Dimensional flow from nonlocality:
some results on a cyclic melonic Tensor Field Theory

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A Joint Work in Progress
with
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Outline

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The concept of dimensionality change

- In theoretical physics: some occurrences of dimensional change

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$\rightarrow$ Kaluza-Klein Theory [1921-1926]

$\rightarrow$ String compactification (extending KK-theory) Compactify some directions (periodic direction): expand the fields in modes along these directions, then let the radius $\rightarrow 0$. Select the modes independent of the directions so that they are not blowing up with the energy;

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$\rightarrow$ Dimensionality reduction: phenomenon at criticality
[Aharony etal (1976). "Lowering of dimensionality in phase transitions with random fields" PRL 37 (20) 1364-1367]
[Parisi, Sourias (1979). "Random Magnetic Fields, Supersymmetry, and Negative Dimensions" PRL 43 (11) 744-745].
"the critical exponents in a d-dimensional ($4 < d < 6$) system with short-range exchange and a random quenched field are the same as those of a $(d-2)$-dimensional pure system."

Dimensional reduction $\Leftrightarrow$ Trade

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$\rightarrow$ PCA and signal detection [refs in talk by Mohamed Ouefelli] via statistical analysis

$\rightarrow$ Clustering algorithms via statistical analysis, delivers also a subset of data that "meaningfully" describes the whole data set.

Dimensional reduction $\Leftrightarrow$ Statistical representativity

Is there a way to SEE (literally) the dimension reducing or even flowing?

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YES !

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Today

• Random tensors in Quantum Gravity, AdS/CFT, Holography, BH, quantum information theory
$\rightarrow$ Flavors in condensed matter model à la SYK

• Random tensors: represent multidimensional data, random noise in Data Sciences, AI, etc...

TFT/TGFT Renormalisation group (RG) analysis

• TFT renormalization perturbative have been worked out since 2011 [BG & Rivasseau 2011]

$T_{a_1 a_2 \dots a_r}$ the indices are propagating themselves.

$\rightarrow$ The field $T : G^r \rightarrow \mathbb{K} = \mathbb{R}, \mathbb{C}$
$\rightarrow$ Kinetic term on G^r & interactions as convolutions/contractions of tensors

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• The Tensor Track for QG and random geometry ©Rivasseau.

Functional Renormalisation Group analysis of TFT/TGFT

• Consider G a compact group and $T : G^r \to \mathbb{K}$

• No possible phase transition as long as G is compact [Benedetti 2014]; in the limit of infinite radius yes.

• 2014: FRG for TFTs and first application with $T : U(1)^3 \to \mathbb{R}$
$\rightarrow$ The system of β-functions was non-autonomous: explicit k in the eq.
$\rightarrow$ due to the radius of the compact manifold
$\rightarrow$ due to the nonlocal interaction
$\rightarrow$ resort in large (integer momentum) mode limit (UV): good notion of scaling dimension of coupling constants;
$\rightarrow$ small mode limit (IR): another notion of scaling dimension of coupling constants;
$\rightarrow$ in each limit you can draw phase diagram: strong evidence of fixed points but the meaning of picture was not clear;

What is small k limit?

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T_{000}? T_{010}?

- Computation at an intermediate/interpolation regime.

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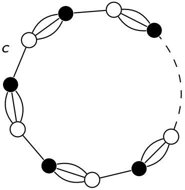
→ Perform a computation of the FRG flow without resorting in any large/small k-limit
Interaction: arbitrary valence of cyclic melonic interactions (nonlocal);

λ_n^c



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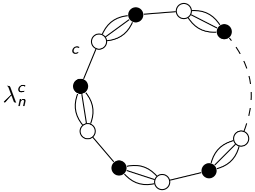

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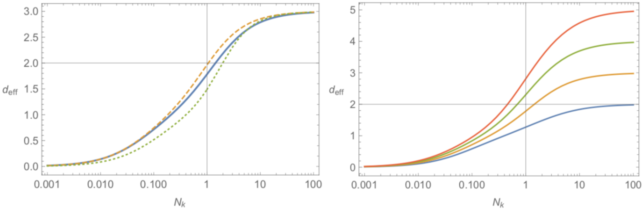

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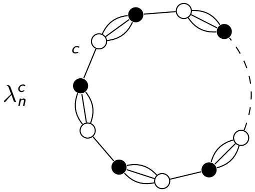

