JT gravity at finite cutoff

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Work in progress, in collaboration with Frank Ferrari (ULB) and Nicolas Delporte (OIST)

18/05/2021

Outline

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1. Short review of JT gravity
   - Motivations
   - JT gravity in the Schwarzian limit
2. From embeddings to immersions
   - Why immersions?
   - What is the boundary of an immersed disk?
3. JT at finite cutoff
   - Conformal gauge
   - The action
   - The path integral
4. Conclusion

Motivations

Jackiw-Teitelboim gravity is a model of 2d dilaton gravity:

- Appears in the dimensional reduction of the NH limit of NEBH.
- Dual to the SYK model: $\mathcal{N}AdS_2/\mathcal{N}CFT_1$ holography.
  $\rightarrow$ Toy model for islands.
- Model of 2d QG different from Liouville or topological gravity.

Outline

Euclidean JT gravity

$$Z = \int \mathcal{D}g_{\mu\nu}\mathcal{D}\phi \exp\left\{\frac{1}{16\pi G_N}\int \mathrm{d}^2x\sqrt{g}\phi(R+2) + \frac{\phi_b}{8\pi G_N}\oint k\mathrm{d}s\right\}. \quad (1)$$

Dilaton fixes $R=-2$, the H^2 metric is: $\mathrm{d}s^2 = \frac{\mathrm{d}t^2+\mathrm{d}x^2}{x^2}$.

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Reparametrization ansatz[1]: cutoff a boundary curve $(t(u), x(u))$, with fixed proper length $l = \frac{\beta}{\epsilon}$,

$$g|_{bdy} = \frac{1}{\epsilon^2}, \qquad \frac{t'^2+x'^2}{x^2} = \frac{1}{\epsilon^2} \to x = \epsilon t' + O(\epsilon^3). \quad (2)$$

[1] Juan Maldacena, Douglas Stanford, and Zhenbin Yang. "Conformal symmetry and its breaking in two dimensional Nearly Anti-de-Sitter space". In: *PTEP* 2016.12 (2016). arXiv: 1606.01857 [hep-th].

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Dilaton profile solution of EOM: $\phi = \frac{\alpha+\gamma t+\delta(t^2+x^2)}{x}$,
$\to$ dilaton at the boundary: $\phi_b = \frac{\phi_r}{\epsilon}$.

[1] Maldacena, Stanford, and Yang, "Conformal symmetry and its breaking in two dimensional Nearly Anti-de-Sitter space".

Schwarzian action

The action reduces to the boundary term:

$$\mathcal{S}_{JT} \to -\frac{1}{8\pi G_N}\frac{\phi_r}{\epsilon}\int_0^\beta \frac{\mathrm{d}u}{\epsilon}k, \tag{3}$$

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with the extrinsic curvature in the limit $\epsilon \to 0$ (or $l = \frac{\beta}{\epsilon} \to \infty$)

$$k = \frac{t'(t'^2 + x'^2 + xx'') - xx't''}{(t'^2 + x'^2)^{\frac{3}{2}}} = 1 + \epsilon^2 \mathrm{Sch}[t, u], \tag{4}$$

and $\mathrm{Sch}[t, u] = \frac{t'''}{t'} - \frac{3t''^2}{2t'^2}$ has a $\mathrm{PSL}(2, \mathbb{R})$ symmetry:
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Metric $g_{\mu\nu} \to$ reparametrization $t(u)$.

Is the reparametrization a good characterisation of the metric at finite cutoff ?

Localization

Noting $t = \tan\frac{g}{2}$, the path integral reduces to

$$Z = \int_{\frac{\mathbf{Diff}(S_1)_+}{\mathbf{PSL}(2,\mathbb{R})}} \mathcal{D}g \exp\left(\frac{\phi_r}{8\pi G_N}\int_0^\beta \mathrm{d}u \,\mathrm{Sch}[\tan\frac{g}{2}, u]\right). \tag{5}$$

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The integration is exact[2]:

[2] Douglas Stanford and Edward Witten. "Fermionic Localization of the Schwarzian Theory". In: *JHEP* 10 (2017). arXiv: 1703.04612 [hep-th].

