_c = s_0 + 4/d_{\mathrm{G}}$

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infinite volume limit: $Q \underset{a\to\infty}{\sim} \dfrac{-4 f_{s_0}\lambda_\gamma}{\mu^2}\left(\dfrac{\xi}{a}\right)^{d_{\mathrm{G}}(s_0-r)} \underset{\xi\to\infty}{\sim} -4 f_{s_0}\lambda_\gamma \dfrac{\xi^{4-d_{\mathrm{G}}(r-s_0)}}{a^{d_{\mathrm{G}}(s_0-r)}}$

critical rank: $r_c = s_0 + 4/d_{\mathrm{G}}$ for $r < r_c$ divergent Q, no phase trans in Gaussian approx

for $r > r_c$ small Q, Gaussian approx holds at phase transition

- note: for melonic interactions: $Q \sim \xi^{4-d_G(r-1)}$; for necklaces interactions: $s_0 = r/2$

Ginzburg criterion for TGFTs

- cannot assume isotropy, choose averaging domain: $\Omega_\xi \sim \xi_{\mathrm{l}}^{d_{\mathrm{l}}} \times \xi_{\mathrm{nl}}^{d_{\mathrm{nl}}}$
- relative strength of fluctuations given by "Q-parameter": $Q \equiv \frac{\int_{\Omega_\xi} \mathrm{d}^r g \; \mathrm{d}^{d_{\mathrm{l}}}\phi \; C(\boldsymbol{g},\boldsymbol{\phi})}{\int_{\Omega_\xi} \mathrm{d}^r g \mathrm{d}^{d_{\mathrm{l}}}\phi \; \Phi_0^2} = \frac{\int_{\Omega_\xi} \mathrm{d}^{d_{\mathrm{nl}}}\theta \; \mathrm{d}^{d_{\mathrm{l}}}\phi \; C(\boldsymbol{\theta},\boldsymbol{\phi})}{\int_{\Omega_\xi} \mathrm{d}^{d_{\mathrm{nl}}}\theta \; \mathrm{d}^{d_{\mathrm{l}}}\phi \; \Phi_0^2}$
- Ginzburg criterion for validity of Gaussian mean field-approx is: $|Q| \ll 1$
- several contributions, from different TGFT interaction vertices, each giving different number of zero-modes
- restrict attention to terms with positive effective mass
  (only interested in vicinity of phase transition, when correlation length becomes divergent, and negative effective mass gives divergent correlation length at any value of mass coupling)

For non-local directions only:

- for quartic TGFT interaction, we get: $Q = -\frac{4\sum_\gamma \lambda_\gamma}{\mu^2} \sum_{s=s_0}^{r} f_s \left(\frac{\xi}{a}\right)^{d_G(s-r)}$

  independent of

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thus critical dimension = 4; same as in local QFT

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- no insight yet (because of simplicity of models) on geometric/spacetime/physics interpretation

Extension to quantum geometric models (in progress)

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case study: Lorentzian Barrett-Crane model coupled to scalar matter

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justification:

impose geometricity of simplicial structures dual to TGFT quanta and Feynman diagrams,
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- can be expanded in modes (group irreps):

$$\Phi(\boldsymbol{\phi}, \boldsymbol{g}) = \int_{\mathbb{H}^3} \mathrm{d}X \, \Phi(\boldsymbol{\phi}, \boldsymbol{g}, X) \qquad = \prod_{i=1}^{4} \left(\int \mathrm{d}\rho_i 4\rho_i^2 \sum_{\substack{j_i, m_i; \\ l_i, n_i}} D^{(\rho_i, 0)}_{j_i m_i l_i n_i}(g_i) \right) B^{\rho_1\rho_2\rho_3\rho_4}_{l_1 n_1 l_2 n_2 l_3 n_3 l_4 n_4} \Phi^{\rho_1\rho_2\rho_3\rho_4}_{j_1 m_1 j_2 m_2 j_3 m_3 j_4 m_4}(\boldsymbol{\phi})$$

$$\Phi^{\rho_1\rho_2\rho_3\rho_4}_{j_1 m_1 j_2 m_2 j_3 m_3 j_4 m_4}(\boldsymbol{\phi}) = \int_{\mathbb{R}^{d_{\rm loc}}} \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^{d_{\rm loc}}} \Phi^{\rho_1\rho_2\rho_3\rho_4}_{j_1 m_1 j_2 m_2 j_3 m_3 j_4 m_4}(\boldsymbol{k}) \mathrm{e}^{i\boldsymbol{\phi}\cdot\boldsymbol{k}}$$

in terms of Barrett-Crane intertwiner $B^{\rho_1\rho_2\rho_3\rho_4}_{j_1 m_1 j_2 m_2 j_3 m_3 j_4 m_4} \equiv \int \mathrm{d}X \prod_{i=1}^{4} D^{(\rho_i, 0)}_{j_i m_i 0 0}(X)$