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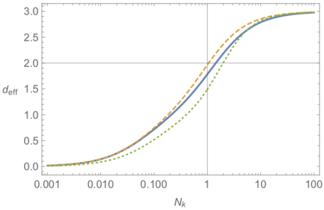

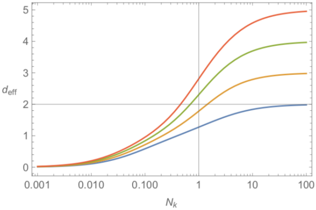

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- The fields: G a Lie group

$$\Phi : \mathbb{R}^d \times G^r \to \mathbb{K} = \mathbb{C}, \mathbb{R} \quad (1)$$
$$(\boldsymbol{x}, \boldsymbol{g}) \mapsto \Phi(\boldsymbol{x}, \boldsymbol{g}) \quad (2)$$

- G is chosen compact $\rightarrow$ Peter-Weyl transform of the field

$$\Phi(\boldsymbol{x}, \boldsymbol{g}) = \int_{\mathbb{R}^d} \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^{d/2}} \mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{x}} \sum_{j_1,\dots,j_r} \left(\prod_{c=1}^{r} d_{j_c} \right) \mathrm{tr}_{\boldsymbol{j}} \left[\Phi_{j_1 j_2 \dots j_r}(\boldsymbol{p}) \bigotimes_{c=1}^{r} D^{j_c}(g_c) \right] \quad (3)$$

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+ 1 new motivation: it will allow a nontrivial dimensional flow towards the IR !

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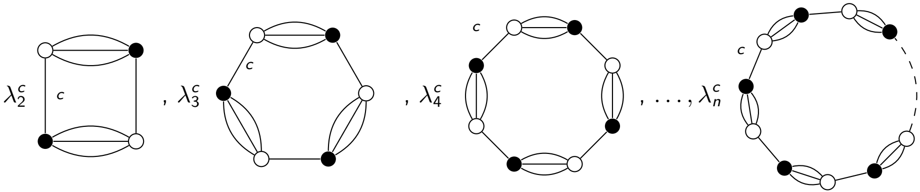

Figure: Rank $d = 4$ cyclic-melonic interactions diagrammatically described by colored graphs.

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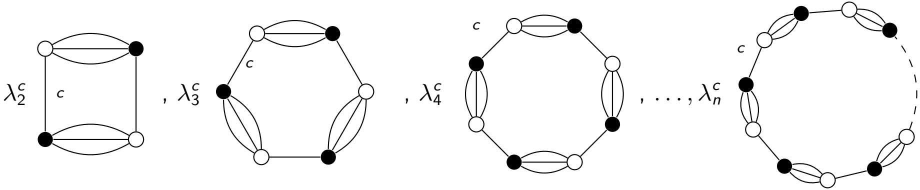

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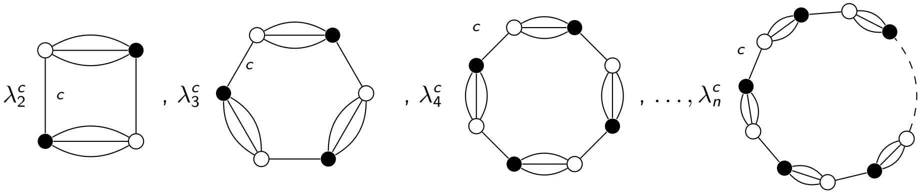

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- Nonlocal and a tractable combinatorics: computable at arbitrary valence.
- $S_{int}(\phi, \bar{\phi}) = \int_{\mathbb{R}^d} \mathrm{d}\boldsymbol{x} \left[\sum_{n=2}^{n_{\max}} \sum_{c=1}^{r} \lambda_n^c \, \mathrm{Tr}_{n;c}(\phi, \bar{\phi})(\boldsymbol{x}) \right]$