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The integration is exact[2]:

- $\frac{\mathrm{Diff}(S_1)_+}{\mathrm{PSL}(2,\mathbb{R})}$ is a coadjoint orbit of Virasoro group.
  $\rightarrow$ symplectic manifold by Kirillov-Kostant-Souriau construction.

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- $\mathcal{D}g$ is the Liouville measure given by the sympletic form.
- Schwarzian is the Hamlitonian $U(1)$ generator on the orbit.
- Duistermaat–Heckman: "stationary phase approximation" is exact.

Other approach without DH : Goldstone vs gauge theory[3].

[2] Stanford and Witten, "Fermionic Localization of the Schwarzian Theory".

[3] Dionysios Anninos, Diego M. Hofman, and Stathis Vitouladitis. "One-dimensional Quantum Gravity and the Schwarzian theory". In: *JHEP* 03 (2022). arXiv: 2112.03793 [hep-th].

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Why immersions?

Disk metric in conformal gauge (more details later)

$$\mathrm{d}s^2 = \frac{4|F'(z)|^2}{(1-|F(z)|^2)^2}|\mathrm{d}z|^2, \tag{6}$$

with $F : \mathcal{D} \to H^2$ holomorphic function.
Well defined metric for $F'(z) \neq 0$ for all $z \in \mathcal{D}$.

Which F are allowed?

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Embedding : globally injective $\implies F'(z) \neq 0$.

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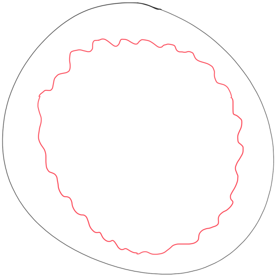
Which F are allowed?
Embedding : globally injective $\implies F'(z) \neq 0$.
Immersion : locally injective $\iff F'(z) \neq 0$.

Metric $\leftrightarrow$ Immersion F

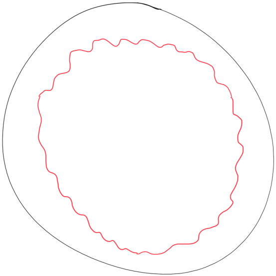
Boundary curves



(a) Reparametrization embedding.

Figure: Different types of metrics obtained from deformed disks.

Boundary curves



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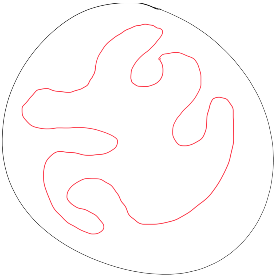


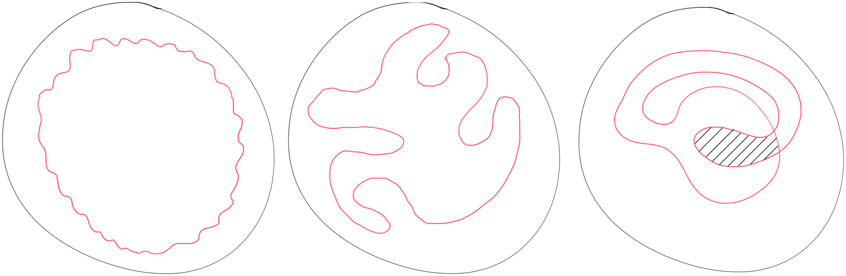
(b) General embedding, self-avoiding loop[4]

Figure: Different types of metrics obtained from deformed disks.

General embedding: $g(u) \notin \text{Diff}(S^1)_+$ boundary curve has turning points.
In Poincaré disk coordinates: $g(u) \to \Phi(u)$ angle.

[4] Douglas Stanford and Zhenbin Yang. "Finite-cutoff JT gravity and self-avoiding loops". In: (Apr. 2020). arXiv: 2004.08005 [hep-th].