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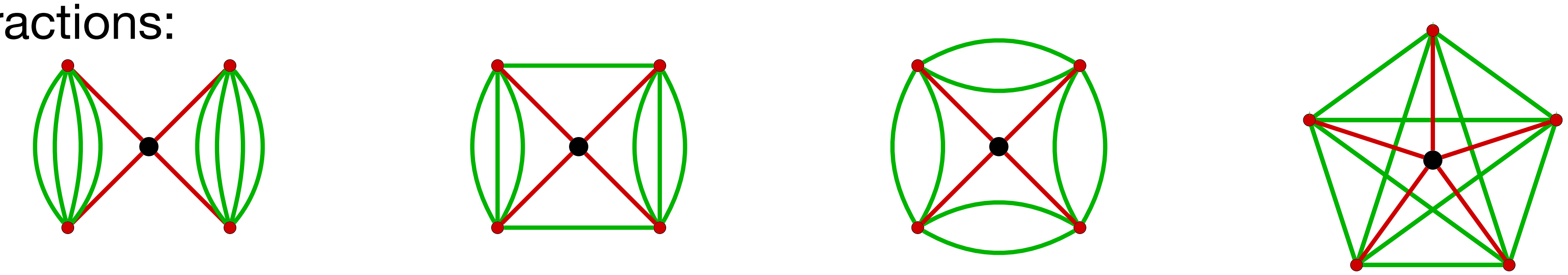
- for example, we can choose interactions:



but if we want to restrict attention to symmetry breaking phase transition for discrete Z2 symmetry, we would pick up only first three

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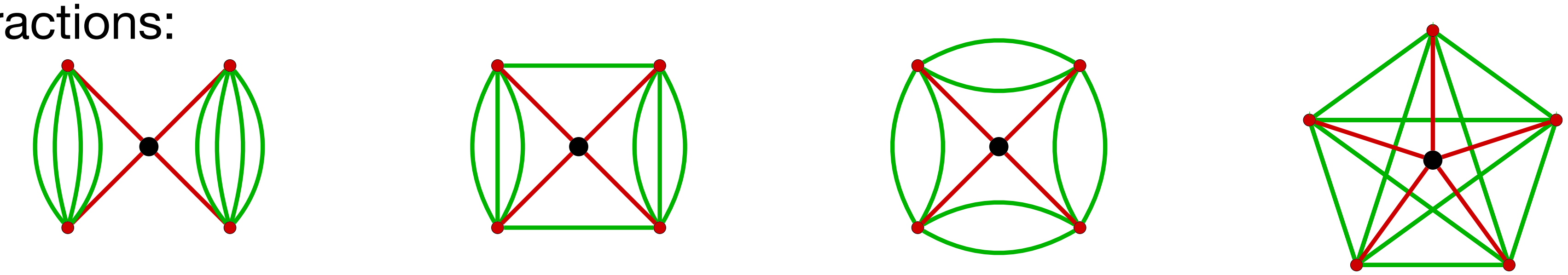
$$= \int_{\mathbb{R}^{d_{\mathrm{loc}}}} \mathrm{d}\boldsymbol{\phi} \int_{\mathrm{SL}(2,\mathbb{C})^4} \mathrm{d}\boldsymbol{g} \int_{\mathbb{H}^3} \mathrm{d}X\, \Phi(\boldsymbol{\phi}, \boldsymbol{g}, X) \left(- \sum_{i=1}^{d_{\mathrm{loc}}} \alpha_i \partial^2_{\phi_i} - \sum_{c=1}^{4} \Delta_c + \mu \right) \Phi(\boldsymbol{\phi}, \boldsymbol{g}, X) +$$

$$\sum_{\gamma} \frac{\lambda_\gamma}{V_\gamma!} \int_{\mathbb{R}^{d_{\mathrm{loc}}}} \mathrm{d}\boldsymbol{\phi} \int_{\mathrm{SL}(2,\mathbb{C})^{4\cdot V_\gamma}} \int_{\mathbb{H}^{3\cdot V_\gamma}} \prod_{i=1}^{V_\gamma} \mathrm{d}\boldsymbol{g}_i \mathrm{d}X_i \prod_{(i,a;j,b)} \delta(g_i^a (g_j^b)^{-1}) \prod_{i=1}^{V_\gamma} \Phi(\boldsymbol{\phi}, \boldsymbol{g}_i, X_i)$$

- for example, we can choose interactions:



but if we want to restrict attention to symmetry breaking phase transition for discrete Z2 symmetry, we would pick up only first three