TFT model: action

- The action

$$S(\phi,\bar\phi) = S_{kin}(\phi,\bar\phi) + S_{int}(\phi,\bar\phi)$$

$$S_{kin}(\phi,\bar\phi) = (\bar\phi, K\phi) = \int_{\mathbb{R}^d\times\mathbb{R}^d} \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' \int_{G^r\times G^r} \mathrm{d}\boldsymbol{g}\mathrm{d}\boldsymbol{g}' \;\; \bar\phi(\boldsymbol{x},\boldsymbol{g})K(\boldsymbol{x},\boldsymbol{g};\boldsymbol{x}',\boldsymbol{g}'))\phi(\boldsymbol{x}',\boldsymbol{g}')$$

$$K(\boldsymbol{x},\boldsymbol{g};\boldsymbol{x}',\boldsymbol{g}') = \delta(\boldsymbol{x}-\boldsymbol{x}')\delta(\boldsymbol{g}\boldsymbol{g}'^{-1})\Big[\Big(-\Delta_x - \kappa^2\sum_{c=1}^{r}(\Delta_g^{(c)})^\zeta\Big) + \mu_k\Big] \quad (5)$$

where
Δ_x is the Laplacian on $\mathbb{R}^d$,
$\Delta_g^{(c)}$) the (colored) Laplacian on G,
$\zeta\in]0,1]$
κ restores the dimension balance.

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Functional Renormalization Group formalism: Wetterich-Morris equation

[Wetterich 92, Morris 93]

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- The generating function all all correlators

$$Z[J, \bar{J}] = \mathrm{e}^{W[J,\bar{J}]} = \int \mathcal{D}\Phi\mathcal{D}\bar{\Phi}\ \mathrm{e}^{-S[\Phi,\bar{\Phi}]+(J,\Phi)+(\Phi,J)} \tag{6}$$

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$$\varphi(\boldsymbol{x},\boldsymbol{g}) := \langle\Phi(\boldsymbol{x},\boldsymbol{g})\rangle = \frac{\delta W[J,\bar{J}]}{\delta\bar{J}(\boldsymbol{x},\boldsymbol{g})} \quad , \quad \bar{\varphi}(\boldsymbol{x},\boldsymbol{g}) := \langle\bar{\Phi}(\boldsymbol{x},\boldsymbol{g})\rangle = \frac{\delta W[J,\bar{J}]}{\delta J(\boldsymbol{x},\boldsymbol{g})}. \tag{7}$$

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- Effective average action: Legendre transform of $W[J,\bar{J}]$

$$\Gamma[\varphi,\bar{\varphi}] = \sup_{\bar{J},J}\{(\varphi,J) + (J,\varphi) - W[\bar{J},J]\} \qquad (8)$$

Generating function of all 1PI correlation functions.

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• Introduce a scale k and an IR (cut-off) regulator $\mathcal{R}_k$ that projects only on field modes relevant to that scale

$$Z_k[J, \bar{J}] = e^{W_k[J,\bar{J}]} = \int \mathcal{D}\varphi \mathcal{D}\bar{\varphi} e^{-S[\varphi,\bar{\varphi}]-(\varphi,\mathcal{R}_k\varphi)+(J,\varphi)+(\varphi,J)}. \quad (9)$$

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$\mathcal{R}_k$ should satisfy specific conditions;
• Scale dependent effective action

$$\Gamma_k[\varphi,\bar{\varphi}] = \sup_{J,\bar{J}} \left[(\varphi,J)+(J,\varphi)-W_k[J,\bar{J}]\right] - (\varphi,\mathcal{R}_k\varphi). \qquad (10)$$

• Expansion for TFT:

$$\Gamma_k[\varphi,\bar{\varphi}] = (\varphi,\mathcal{K}_k\varphi) + \sum_\gamma \lambda_{\gamma;k}\mathrm{Tr}_\gamma[\varphi,\bar{\varphi}],$$

$$\mathcal{K}_k = Z_k\Big(-\Delta_x - \kappa^2\sum_{c=1}^{r}(\Delta_g^{(c)})^\zeta\Big) + \mu_k \qquad (11)$$

FRG formalism for TFT: Wetterich-Morris equation

- Flow equation for the effective average action: The Wetterich-Morris equation

$$(k\partial_k)\,\Gamma_k[\varphi,\bar\varphi] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\mathbb{I}_2\right)^{-1}(k\partial_k)\,\mathcal{R}_k\right], \tag{12}$$

where STr is a supertrace (all configuration space variables integrated), $\Gamma_k^{(2)}$ is the Hessian matrix of Γ_k

$$\begin{aligned}
\Gamma_k^{(2)}[\varphi,\bar\varphi](\boldsymbol{x},\boldsymbol{g};\boldsymbol{y},\boldsymbol{h}) &:= \frac{\delta^2\Gamma_k[\varphi,\bar\varphi]}{\delta\varphi(\boldsymbol{x},\boldsymbol{g})\delta\bar\varphi(\boldsymbol{y},\boldsymbol{h})}\\
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\Gamma_k^{(2)}[\bar\varphi,\bar\varphi](\boldsymbol{x},\boldsymbol{g};\boldsymbol{y},\boldsymbol{h}) &:= \dots
\end{aligned} \tag{13}$$

- Results are dependent on $\mathcal{R}_k$ and the ansatz for Γ_k;

$\Rightarrow$ Prove that the results holds for classes of regulators and an enlarged truncation helps in gaining confidence in the results.