Boundary curves



(a) Reparametrization embedding.

(b) General embedding, self-avoiding loop[4]

(c) Immersion with self-overlap.

Figure: Different types of metrics obtained from deformed disks.

At finite cut-off, can we describe the metrics/immersions by their boundary curve?

$\rightarrow$ **What is the boundary of an immersed disk?**

[4]Stanford and Yang, "Finite-cutoff JT gravity and self-avoiding loops".

Outline

What is the boundary of an immersed disk?

Boundary curve = self-overlapping curve[5]

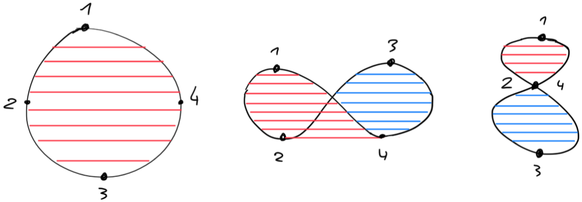


Figure: Immersed disk: Deformed without folding or twisting[6].
→ Not all self-intersecting curves bound a disk.

[5] Valentin Poénaru. "Extension des immersions en codimension 1". In: *Séminaire Bourbaki : années 1966/67 1967/68, exposés 313-346*. Séminaire Bourbaki 10. talk:342. Société mathématique de France, 1968.

[6] Jack E. Graver and Gerald T. Cargo. "When Does a Curve Bound a Distorted Disk?" In: *SIAM Journal on Discrete Mathematics* 25.1 (2011). eprint: https://doi.org/10.1137/090767716.

What is the boundary of an immersed disk?

Boundary curve = self-overlapping curve[5]

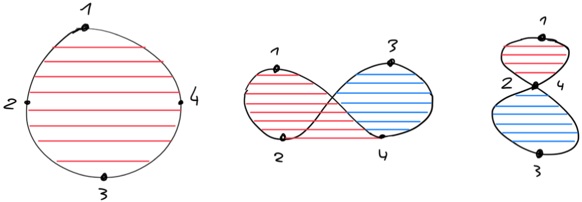


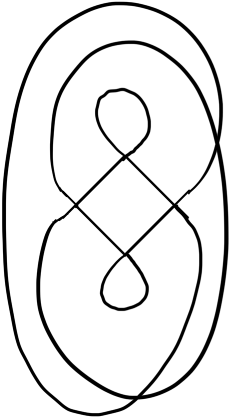
Figure: Immersed disk: Deformed without folding or twisting[6].
→ Not all self-intersecting curves bound a disk.

Subtlety: one curve can bound several inequivalent immersed disks.

→ Simplest example: Milnor's curve.

[5] Poénaru, "Extension des immersions en codimension 1".
[6] Graver and Cargo, "When Does a Curve Bound a Distorted Disk?"

Milnor's curve



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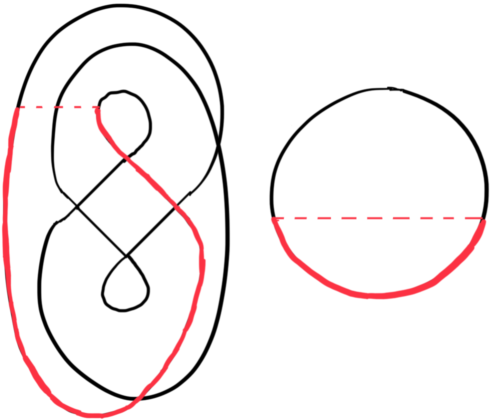

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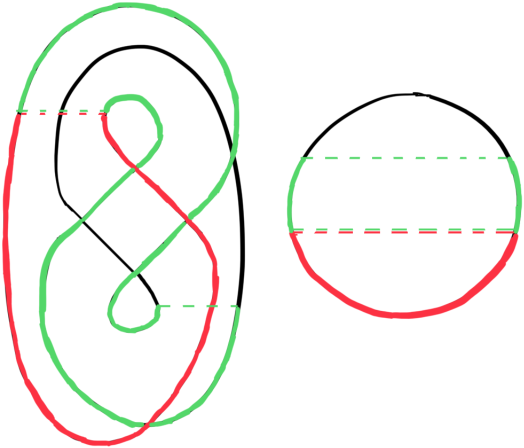