- giving eqns motion for constant fields (pending regularization):

$$0 = \mu \left(a^3_{\mathbb{H}^3} \mathrm{vol}_{A+} \right) \Phi_0 + \sum_{\gamma} \frac{\lambda_\gamma}{(V_\gamma - 1)!} \left(a^3_{\mathbb{H}^3} \mathrm{vol}_{A+} \right)^{4\frac{V_\gamma - 2}{2} + V_\gamma} \Phi_0^{V_\gamma - 1}$$

$$= \left(\mu + \sum_{\gamma} \frac{\lambda_\gamma}{(V_\gamma - 1)!} \left(a^3_{\mathbb{H}^3} \mathrm{vol}_{A+} \right)^{4\frac{V_\gamma - 2}{2} + V_\gamma - 1} \Phi_0^{V_\gamma - 2} \right) \left(a^3_{\mathbb{H}^3} \mathrm{vol}_{A+} \right) \Phi_0$$

Extension to quantum geometric models (in progress)

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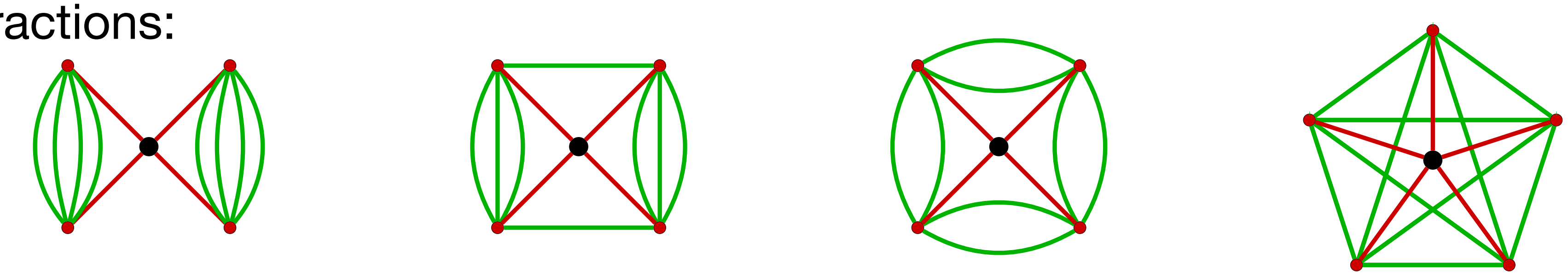

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- and solutions: $$\left(a^3_{\mathbb{H}^3}\mathrm{vol}_{A^+}\right)^{\frac{4}{2}+\frac{V_\gamma-1}{V_\gamma-2}}\Phi_0=\zeta_i\left(-\frac{\mu}{\sum_\gamma\frac{\lambda_\gamma}{(V_\gamma-1)!}}\right)^{\frac{1}{V_\gamma-2}}$$

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dynamics becomes: $\left(\int_{\mathbb{H}^3} \mathrm{d}X \mathcal{K} + F[\Phi_0]\right) \delta\Phi(\boldsymbol{\phi}, \boldsymbol{g}, X) = 0$ with operators (in representation space):

$$\hat{F}[\Phi_0](\boldsymbol{k}, \boldsymbol{\rho}, \boldsymbol{j}, \boldsymbol{m}; \boldsymbol{k'}, \boldsymbol{\rho'}, \boldsymbol{j'}, \boldsymbol{m'}) = -\mu\delta(\boldsymbol{k}+\boldsymbol{k'})\sum_\gamma \tilde{\lambda}_\gamma \hat{\mathcal{X}}_\gamma(\boldsymbol{\rho}, \boldsymbol{j}, \boldsymbol{m}) \prod_{c=1}^{4} \delta(\rho_c - \rho'_c)\mathbb{1}_{\rho_c}\delta_{j_c,j'_c}\mathbb{1}_{j_c}\delta_{m_c,m'_c}\mathbb{1}_{m_c}$$