Outline

The cyclic melonic potential approximation

- Now for simplicity we will restrict to $G = U(1)$.
- We project on constant and uniform fields

$$\varphi(\boldsymbol{x}, \boldsymbol{g}) = \chi \tag{14}$$

$$\Gamma_k[\varphi, \bar{\varphi}] = \Gamma_k(\rho) = U_k(\rho) = a_{\mathbb{R}}^d a_G^r \mu_k \chi^2 + a_{\mathbb{R}}^d \sum_{n=2}^{n_{\max}} \Big(\sum_{\gamma | V_\gamma = 2n} \lambda_{\gamma;k} \Big) (a_G^r \chi^2)^n ,$$
$$\rho := a_G^r \chi^2 \tag{15}$$

where $a_{\mathbb{R}}$ is the formal volume of $\mathbb{R}$ and a_G the volume of the G (note that we do not use Haar measure);

$\rightarrow$ For the cyclic melonic potential: $\sum_{\gamma | V_\gamma = 2n} = \sum_{c=1}^{r}$

The cyclic melonic potential approximation

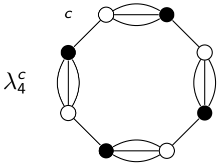
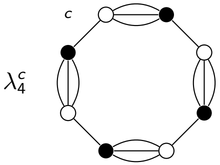


Figure: 2nd order derivative of a rank $d = 4$ cyclic-melonic interaction $2n = 8$.

- Second field derivative of the interacting part: example

$$F_2[\varphi, \bar{\varphi}](\boldsymbol{x}, \boldsymbol{g}; \boldsymbol{y}, \boldsymbol{h}) = \sum_{c=1}^{r} \sum_{n=2}^{n_{\max}} \frac{n}{n!} \lambda_{n,k}^{c} \Big[$$

The cyclic melonic potential approximation



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$$\Big[\prod_{b \neq c} \delta(g_b, h_b)\Big] \left(\bar{\varphi} \cdot_{\hat{c}} \varphi\right)^{n-1} (g_c, h_c)$$

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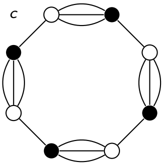

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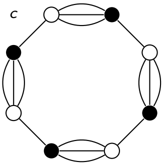
$$F_2[\varphi,\bar\varphi](\boldsymbol{x},\boldsymbol{g};\boldsymbol{y},\boldsymbol{h}) = \sum_{c=1}^{r}\sum_{n=2}^{n_{\max}} \frac{n}{n!}\lambda^c_{n,k}\Big[$$

$$\Big[\prod_{b\neq c}\delta(g_b,h_b)\Big]\,(\bar\varphi\cdot_{\hat c}\varphi)^{n-1}(g_c,h_c) + \delta(g_c,h_c)(\bar\varphi\cdot_c\varphi)^{n-1}(\hat{\boldsymbol{g}}_c,\hat{\boldsymbol{h}}_c)$$

The cyclic melonic potential approximation



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- Second field derivative of the interacting part: example

$$F_2[\varphi,\bar\varphi](\boldsymbol{x},\boldsymbol{g};\boldsymbol{y},\boldsymbol{h}) = \sum_{c=1}^{r}\sum_{n=2}^{n_{\max}} \frac{n}{n!}\lambda^c_{n,k}\Big[\Big[\prod_{b\neq c}\delta(g_b,h_b)\Big](\bar\varphi\cdot_{\hat c}\varphi)^{n-1}(g_c,h_c) + \delta(g_c,h_c)(\bar\varphi\cdot_c\varphi)^{n-1}(\hat{\boldsymbol{g}}_c,\hat{\boldsymbol{h}}_c) + \sum_{p=1}^{n-2}(\bar\varphi\cdot_{\hat c}\varphi)^p(g_c,h_c)(\bar\varphi\cdot_c\varphi)^{n-p-1}(\hat{\boldsymbol{g}}_c,\hat{\boldsymbol{h}}_c)\Big]. \quad (16)$$