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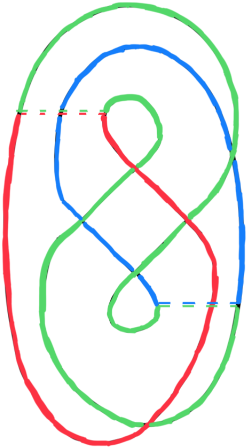

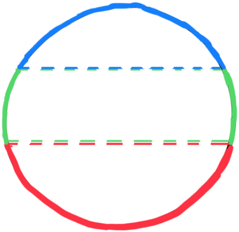

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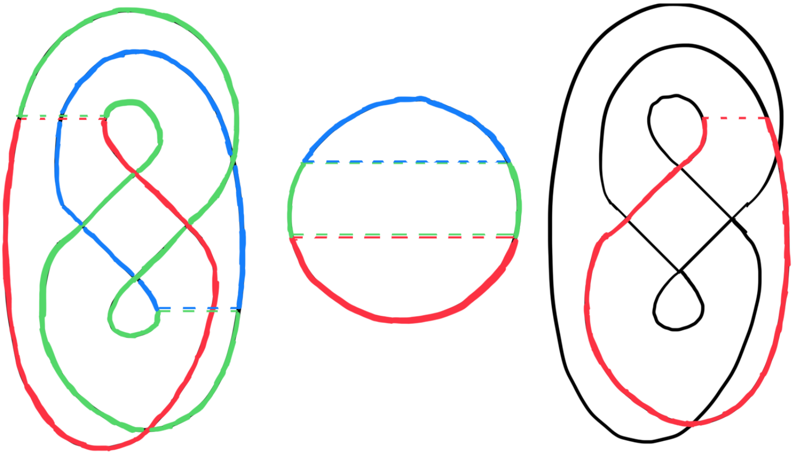

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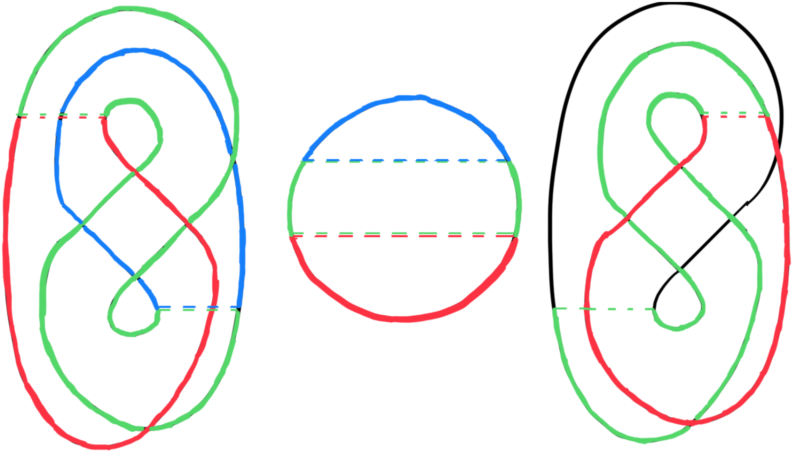