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- and the correlation function is given by:

$$C(\boldsymbol{\phi}, \boldsymbol{g}) = \int_{\mathbb{H}^3} \mathrm{d}X\, C(\boldsymbol{\phi}, \boldsymbol{g}, X) = \int_{\mathbb{R}^{d_{\mathrm{loc}}}} \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^{d_{\mathrm{loc}}}} \mathrm{e}^{i\boldsymbol{\phi}\cdot\boldsymbol{k}} \prod_{i=1}^4 \left(\int \mathrm{d}\rho_i 4\rho_i^2 \sum_{\substack{j_i, m_i;\\ l_i, n_i}} D^{(\rho_i, 0)}_{j_i m_i l_i n_i}(g_i)\right) B^{\rho_1\rho_2\rho_3\rho_4}_{l_1n_1l_2n_2l_3n_3l_4n_4} \hat{C}^{\rho_1\rho_2\rho_3\rho_4}_{j_1m_1j_2m_2j_3m_3j_4m_4}(\boldsymbol{k})$$

$$\hat{C}^{\rho_1\rho_2\rho_3\rho_4}_{j_1m_1j_2m_2j_3m_3j_4m_4}(\boldsymbol{k}) = \hat{C}(\boldsymbol{k}, \boldsymbol{\rho}, \boldsymbol{j}, \boldsymbol{m}) = \frac{1}{\alpha(\boldsymbol{\rho}, \boldsymbol{j}, \boldsymbol{m})\sum_i k_i^2 + \frac{1}{a^2_{\mathbb{H}^3}}\sum_c \mathrm{Cas}_{1,\rho_c} + b_{\boldsymbol{\rho}, \boldsymbol{j}, \boldsymbol{m}}}$$

with effective mass: $b_{\boldsymbol{\rho}, \boldsymbol{j}, \boldsymbol{m}} := \mu\left(1 - \sum_\gamma \tilde{\lambda}_\gamma \hat{\mathcal{X}}_\gamma(\boldsymbol{\rho}, \boldsymbol{j}, \boldsymbol{m})\right)$

Extension to quantum geometric models (in progress)

case study: Lorentzian Barrett-Crane model coupled to scalar matter

- calculations require regularization: "Wick rotation" (incl. compactification) to Riemannian BC model

P. Dona', F. Gozzini, A. Nicotra, '21

Lorentz group mapped to Spin(4); 3-hyperboloid mapped to 3-sphere; irreps of Lorentz mapper to irreps of Spin(4); ...

need also generalised regularization including different size of "boost direction"

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a) via the reciprocal value of the logarithm of the asymptotic correlation function in direct space

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- technical challenges:

closure and simplicity (thus projection onto homogeneous space) constraints

non-compactness and non-abelian nature (curvature) of Lorentz group

intricacies of representation theory for Lorentz group

curvature requires taking into account contribution from integration measure

expansion in moments requires care with non-commutative plane waves and/or representation functions

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- value of correlation length dictated by zero modes contributing to correlation function as:

$$C_s(g_{c_1},\ldots,g_{c_{\varrho-s}}) = \sum_{m_u\neq 0}\sum_{j_u\neq 0}\prod_{u=1}^{\varrho-s}\left[\int \mathrm{d}\rho_u 4\rho_u^2\, D^{(\rho_u,0)}_{j_u m_u 00}(g_{c_u})\right]\frac{B^{\rho_{c_1},\ldots,\rho_{c_{\varrho-s}}}_{0,\ldots,0}}{\frac{1}{a^2}\sum_{v=1}^{\varrho-s}(\rho_u^2+1)+b_{c_1,\ldots,c_s}}$$

and we are interested in large "distances" on the group (large boosts)

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- so, full correlation length determined by the largest contribution by zero modes

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this is what will be taken to infinity in thermodynamic limit, recovering the full SL(2,C)

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- Ginzburg parameter is then given by:

$$Q \sim \frac{\lambda}{\mu}\mathrm{Vol}(A_\Lambda^+)^3 \sum_{s=s_0}^{\varrho} e^{2(\varrho-s)(\Lambda-\xi)/a} \sum_{c_1,\dots c_s} \frac{1}{b_{c_1,\dots,c_s}} \sim \frac{\lambda \mathrm{Vol}(A_\Lambda^+)^3}{\mu^2} e^{2(\varrho-s_0)(\Lambda-\xi)/a} f_{c_1,\dots,c_{s_0}} \sim \xi^2 \tilde{\lambda} e^{2(\varrho-s_0)(\Lambda-\xi)/a}$$

thus we find exponential suppression, dominated by lowest zero ode

$d_{\mathrm{G}} s_0$ = minimal number of delta functions in interactions

Conclusions

- Landau-Ginzburg analysis can be generalised to TGFTs (with both local and non-local directions)
- results consistent with full FRG results (when available)
- additional local directions ("matter components") affect non-trivially the results ("matter matters")
- in non-compact abelian case, critical dimension depends on: rank, order of interaction, minimal number of zero modes corresponding to interaction
- mean field critical behaviour of TGFT is like local QFT in effective dimension
- melonic interactions drive the critical behaviour
- gauge constraint gives simple rescaling of critical dimension
- in compact abelian case, TGFT at criticality behaves like local QFT driven by local directions only; non-local directions are negligible (no phase transition in purely non-local compact TGFTs)
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.... stay tuned ....

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