The cyclic melonic potential approximation: Projection on local fields

- Projection on local fields after derivation:

$$
\begin{aligned}
&F_2[\bar\chi,\chi](\boldsymbol{x},\boldsymbol{g};\boldsymbol{y},\boldsymbol{h}) = \\
&a_{\mathbb{R}}^d \sum_{c=1}^{r}\sum_{n=2}^{n_{\max}} \frac{n}{n!}\lambda_n^c a_{\mathrm{G}}^{(n-2)r}\left(\prod_{b\neq c} a_g\delta(g_b,h_b) + a_G\delta(g_c,h_c) + n-2\right)(\bar\chi\chi)^{n-1} \\
&= a_{\mathbb{R}}^d a_{\mathrm{G}}^{-r}\sum_{c=1}^{r}\left[\left(a_G\prod_{b\neq c}\delta(g_b,h_b) + a_G\delta(g_c,h_c) - 1\right)V_k^{c\,\prime}(\rho) + \rho V''(\rho)\right] \\
&V_k^c(z) = \sum_{n=2}^{n_{\max}}\frac{1}{n!}\lambda_{n,k}^c z^n
\end{aligned}
\tag{17}
$$

- Regulator in momentum space

$$
\mathcal{R}_k(\boldsymbol{p},\boldsymbol{j}) = Z_k\left(k^2 - p^2 - \kappa^2\frac{C_{\boldsymbol{j}}^{(\zeta)}}{a_{\mathrm{G}}^{2\zeta}}\right)\theta\left(k^2 - p^2 - \kappa^2\frac{C_{\boldsymbol{j}}^{(\zeta)}}{a_{\mathrm{G}}^{2\zeta}}\right) \tag{18}
$$

where $C_{\boldsymbol{j}}^{(\zeta)}$ is the fractional Casimir of G^r (think about $C_{\boldsymbol{j}}$ as $\sum_{j_c} j_c(j_c+1)$ for $SU(2)$ or $\sum_c j_c^2$ for $U(1)^r$).

The full non autonomous system

• Scale $t = \log k$ then $\partial_t = k\partial_k$

$$\partial_t U_k(\rho) = \tag{19}$$

$$\frac{1}{2}\int_{\mathbb{R}^d}\frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^{d/2}}\sum_{\{j_c\}\in\mathbb{Z}^r}\left[\frac{\partial_t\mathcal{R}_k(\boldsymbol{p},\boldsymbol{j})}{P_{\mathrm{R}}+\sum_c \mathcal{O}^c_{\boldsymbol{j}} V^{c\,\prime}_k(\rho)}+\frac{\partial_t\mathcal{R}_k(\boldsymbol{p},\boldsymbol{j})}{P_{\mathrm{R}}+\sum_c \mathcal{O}^c_{\boldsymbol{j}} V^{c\,\prime}_k(\rho)+2\rho\mathcal{O}_{0\boldsymbol{j}}\sum_c V^{c\,\prime\prime}_k(\rho)}\right],$$

where the $\mathcal{O}^c_{\boldsymbol{j}}$ and$\mathcal{O}_{0\boldsymbol{j}}$ encodes now nonlocality

$$\mathcal{O}^c_{\boldsymbol{j}} := \delta_{0j_c} + (1-\delta_{0j_c})\prod_{b\neq c}\delta_{0j_b} \quad , \quad \mathcal{O}_{0\boldsymbol{j}} = \prod_c \delta_{0j_c} \tag{20}$$

and assuming $\theta\left(k^2 - p^2 - \kappa^2\frac{C^{(\zeta)}_{\boldsymbol{j}}}{a^{2\zeta}_{\mathrm{G}}}\right) = 1$ holds:

$$P_{\mathrm{R}} = Z_k\left(p^2 + \kappa^2\frac{C^{(\zeta)}_{\boldsymbol{j}}}{a^{2\zeta}_{\mathrm{G}}}\right) + \mu_k + \mathcal{R}_k(\boldsymbol{p},\boldsymbol{j}) = Z_k k^2 + \mu_k \tag{21}$$