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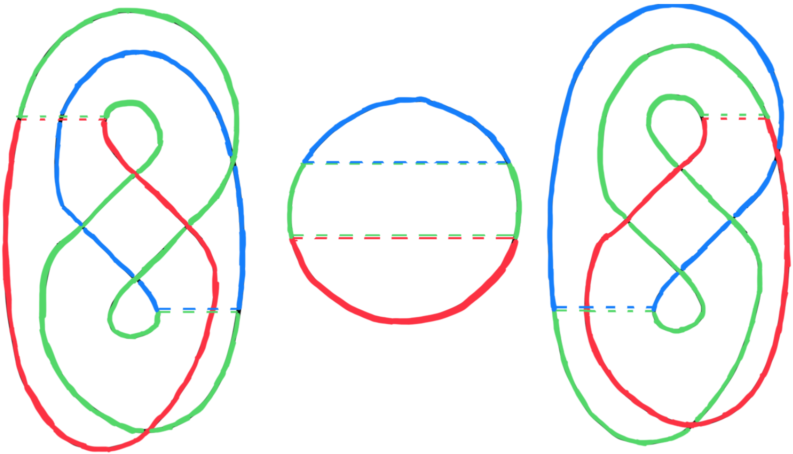

→ Two inequivalent immersed disks.

1 boundary curve $\not\leftrightarrow$ 1 immersed disk

Algorithms to count how many disks are bounded by the same self-overlapping curve[7,8].

$\rightarrow$ disks/metrics are not well characterised by boundary curves.

Then what characterises metrics?

[7] Peter W. Shor and Christopher J. Van Wyk. "Detecting and decomposing self-overlapping curves". In: *Computational Geometry* 2.1 (1992).

[8] Uddipan Mukherjee. "Self-overlapping curves: Analysis and applications". In: *Comput. Aided Des.* 46 (2014).

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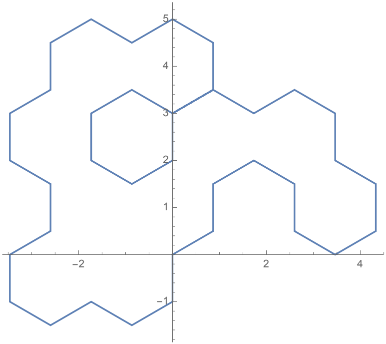
Curves studied in the JT literature are the reparametrization ansatz, closed Brownian paths[9,10] or self-avoiding loops[11], but not self-overlapping curves. **Can we generate them?**

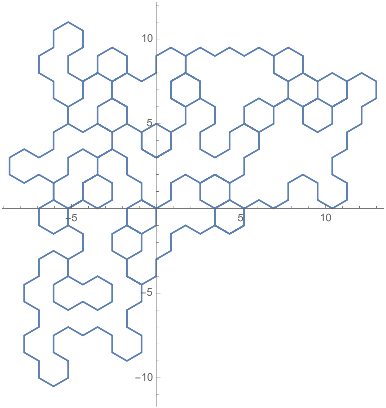
---

[7] Shor and Van Wyk, "Detecting and decomposing self-overlapping curves".
[8] Mukherjee, "Self-overlapping curves: Analysis and applications".
[9] Alexei Kitaev and S. Josephine Suh. "Statistical mechanics of a two-dimensional black hole". In: *JHEP* 05 (2019). arXiv: 1808.07032 [hep-th].
[10] Zhenbin Yang. "The Quantum Gravity Dynamics of Near Extremal Black Holes". In: *JHEP* 05 (2019). arXiv: 1809.08647 [hep-th].
[11] Stanford and Yang, "Finite-cutoff JT gravity and self-avoiding loops".

Discrete self-overlapping curves 1



(a) 10 hexagons, perimeter 38.



(b) 100 hexagons, perimeter 272.

Discrete self-overlapping curves 2

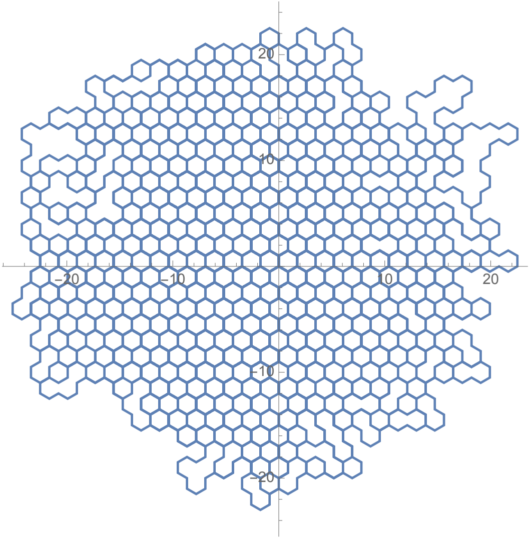


Figure: 3000 hexagons, perimeter 7472.