The cyclic melonic potential approximation: isotropic sector

- We consider the isotropic sector: $\lambda^c_{n,k} = \lambda_{n,k}/r \;,\; \forall c = 1, \dots, r.$

$$U_k(\rho) = \mu_k\,\rho + \sum_{n=2}^{n_{\max}} \Big(\sum_{\gamma | V_\gamma = 2n} \lambda_{\gamma;k} \Big) \rho^n = \mu_k\,\rho + \sum_{n=2}^{n_{\max}} \sum_{c=1}^{r} \lambda^c_{n,k} \rho^n$$
$$= \mu_k \rho + \sum_{n=2}^{\infty} \frac{1}{n!} \lambda_{n,k} \rho^n \tag{22}$$

- The FRG equation becomes:

$$k\partial_k U_k(\rho) = \frac{I^{(d,0)}_{\eta_k}(k)}{k^2 Z_k + U'_k(\rho) + 2\rho\, U''_k(\rho)} + \frac{I^{(d,0)}_{\eta_k}(k) + 2r I^{(d,1)}_{\eta_k}(k)}{k^2 Z_k + U'_k(\rho)}$$
$$+ 2 \sum_{s=2}^{r} \binom{r}{s} \frac{I^{(d,s)}_{\eta_k}(k)}{k^2 Z_k + \mu_k + \frac{r-s}{r} V'_k(\rho)} \tag{23}$$

Threshold spectral sums in rank $s \leq r$

- The master: $\eta_k = -\partial_t \log Z_k$

$$I_{\eta_k}^{(d,s)}(k) = k^2 Z_k \left(1 - \frac{\eta_k}{2}\right) I_0^{(d,s)} + Z_k \frac{\eta_k}{2} \left(I_1^{(d,s)} + I_2^{(d,s)}\right) \tag{24}$$

- The threshold (discrete-volume) functions: setting $\xi = 0, 1$

$$I_\xi^{(d,s)}(k) = \int_{\mathbb{R}^d} \mathrm{d}\boldsymbol{p}\; p^{2\xi} \sum_{\boldsymbol{j} \in (\mathbb{Z}\setminus\{0\})^s} \theta\left(k^2 - p^2 - \frac{\kappa^2}{a_{\mathrm{G}}^{2\zeta}} C_{\boldsymbol{j}}^{(\zeta)}\right) \tag{25}$$

$$I_\xi^{(d,0)}(k) = \int_{\mathbb{R}^d} \mathrm{d}\boldsymbol{p}\; p^{2\xi}\, \theta\left(k^2 - p^2\right) = \frac{1}{d + 2\xi} v_d k^{d+2\xi} \tag{26}$$

$$I_2^{(d,s)}(k) = \int_{\mathbb{R}^d} \mathrm{d}\boldsymbol{p} \sum_{\boldsymbol{j} \in (\mathbb{Z}\setminus\{0\})^s} \sum_{c=1}^{s} \frac{\kappa^2}{a_{\mathrm{G}}^{2\zeta}} (C_{j_c})^\zeta\, \theta\left(k^2 - p^2 - \frac{\kappa^2}{a_{\mathrm{G}}^{2\zeta}} C_{\boldsymbol{j}}^{(\zeta)}\right) \tag{27}$$

$$I_2^{(d,0)}(k) = 0 \tag{28}$$

$\rightarrow$ The sums over discrete volumes have a long history [trace back to polytope volumes, combinatorics and asymptotics Birkhoff].
$\rightarrow$ Difficult to handle in full generality.
$\rightarrow$ Hopefully: no need of an explicit expression, but just their behavior !

Threshold spectral sums in rank $s \leq r$

• We set $\zeta = 1/2$: (Strong constraint)

$$I_\xi^{(d,s)}(k) \approx 2^s \frac{v_d}{s!} k^{d+2\xi} \Big(\frac{1}{2\xi + d} + (\dots)(\frac{a_G k}{\kappa})^2 + (\dots)(\frac{a_G k}{\kappa})^4 + \dots + \frac{(-1)^s}{2\xi + d + 2s}(\frac{a_G k}{\kappa})^{2s} \Big)$$
$$I_2^{(d,s)}(k) \approx 2^s \frac{v_d}{(s-1)!} k^{d+2} \Big(c_1 + (\dots)\Big(\frac{a_G k}{\kappa^2}\Big)^2 + \dots + \frac{(-1)^{2s}}{d+2s+2}\Big(\frac{a_G k}{\kappa^2}\Big)^{2s} \Big) \tag{29}$$

• Coefficients of the polynomials are not relevant for the dimensionless flow equations: eventually as they can be eliminated by rescaling.

• The fact that they are polynomial is what truly matters in the IR: $I_l^{(d,s)}(k) = \sum_{i=0}^{d+s} v_{l,i}\, k^i$, $\quad l = 0, 1, 2$, with a particular expansion for $i < d$.

• $\zeta = 1/2$ a strong constraint: a priviledge model?

The full β-functions

- The dimensionful β-functions

$$\beta_{n,k}(\mu,\lambda_i) = \beta_n^{\mathrm{v}1}(\mu_k,\lambda_i) I_{\eta_k}^{(d,0)}(k) + 2\sum_{l=1}^{n} \beta_{n,l}^{\mathrm{v}2}(\mu_k,\lambda_i) F_{\eta_k,l}^{(d,r)}(k) \tag{30}$$

where $\beta_{n,k} = \partial_t \lambda_{n,k},\ n \geq 2$ and $\partial_t \mu_k$ for $n = 1$;

- Coeff type 1

$$\beta_0^{\mathrm{v}1}(\mu_k,\lambda_i) = \frac{1}{Z_k k^2 + \mu_k} \tag{31}$$

$$\beta_n^{\mathrm{v}1}(\mu_k,\lambda_i) = \sum_{l=1}^{n} \frac{(-1)^l l!}{(Z_k k^2 + \mu_k)^{l+1}} B_{n,l}\left(3\lambda_2, 5\lambda_3, \ldots, (2n-2l+3)\lambda_{n-l+2}\right) . \tag{32}$$

- Coeff type 2

$$\beta_{n,l}^{\mathrm{v}2}(\mu_k,\lambda_i) = \frac{(-1)^l l!}{(Z_k k^2 + \mu_k)^{l+1}} B_{n,l}\left(\lambda_2, \lambda_3, \ldots, \lambda_{n-l+2}\right)$$

with $B_{n,l}(x_1, \ldots, x_{n-l+1})$ are the so-called Bell polynomials;

- and the non-autonomous part:

$$F_{\eta_k,l}^{(d,r)}(k) := \frac{1}{2} I_{\eta_k}^{(d,0)}(k) + r I_{\eta_k}^{(d,1)}(k) + \sum_{s=2}^{r} \binom{r}{s} \left(\frac{r-s}{r}\right)^l I_{\eta_k}^{(d,s)}(k)$$

The full β-functions

- At the first order $n \leq 2$, i.e. φ^4 truncation:

Proposition

$$\partial_t \mu_k = \frac{(-\lambda_2)}{(Z_k k^2 + \mu_k)^2} \left[3 I_{\eta_k}^{(d,0)}(k) + 2 F_{\eta_k,1}^{(d,r)}(k) \right] \tag{33}$$

$$\partial_t \lambda_2 = \frac{2(\lambda_2)^2}{(Z_k k^2 + \mu_k)^3} \left[9 I_{\eta_k}^{(d,0)}(k) + 2 F_{\eta_k,2}^{(d,r)}(k) \right] \tag{34}$$

- At rank $r = 0$: No-Nonlocality (usual $\sum_{n=2}^{n_{\max}} |\varphi|^{2n}$ model on $\mathbb{R}^d$)

$$\beta_{n,k}(\mu, \lambda_i) = \beta_n^{\mathrm{V}1}(\mu_k, \lambda_i) I_{\eta_k}^{(d,0)}(k) + 2 \sum_{l=1}^{n} \beta_{n,l}^{\mathrm{V}2}(\mu_k, \lambda_i) F_{\eta_k,l}^{(d,r=0)}(k) \tag{35}$$

$$\begin{aligned} I_{\eta_k}^{(d,0)}(k) &= k^2 Z_k \left(1 - \frac{\eta_k}{2}\right) I_0^{(d,0)} + Z_k \frac{\eta_k}{2} I_1^{(d,0)} \\ &= Z_k k^2 \frac{v_d}{d} k^d \left(1 - \frac{\eta_k}{d+2}\right) \\ F_{\eta_k,l}^{(d,r=0)}(k) &= \frac{1}{2} I_{\eta_k}^{(d,0)}(k) \end{aligned} \tag{36}$$

- $F_{\eta_k,l}^{(d,r)} = Z_k k^2 F_l^{(d,r)} + Z_k \frac{\eta_k}{2} G_l^{(d,r)}$

are dimensionful quantities and encode the scaling dimension of the coupling constants.