Outline

Conformal gauge and Liouville field[12]

Conformal gauge: $\mathrm{d}s^2 = e^{2\sigma}|\mathrm{d}z|^2$.

Constraint of constant curvature: $R = -2$.
$\rightarrow$ Liouville field solution of $\Delta\sigma = -2e^{2\sigma}$.

[12] Daniela Kraus and Oliver Roth. *Conformal Metrics*. 2008. arXiv: 0805.2235 [math.CV].

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Theorem 1: Let $\sigma_b : S^1 \rightarrow R$ be a continuous function defined on the boundary of the disk. Then there exists a unique solution σ of the Liouville equation such that $\sigma = \sigma_b$ on the boundary.

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Theorem 2: The most general solution to the Liouville equation is of the form $e^{\sigma} = \frac{2|F'(z)|}{1-|F(z)|^2}$, where $F : \mathcal{D}^0 \rightarrow \mathcal{D}^0$ is a locally univalent holomorphic function (unique up to $\mathrm{PSL}(2,\mathbb{R})$ disk automorphisms).

Then Liouville field at the boundary characterises the metric!

[12] Kraus and Roth, *Conformal Metrics*.

Outline

The action: ϵ-expansion beyond the Schwarzian limit

We can parameterise the boundary Liouville field as a diffeomorphism of the circle:

$$e^{\sigma_b} = \frac{2|F'|}{1-|F|^2} = \frac{\beta}{2\pi\epsilon}\frac{1}{f'}, \tag{7}$$

with $f \in \mathrm{Diff}(S^1)_+$, $F(0) = 0$ and $F'(0) > 0$ which fixes F uniquely.

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We can parameterise the boundary Liouville field as a diffeomorphism of the circle:

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In the Schwarzian limit $\epsilon \to 0$, writing an ϵ-expansion for F, we obtain the "reparametrization ansatz" g in terms on the diffeomorphism f

$$g = f + \sum_{n>0}\left(\frac{2\pi\epsilon}{\beta}\right)^n f_n. \tag{8}$$

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We then compute the ϵ-expansion of the extrinsic curvature to get

$$k = 1 + \left(\frac{2\pi\epsilon}{\beta}\right)^2 \mathrm{Sch}[\tan\frac{f}{2}] + \sum_{n\geq 3} \left(\frac{2\pi\epsilon}{\beta}\right)^n k_n. \tag{9}$$

Reparametrization ansatz in light-cone coordinates

Introducing the coordinates $z = -ix + t$, $\bar{z} = ix + t$, the extrinsic curvature writes[13]

$$k = \frac{2z'^2\bar{z}' + (\bar{z} - z)\bar{z}'z'' + z'(2\bar{z}'^2 + (z - \bar{z})\bar{z}'')}{4(z'\bar{z}')^{\frac{3}{2}}}, \tag{10}$$

$$= 1 + \left(\frac{2\pi\epsilon}{\beta}\right)^2 \mathrm{Sch}[z,u] - i\left(\frac{2\pi\epsilon}{\beta}\right)^3 \partial_u \mathrm{Sch}[z,u]$$

$$+ \left(\frac{2\pi\epsilon}{\beta}\right)^4 \left(-\frac{1}{2}\mathrm{Sch}[z,u]^2 + \partial_u^2 \mathrm{Sch}[z,u]\right) + O(\epsilon^5) \tag{11}$$

Expressed only in function of the Schwarzian and its derivatives.

$\rightarrow z(u)$ can be expressed in terms of f.