The matter of dimension and (re-)scaling

- Dimensionless couplings

$$\mu_k = Z_k k^2 \tilde{\mu}_k \qquad \lambda_n = Z_k^n k^{2n} \left(F_1^{(d,r)}(k)\right)^{1-n} \tilde{\lambda}_n \quad \text{for } n \geq 2$$

$$Z_k^n k^{2n} \left(F_1^{(d,0)}(k)\right)^{1-n} = Z_k^n k^{2n} \left(k^d\right)^{1-n} = Z_k^n k^{d-(d-2)n} \tag{37}$$

- Effective dimension

$$d_{\rm eff}(k) := \frac{\partial \log F_1^{(d,r)}(k)}{\partial \log k} \tag{38}$$

- Coupling constant equation: $n \geq 2$

$$\partial_t \tilde{\lambda}_n = - d_{\rm eff}(k) \tilde{\lambda}_n + n(d_{\rm eff}(k) - 2 + \eta_k) \tilde{\lambda}_n \tag{39}$$

$$+ \frac{\left(1 - \frac{\eta_k}{2}\right) I_0^{(d,0)}(k) + \frac{\eta_k}{2} \frac{I_1^{(d,0)}(k)}{k^2}}{F_1^{(d,r)}(k)} \beta_n^{\rm V1}(\tilde{\lambda}_i)$$

$$+ 2 \sum_{l=1}^{n} \left(\frac{F_l^{(d,r)}(k)}{F_1^{(d,r)}(k)} - \frac{\eta_k}{2} \frac{G_l^{(d,r)}(k)}{k^2 F_1^{(d,r)}(k)} \right) \beta_{n,l}^{\rm V2}(\tilde{\lambda}_i) \ .$$

Flow of dimension

- Limits

$$d_{\text{eff}}(k \gg 1) = d + r - 1 \qquad d_{\text{eff}}(k \ll 1) = d \tag{40}$$

- At finite k: $F_1^{(d,r)}(k)$ is a polynomial in k;

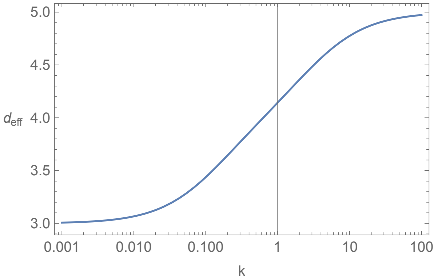

Figure: Flow of effective dimension for $d = r = 3$ for φ^4-model (with $a_{\mathrm{G}} = 1$) using the integral approximation to the threshold function $I_0^{(3,3)}$.

Fixed points, phase transition and symmetry broken

• Fixed points *to* work in progress: We have hints that we recover the structure of fixed of a ϕ^4 in in the IR; but in the UV?

• Numerics: symmetry may be restored in the IR, for a choice of $\mu_k < 0$

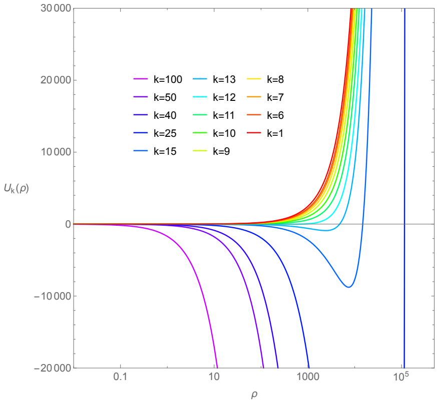

Figure: Symmetry restoration in the IR for $d = r = 3$ for φ^6-model.

Fixed points, phase transition and symmetry broken

• Numerics: we see symmetry is still broken in the IR (thus phase transition): for another choice $\mu_k < 0$ (15% off the previous choice)

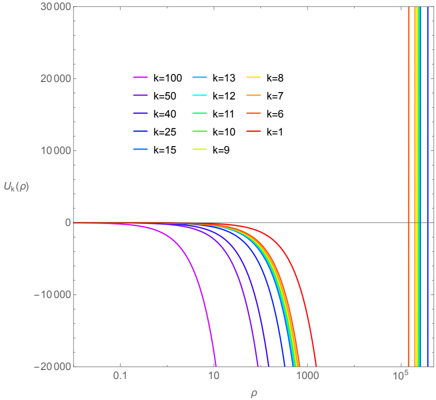

Figure: Symmetry remains broken in the IR for $d = r = 3$ for φ^6-model.

Outline

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Thank you !