[13] Luca V. Iliesiu et al. "JT gravity at finite cutoff". In: *SciPost Phys.* 9 (2020). arXiv: 2004.07242 [hep-th].

Extrinsic curvature

The first correction k_3, is manifestly $\mathrm{PSL}(2,\mathbb{R})$ invariant

$$k_3[f](\vartheta) = \frac{6}{\pi} \lim_{\delta \to 0} \left(\left(\int_0^{\vartheta - \delta} + \int_{\vartheta+\delta}^{2\pi} \right) \mathcal{O}_2(\vartheta, \vartheta') - \frac{2}{3\delta^3} - \frac{2}{3\delta} \mathrm{Sch}[\tan \frac{f}{2}] \right), \tag{12}$$

with the bilocal $\mathrm{PSL}(2,\mathbb{R})$-invariant operator

$$\mathcal{O}_k(\vartheta, \vartheta') = \left(\frac{f'(\vartheta) f'(\vartheta')}{4 \sin^2 \left(\frac{f(\vartheta) - f(\vartheta')}{2} \right)} \right)^k . \tag{13}$$

Extrinsic curvature

The first correction k_3, is manifestly $\mathrm{PSL}(2,\mathbb{R})$ invariant

$$k_3[f](\vartheta) = \frac{6}{\pi}\lim_{\delta\to 0}\left(\left(\int_0^{\vartheta-\delta} + \int_{\vartheta+\delta}^{2\pi}\right)\mathcal{O}_2(\vartheta,\vartheta') - \frac{2}{3\delta^3} - \frac{2}{3\delta}\mathrm{Sch}[\tan\frac{f}{2}]\right), \tag{12}$$

with the bilocal $\mathrm{PSL}(2,\mathbb{R})$-invariant operator

$$\mathcal{O}_k(\vartheta,\vartheta') = \left(\frac{f'(\vartheta)f'(\vartheta')}{4\sin^2\left(\frac{f(\vartheta)-f(\vartheta')}{2}\right)}\right)^k. \tag{13}$$

→ k_4 is a work in progress.

We get a different ϵ-expansion compared to the expansion obtain from considering only embeddings[14].

[14] Iliesiu et al., "JT gravity at finite cutoff".

Outline

Closed manifold

In the spirit of D'Hoker and Phong[15], in conformal gauge, the JT path integral for a closed manifold[16] reduces to

$$Z_{JT} = \int_{moduli} \mathrm{d}(\text{Weil-Pet.}) \det(P_1^\dagger P_1) \int \mathcal{D}\sigma\mathcal{D}\phi \, e^{-26\mathcal{S}_L[\sigma]} e^{\frac{1}{16\pi G_N} \int \mathrm{d}x^2 \phi(R+2)}, \tag{14}$$

with $\mathcal{S}_L[\sigma]$ the Liouville action.
The dilaton fixes the Liouville field to be $\sigma = 0$.

Work in progress : the case of the disk.
The moduli space is trivial. However the Liouville field is not fixed but determined by the boundary Liouville field σ_b.

[15] Eric D'Hoker and D. H. Phong. "The geometry of string perturbation theory". In: *Rev. Mod. Phys.* 60 (4 1988).
[16] Phil Saad, Stephen H. Shenker, and Douglas Stanford. "JT gravity as a matrix integral". In: (Mar. 2019). arXiv: 1903.11115 [hep-th].

Conclusion

We propose a formulation of JT gravity at finite cutoff and aim to answer the following questions:

- Can we derive the path integral measure?
- Can we formulate perturbation expansion in powers of the cutoff (for the extrinsic curvature and the partition function)?
- Can we study the properties of self-overlapping curves?

Conclusion

We propose a formulation of JT gravity at finite cutoff and aim to answer the following questions:

- Can we derive the path integral measure?
- Can we formulate perturbation expansion in powers of the cutoff (for the extrinsic curvature and the partition function)?
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We are also interested in:

- Gauge theory formulation of JT.
- Flat JT gravity.